Week7
Standard Model-2

Class Schedule

Date	Topic	
Week 1 (9/22/18)	Introduction	YJ
Week 2 (9/29/18)	History of Particle Physics	YJ
Week 3 (10/6/18)	Special Relativity	Ed
Week 4 (10/13/18)	Quantum Mechanics	Ed
Week 5 (10/20/18)	Experimental Methods	Ed
Week 6 (10/27/18)	The Standard Model - Overview	YJ
Week 7 (11/3/18)	The Standard Model - Limitations	YJ
Week 8 (11/10/18)	Neutrino Theory	Ed
Week 9 (11/17/18)	Neutrino Experiment	Ed
Week 10 (12/1/18)	LHC and Experiments	YJ
Week 11 (12/8/18)	The Higgs Boson and Beyond	YJ
Week 12 (12/15/18)	Particle Cosmology	Ed

Class Policy

- Classes from 10:00 AM to 12:30 PM (10 min break at ~ 11:10 AM).
- Attendance record counts.
- Up to four absences
- Lateness or leaving early counts as half-absence.
- Send email notifications of all absences to shpattendance@columbia.edu.

Class Policy

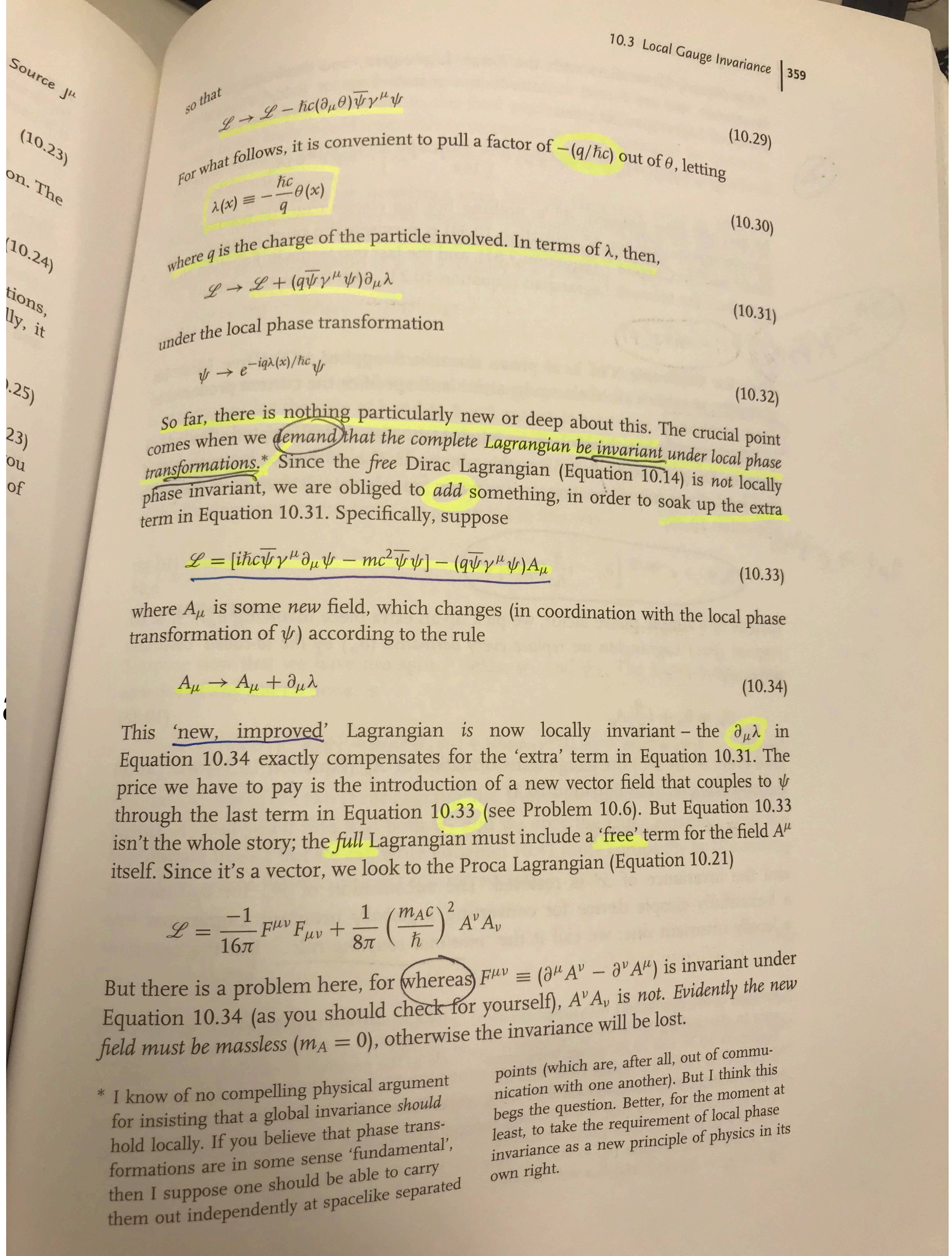
- No cell phone uses during the class.
- Feel free to step outside to the hall way in case of emergencies, bathrooms, starvations.
- Feel free to stop me and ask questions / ask for clarifications.
- Resources for class materials, Research Opportunities + Resources to become a particle physicist

https://twiki.nevis.columbia.edu/twiki/pub/Main/ScienceHonorsProgram

Why

- Do we care about **local** gauge symmetry?

Why


10.3 Local Gauge Invariance 359

so that

$$\mathscr{L} \rightarrow \mathscr{L} - \hbar c (\partial_\mu \theta) \bar{\psi} \gamma^\mu \psi \quad (10.29)$$

For what follows, it is convenient to pull a factor of $-(q/\hbar c)$ out of θ, letting

$$\lambda(x) \equiv -\frac{\hbar c}{q} \theta(x) \quad (10.30)$$

where q is the charge of the particle involved. In terms of λ, then,

$$\mathscr{L} \rightarrow \mathscr{L} + (q \bar{\psi} \gamma^\mu \psi) \partial_\mu \lambda \quad (10.31)$$

under the local phase transformation

$$\psi \rightarrow e^{-iq\lambda(x)/\hbar c} \psi \quad (10.32)$$

So far, there is nothing particularly new or deep about this. The crucial point comes when we *demand that the complete Lagrangian be invariant under local phase transformations*.* Since the *free* Dirac Lagrangian (Equation 10.14) is *not* locally phase invariant, we are obliged to *add* something, in order to soak up the extra term in Equation 10.31. Specifically, suppose

$$\mathscr{L} = [i\hbar c \bar{\psi} \gamma^\mu \partial_\mu \psi - mc^2 \bar{\psi} \psi] - (q \bar{\psi} \gamma^\mu \psi) A_\mu \quad (10.33)$$

where A_μ is some *new* field, which changes (in coordination with the local phase transformation of ψ) according to the rule

$$A_\mu \rightarrow A_\mu + \partial_\mu \lambda \quad (10.34)$$

This 'new, improved' Lagrangian *is* now locally invariant – the $\partial_\mu \lambda$ in Equation 10.34 exactly compensates for the 'extra' term in Equation 10.31. The price we have to pay is the introduction of a new vector field that couples to ψ through the last term in Equation 10.33 (see Problem 10.6). But Equation 10.33 isn't the whole story; the *full* Lagrangian must include a 'free' term for the field A^μ itself. Since it's a vector, we look to the Proca Lagrangian (Equation 10.21)

$$\mathscr{L} = \frac{-1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi} \left(\frac{m_A c}{\hbar} \right)^2 A^\nu A_\nu$$

But there is a problem here, for whereas $F^{\mu\nu} \equiv (\partial^\mu A^\nu - \partial^\nu A^\mu)$ is invariant under Equation 10.34 (as you should check for yourself), $A^\nu A_\nu$ is *not*. *Evidently the new field must be massless* ($m_A = 0$), otherwise the invariance will be lost.

* I know of no compelling physical argument for insisting that a global invariance *should* hold locally. If you believe that phase transformations are in some sense 'fundamental', then I suppose one should be able to carry them out independently at spacelike separated points (which are, after all, out of communication with one another). But I think this begs the question. Better, for the moment at least, to take the requirement of local phase invariance as a new principle of physics in its own right.

- Do we care a

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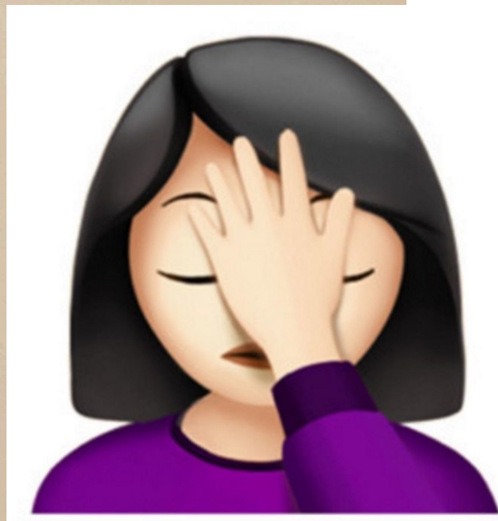

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Why

- Are there 8 gluons? Not 6 or 9?

Why

2.15 Color Factors

We have already mentioned some of the evidence for color. We saw that there are good reasons to believe that each of the N flavors (u, d, ...) of quark comes in three colors which we called R, G, and B. To be precise, the quarks are assigned to a triplet of an $SU(3)$ color group (see Fig. 2.3). Unlike $SU(N)$ flavor symmetry, $SU(3)$ color symmetry is expected to be exactly conserved. A glance back at Fig. 1.4 reminds us that the gluons, which mediate the QCD force between color charges, come in eight different color combinations:

$$R\bar{G}, R\bar{B}, G\bar{R}, G\bar{B}, B\bar{R}, B\bar{G}, \sqrt{\tfrac{1}{2}}\,(R\bar{R} - G\bar{G}), \sqrt{\tfrac{1}{6}}\,(R\bar{R} + G\bar{G} - 2B\bar{B}). \tag{2.93}$$

In other words, the gluons belong to an $SU(3)$ color octet [recall the $SU(3)$ flavor analogy of (2.49), (2.50) and Fig. 2.5]. The remaining combination, the $SU(3)$ color singlet,

$$\sqrt{\tfrac{1}{3}}\,(R\bar{R} + G\bar{G} + B\bar{B}), \tag{2.94}$$

does not carry color and cannot mediate between color charges.

In QED, the strength of the electromagnetic coupling between two quarks is given by $e_1 e_2 \alpha$, where e_i is the electric charge in units of e (that is, $e_i = +\frac{2}{3}$ or $-\frac{1}{3}$) and α is the fine structure constant. Similarly, in QCD, the strength of the (strong) coupling for single-gluon exchange between two color charges is $\frac{1}{2} c_1 c_2 \alpha_s$, ... fficients associated with the verti...

Why

$$R\overline{G}, R\overline{B}, G\overline{R}, G\overline{B}, B\overline{R}, B\overline{G}, \sqrt{\tfrac{1}{2}}\,(R\overline{R} - G\overline{G}), \sqrt{\tfrac{1}{6}}\,(R\overline{R} + G\overline{G} - 2B\overline{B}).$$

$$\sqrt{\tfrac{1}{3}}\,(R\overline{R} + G\overline{G} + B\overline{B}),$$

Accidental symmetry

- A rule of thumb (R. Feynman) : a reaction will be observed unless it is expressly forbidden by a conservation law.

- Baryon number B, Lepton number L

About Last week ...
Particle/Field formulation

- How do we describe interactions and fields mathematically?
- Classically,

Lagrangian L = kinetic energy - potential energy

L = T - V

- Particle physics:
  - Same concept, using Dirac equation to describe free spin-1/2 particles:

$$L = \bar{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi$$

↑ field

Ψ = wavefunction

m = mass

γ^{μ} = μ^{th} gamma matrix

∂_{μ} = partial derivative

Particle/Field formulation

- In particle physics, we define fields like $\phi(x,t)$ at every point in spacetime.
- These fields don't just sit there; they fluctuate harmonically about some minimum energy state.
- The oscillations combine to form wave packets.
- The wave packets move around in the field and interact with each other. We interpret them as elementary particles.
- Terminology: the wave packets are called the quanta of the field $\phi(x,t)$.

Particle/Field formulation

- The Higgs mechanism is described in terms of the Lagrangian of the Standard Model. In quantum mechanics, single particles are described by wavefunctions that satisfy the appropriate wave equation.
- In Quantum Field Theory (QFT), *particles* are described by excitations of a quantum field that satisfies the appropriate quantum mechanical field equations.
- The dynamics of a quantum field theory can be expressed in terms of the Lagrangian density. Lagrangian formalism is necessary for the discussion of the Higgs mechanism.

About Last week …

- We got QED Lagrangian from 1 fermion Lagrangian (Dirac lagrangian) by accommodating local gauge invariance, thus new gauge field A; which is photon field.

- QED : Theory of charged (EM) fermion and photon.

Final lagrangian (for QED!):

$$L = -1/4 F_{\mu\nu} F^{\mu\nu} + \bar{\Psi}(i\gamma^{\mu} D_{\mu} - m)\Psi$$

About Last week …

$$\mathcal{L}_{\text{QED}} \rightarrow \overline{\psi}(i\gamma^\mu\partial_\mu - m_\text{e})\psi + e\overline{\psi}\gamma^\mu A_\mu\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_\gamma^2 A_\mu A^\mu.$$

For the U(1) local gauge transformation of (17.11), the photon field transforms as

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu\chi,$$

and the new mass term becomes

$$\tfrac{1}{2}m_\gamma^2 A_\mu A^\mu \rightarrow \tfrac{1}{2}m_\gamma^2 \left(A_\mu - \partial_\mu\chi\right)(A^\mu - \partial^\mu\chi) \neq \tfrac{1}{2}m_\gamma^2 A_\mu A^\mu,$$

Mass function is not gauge invariant, same for weak interaction and QCD.

About Last week ...

$$\mathcal{L}_{\mathrm{QED}} \to \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m_{\mathrm{e}})\psi + e\overline{\psi}\gamma^{\mu}A_{\mu}\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_{\gamma}^{2}A_{\mu}A^{\mu}.$$

For the U(1) local gauge transformation of (17.11), the photon field transforms as

$$A_{\mu} \to A'_{\mu} = A_{\mu} - \partial_{\mu}\chi,$$

and the new mass term becomes

$$\tfrac{1}{2} \ldots ,$$

Mass fu… for weak

What about massive gauge bosons?

Brian Greene
https://youtu.be/Ni-Lf1y51Dc

Introducing a Scalar Field (Simplified; REAL)

- Consider a scalar field ϕ with the potential

$$V(\phi) = \tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda\phi^4.$$

- The corresponding Lagrangian is given by

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi)\\ &= \tfrac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda\phi^4.\end{aligned}$$

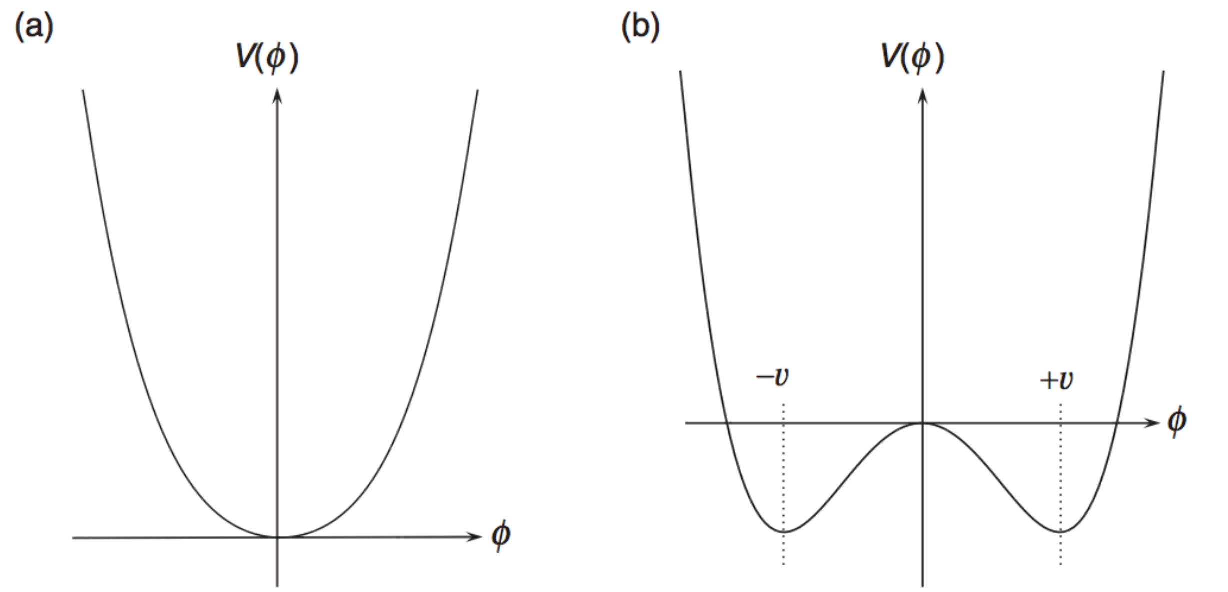
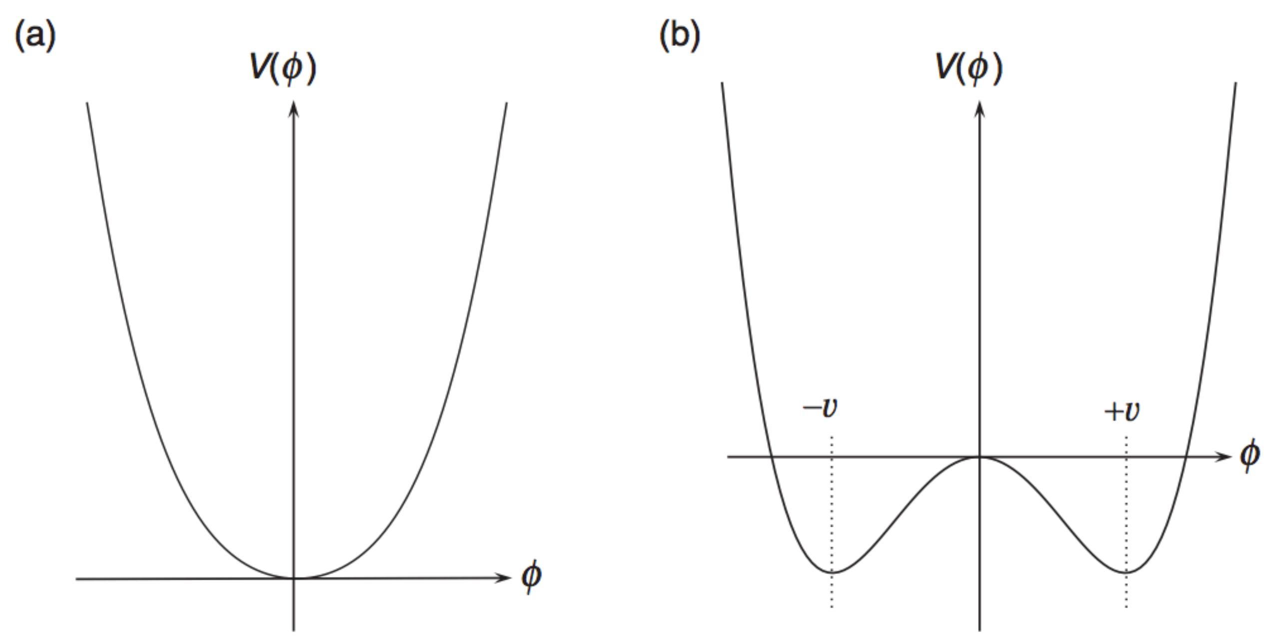


Fig. 17.5 The one-dimensional potential $V(\phi) = \mu^2\phi^2/2 + \lambda\phi^4/4$ for $\lambda > 0$ and the cases where (a) $\mu^2 > 0$ and (b) $\mu^2 < 0$.

Introducing a Scalar Field (Simplified; REAL)

- The vacuum state is the lowest energy state of the field ϕ;

  same to the minimum of the potential.
- Two choices of minimum potential, where it is gonna be?
- **-v or +v?** - > Spontaneous symmetry breaking;
- There was equal chance that ϕ "picks a ground state"
- (i.e. when the gauge symmetry of a Lagrangian is spontaneously broken)



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Introducing a Scalar Field (Simplified; REAL)

- The vacuum state is the lowest energy state of the field ϕ; same to the minimum of the potential
- Two choices of minimum potential, where it is gonna be?
- - > Spontaneous symmetry breaking
- Without losing any generality we can choose positive solution

$$\phi(x) = v + \eta(x).$$

Introducing a Scalar Field (Simplified; REAL)

- Expanding the lagrangian density around the minimum,
- $$\phi(x) = v + \eta(x).$$

$$\begin{aligned}\mathcal{L}(\eta) &= \tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - V(\eta) \\ &= \tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \tfrac{1}{2}\mu^2(v+\eta)^2 - \tfrac{1}{4}\lambda(v+\eta)^4.\end{aligned}$$

Introducing a Scalar Field (Simplified; REAL)

- Expanding the lagrangian density around the minimum,
- $$\phi(x) = v + \eta(x).$$

$$\mathcal{L}(\eta) = \tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - V(\eta)$$
$$= \tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \tfrac{1}{2}\mu^2(v+\eta)^2 - \tfrac{1}{4}\lambda(v+\eta)^4.$$

$$\mathcal{L}(\eta) = \tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) \boxed{- \lambda v^2 \eta^2} - \lambda v \eta^3 - \tfrac{1}{4}\lambda \eta^4 + \tfrac{1}{4}\lambda v^4.$$

$$m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2},$$

V(ϕ) and spontaneous symmetry breaking

- The phenomenon we have just considered is called *spontaneous symmetry breaking*.
- Why symmetry breaking? Our choice of a ground state “breaks" the obvious reflection symmetry of the original Lagrangian; The two minimum solutions could equally be chosen (the symmetry), one is fated to be chosen (broken symmetry)

- What about the spontaneous part?
  - The choice of a ground state is arbitrary in this system. There is no external agency that favors one over the other, or even forces the choice to begin with.

Introducing a complex scalar field

- Consider a *complex* scalar field ϕ with the potential

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2),$$

- The corresponding Lagrangian is given by

$$\boxed{\mathcal{L} = (\partial_\mu \phi)^*(\partial^\mu \phi) - V(\phi)} \qquad \boxed{V(\phi) = \mu^2(\phi^*\phi) + \lambda(\phi^*\phi)^2.}$$

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi_1)(\partial^\mu \phi_1) + \tfrac{1}{2}(\partial_\mu \phi_2)(\partial^\mu \phi_2) - \tfrac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \tfrac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2.$$

- The minumum happens at

$$\phi_1^2 + \phi_2^2 = \frac{-\mu^2}{\lambda} = v^2,$$

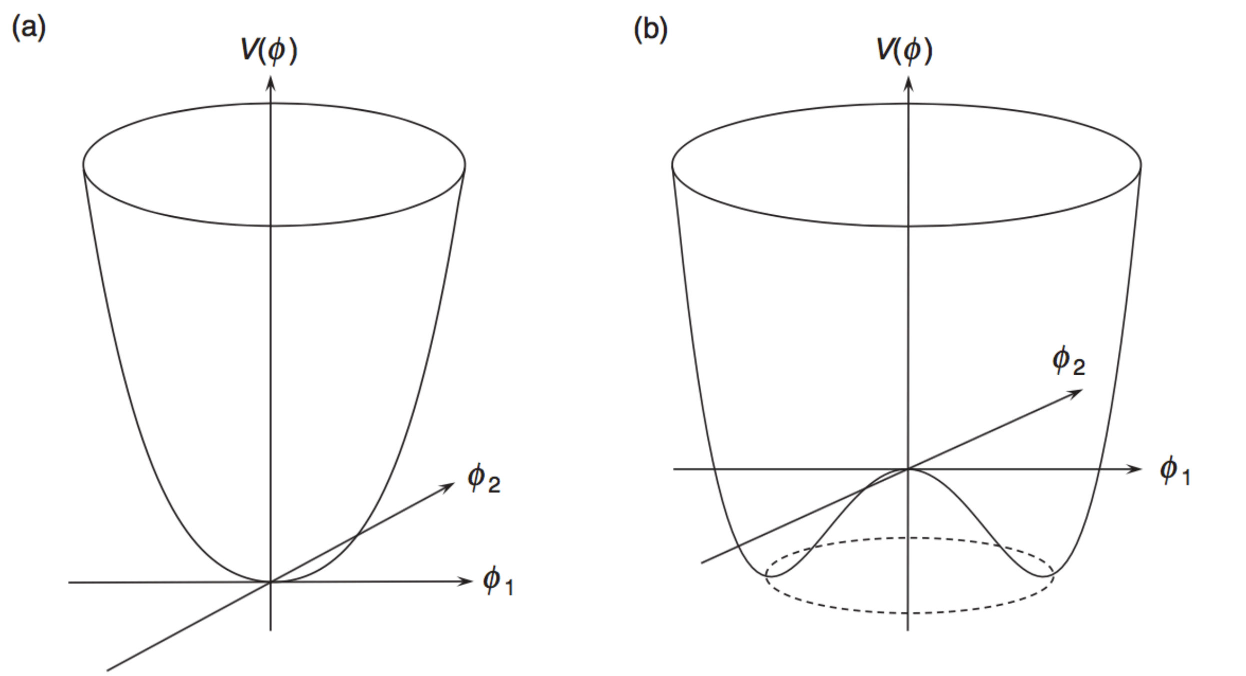


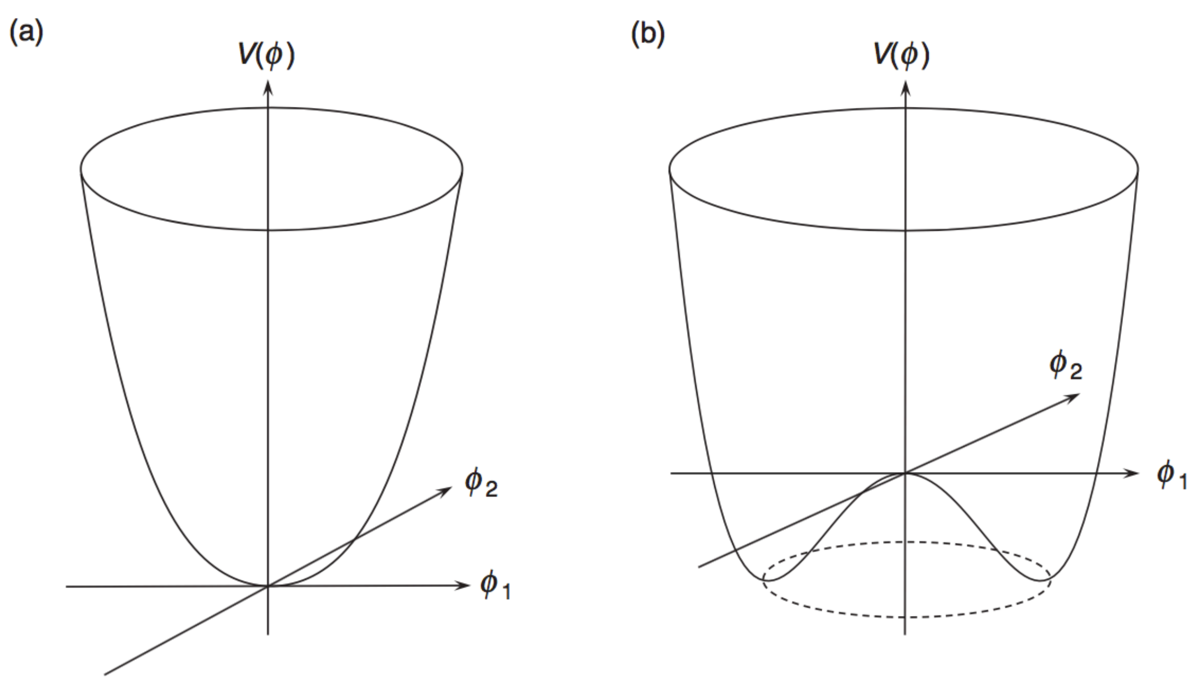
Fig. 17.7 The $V(\phi) = \mu^2(\phi^*\phi) + \lambda(\phi^*\phi)^2$ potential for a complex scalar field for (a) $\mu^2 > 0$ and (b) $\mu^2 < 0$.

Introducing a complex scalar field

- Any point on the bottom circle can be chosen to be the vacuum state. $(\phi_1, \phi_2) = (v, 0)$,
- By writing $\phi_1(x) = \eta(x) + v$ and $\phi_2(x) = \xi(x)$,

$$\phi = \frac{1}{\sqrt{2}}(\eta + v + i\xi).$$

- Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) - V(\eta, \xi),$



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Introducing a complex scalar field

- Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) - V(\eta, \xi),$
- Again, expanding around the vacuum state $\phi = \frac{1}{\sqrt{2}}(\eta + v + i\xi).$

- By writing the potential and lagrangian in terms of η and ξ, using

$$\mu^2 = -\lambda v^2, \quad V(\eta, \xi) = \mu^2\phi^2 + \lambda\phi^4 \quad \text{with} \quad \phi^2 = \phi\phi^* = \frac{1}{2}\left[(v+\eta)^2 + \xi^2\right].$$

- Lagrangian becomes,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) \boxed{- \frac{1}{2}m_\eta^2\eta^2} + \frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) - V_{int}(\eta, \xi), \quad \text{with } m_\eta = \sqrt{2\lambda v^2}$$

- With interaction term $V_{int}(\eta, \xi) = \lambda v\eta^3 + \frac{1}{4}\lambda\eta^4 + \frac{1}{4}\lambda\xi^4 + \lambda v\eta\xi^2 + \frac{1}{2}\lambda\eta^2\xi^2.$

Local gauge invariance again: Toy model Higgs mechanism

- Complex scalar with potential $V(\phi) = \mu^2\phi^2 + \lambda\phi^4,$

- Local U(1) gauge transformation $\phi(x) \to \phi'(x) = e^{ig\chi(x)}\phi(x).$

- New Gauge field B $\partial_\mu \to D_\mu = \partial_\mu + igB_\mu.$

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - \mu^2\phi^2 - \lambda\phi^4,$$

- Another way of writing Lagrangian

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + (\partial_\mu\phi)^*(\partial^\mu\phi) - \mu^2\phi^2 - \lambda\phi^4$$
$$- igB_\mu\phi^*(\partial^\mu\phi) + ig(\partial_\mu\phi^*)B^\mu\phi + g^2B_\mu B^\mu\phi^*\phi.$$

Local gauge invariance again: Toy model Higgs mechanism

- Lagrangian

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + (\partial_\mu\phi)^*(\partial^\mu\phi) - \mu^2\phi^2 - \lambda\phi^4$$
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- Expand around vacuum state

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x)).$$

- Lagrangian becomes

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \lambda v^2\eta^2}_{\text{massive }\eta} + \underbrace{\tfrac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi)}_{\text{massless }\xi} - \underbrace{\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}g^2v^2B_\mu B^\mu}_{\text{massive gauge field}} - V_{int} + gvB_\mu(\partial^\mu\xi),$$

Local gauge invariance again: Toy model Higgs mechanism

- Lagrangian

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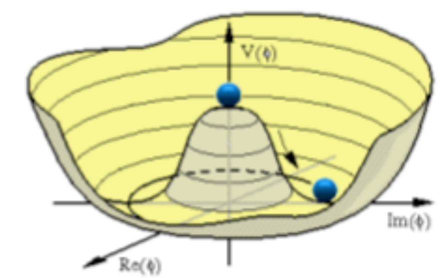
Standard model gauge bosons, fermions get mass like this too.

Higgs Mechanism

Yukawa Couplings

Higgs Field

$$\boxed{\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\phi'}$$



$$\mathcal{L}_\phi = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - V(\phi)$$

Higgs Potential

$$\mathcal{L}_{\text{Yuk}} = c_f(\bar{\psi}_L \psi_R \phi + \bar{\psi}_R \psi_L \phi)$$

Higgs Fermion Interaction

H.-C. Schultz-Coulon

When ϕ “picks a ground state” (i.e. when the gauge symmetry of a Lagrangian is spontaneously broken), all fermion fields and weak bosons become massive!

Physically, the expected massless bosons acquire mass by interacting with a newly apparent massive scalar field called the Higgs field. Hence, this process is called the Higgs mechanism.

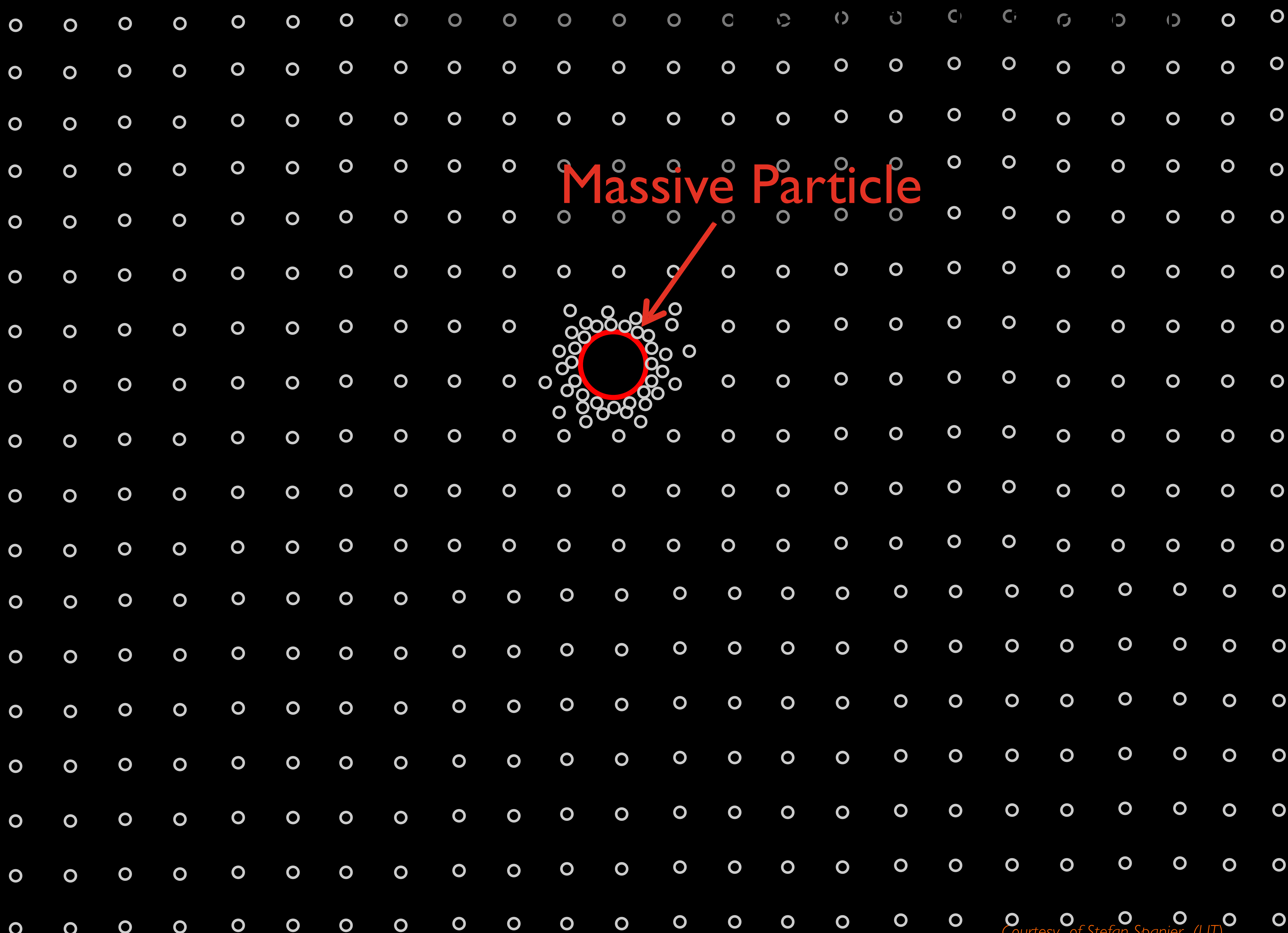
Electroweak unification

- In the end, the SU(2)xU(1) part of the Standard Model is called the *electroweak theory*, because electromagnetism and the weak force start out mixed together in this overall gauge symmetry.
- SU(2)xU(1) predicts four massless bosons, which are not apparent at everyday energies.
- Analogous to our simple example, the ground state of the SU(2)xU(1) theory (where we live) is one in which this gauge symmetry is hidden.
- Result: the four massless gauge bosons appear to us as the massive W^+, W^-, Z and the massless photon. The explicit U(1) symmetry of QED is preserved.

Higgs Mechanism

The idea:

A force field that permeates the Universe and slows particles down to below the speed of light. This is the equivalent to having mass.



Courtesy of Stefan Spanier (UT)

The Higgs particle

- Spin-0 particle which gives other particles and massive force mediators their mass.

The Higgs Mechanism. Source: CERN

To understand the Higgs mechanism, imagine that a room full of physicists chattering quietly is like space filled with the Higgs field ...

The Higgs particle

- Spin-0 particle which gives other particles and massive force mediators their mass.

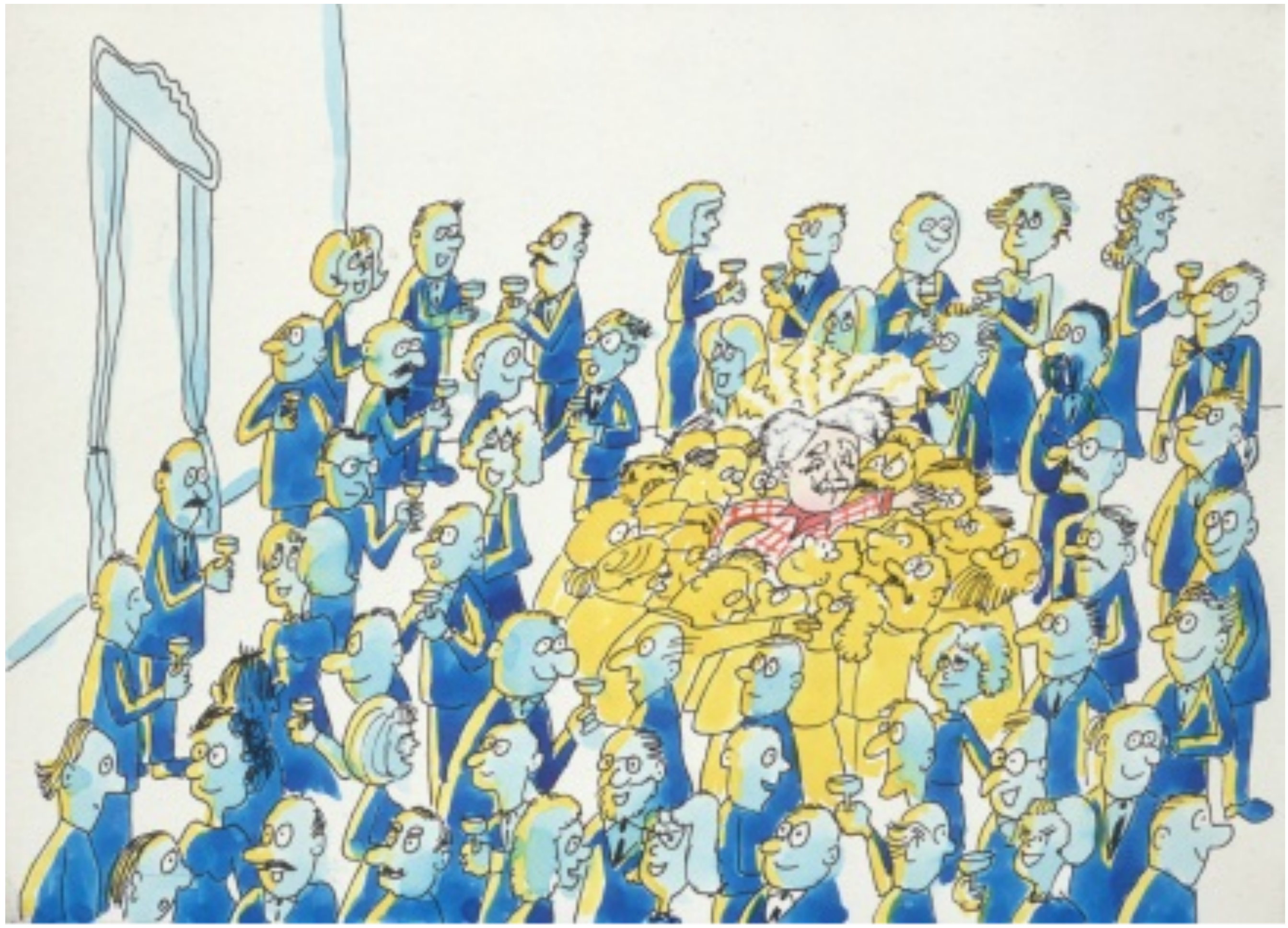
The Higgs Mechanism. Source: CERN

... a well-known scientist walks in, creating a disturbance as he moves across the room and attracting a cluster of admirers with each step ...

The Higgs particle

- Spin-0 particle which gives other particles and massive force mediators their mass.

The Higgs Mechanism. Source: CERN



... this increases his resistance to movement, in other words, he acquires mass, just like a particle moving through the Higgs field...

The Higgs boson discovery

Higgs Boson
Predicted in 1964 and discovered at the LHC in 2012!

VOLUME 13, NUMBER 9 PHYSICAL REVIEW LETTERS 31 AUGUST 1964

BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

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(Received 26 June 1964)

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BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

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(Received 31 August 1964)

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GLOBAL CONSERVATION LAWS AND MASSLESS PARTICLES*

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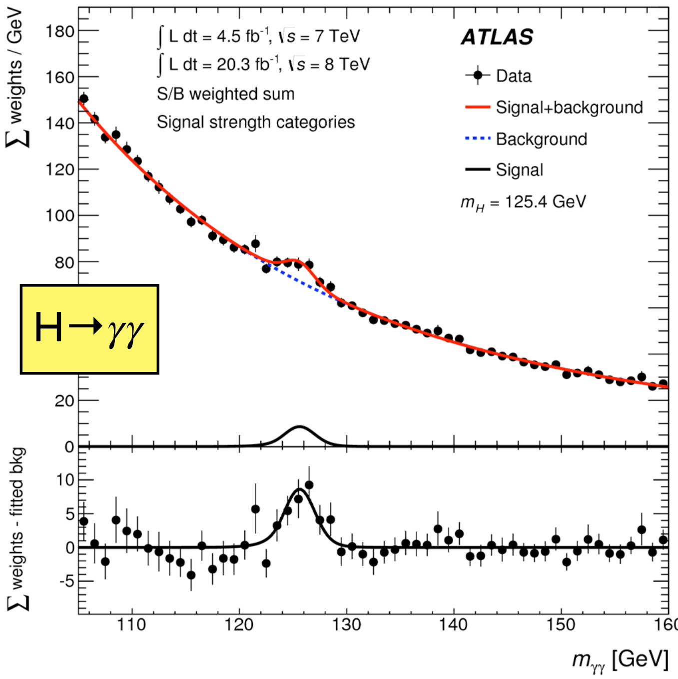
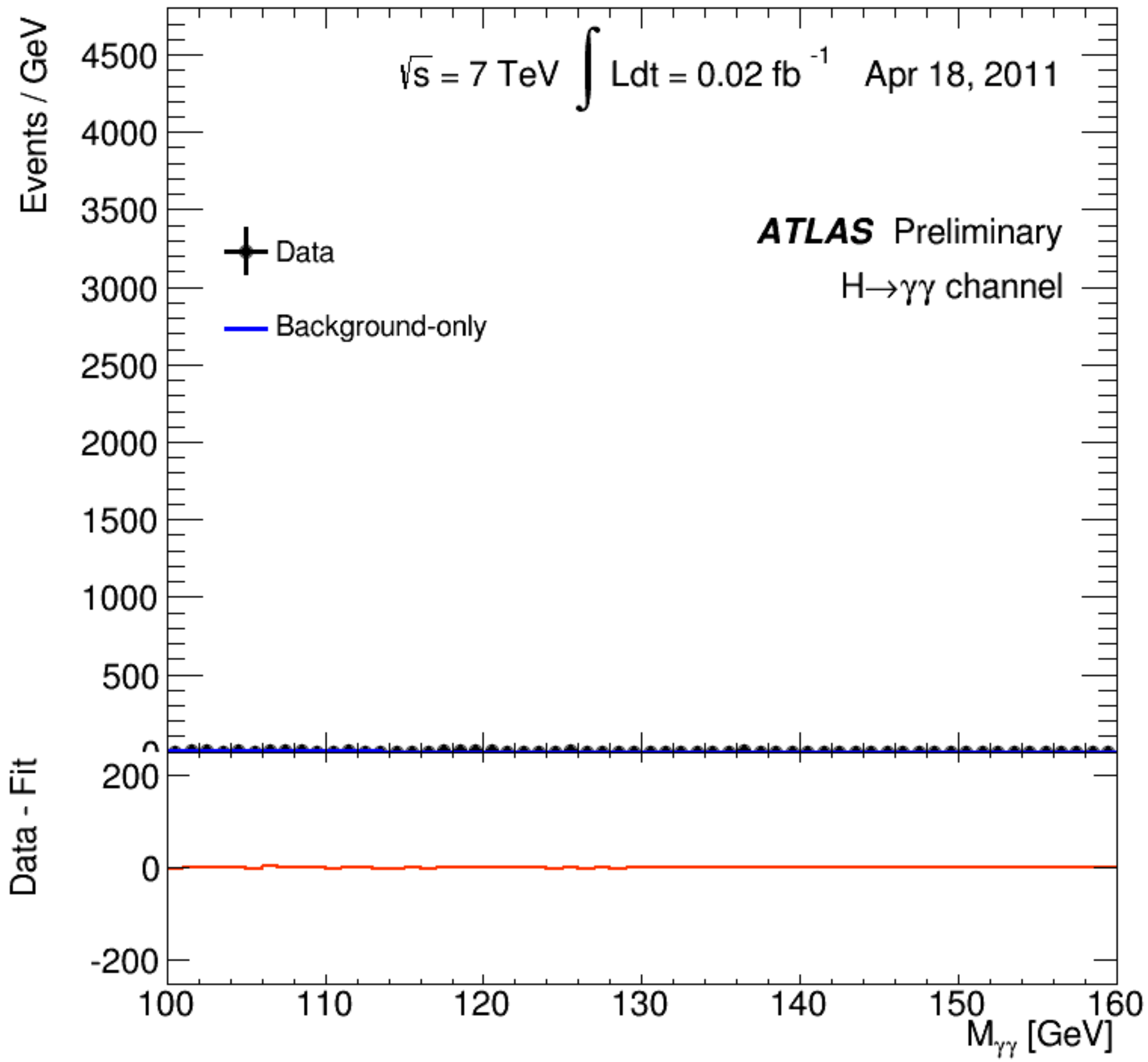
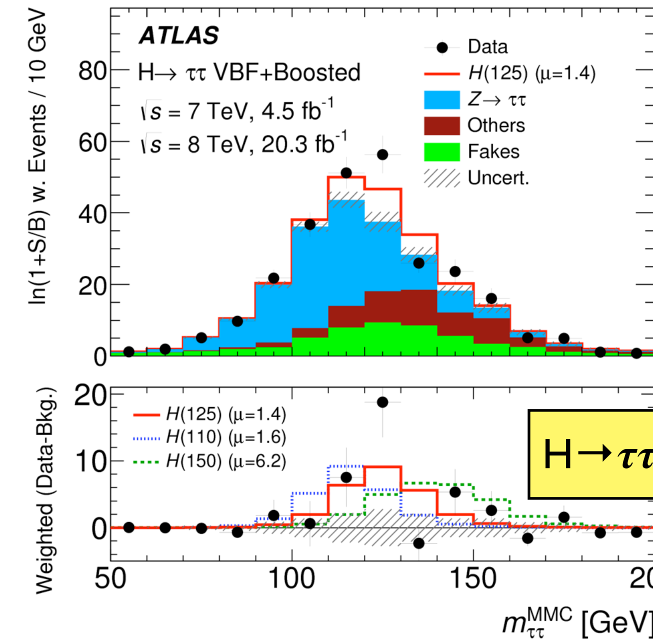
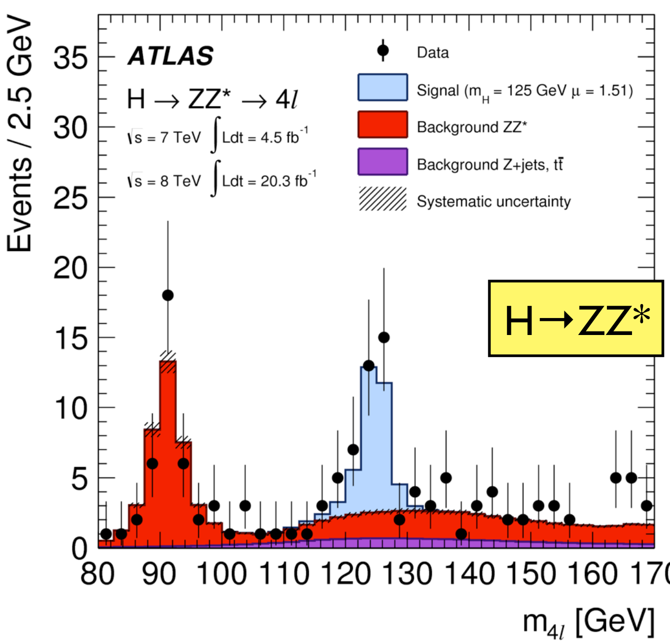
(Received 12 October 1964)

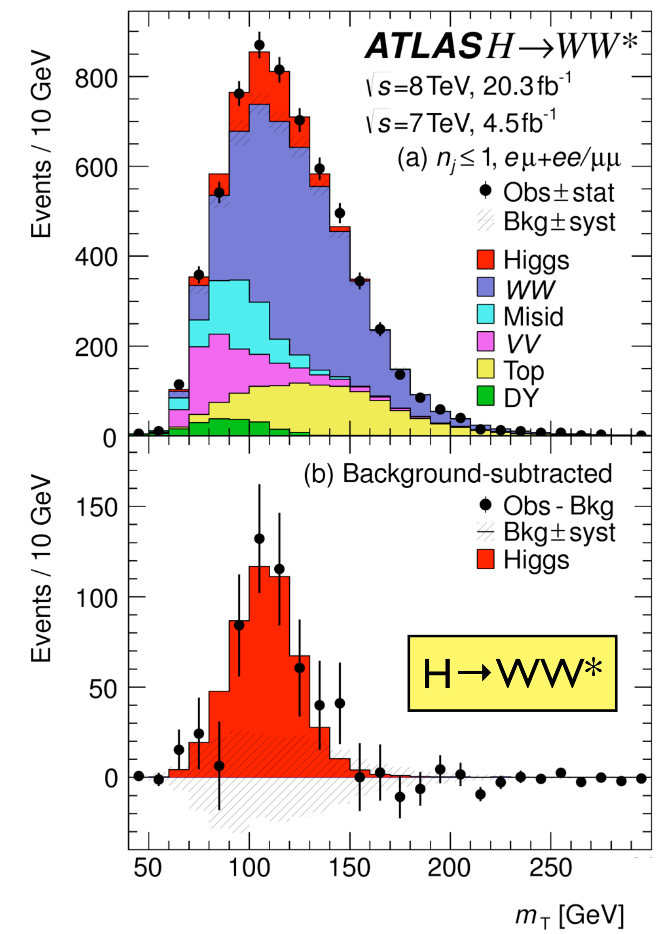
The Higgs Discovery (by ATLAS+CMS)











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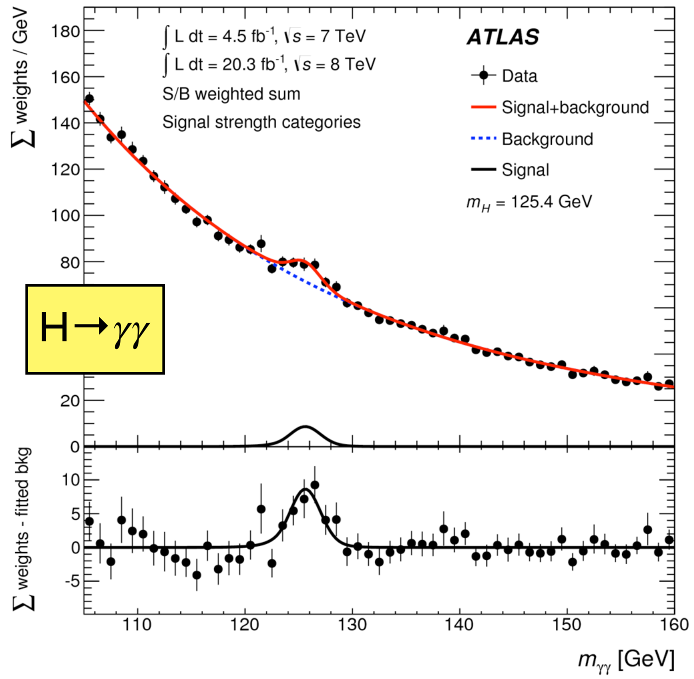

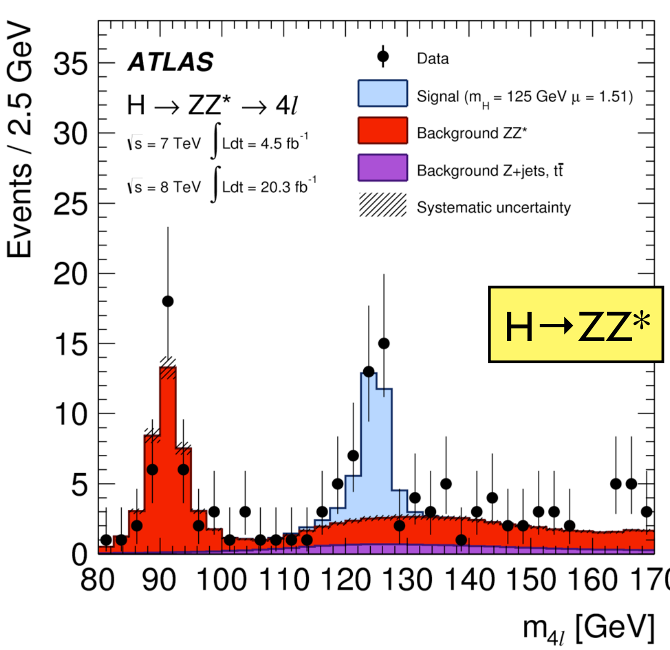

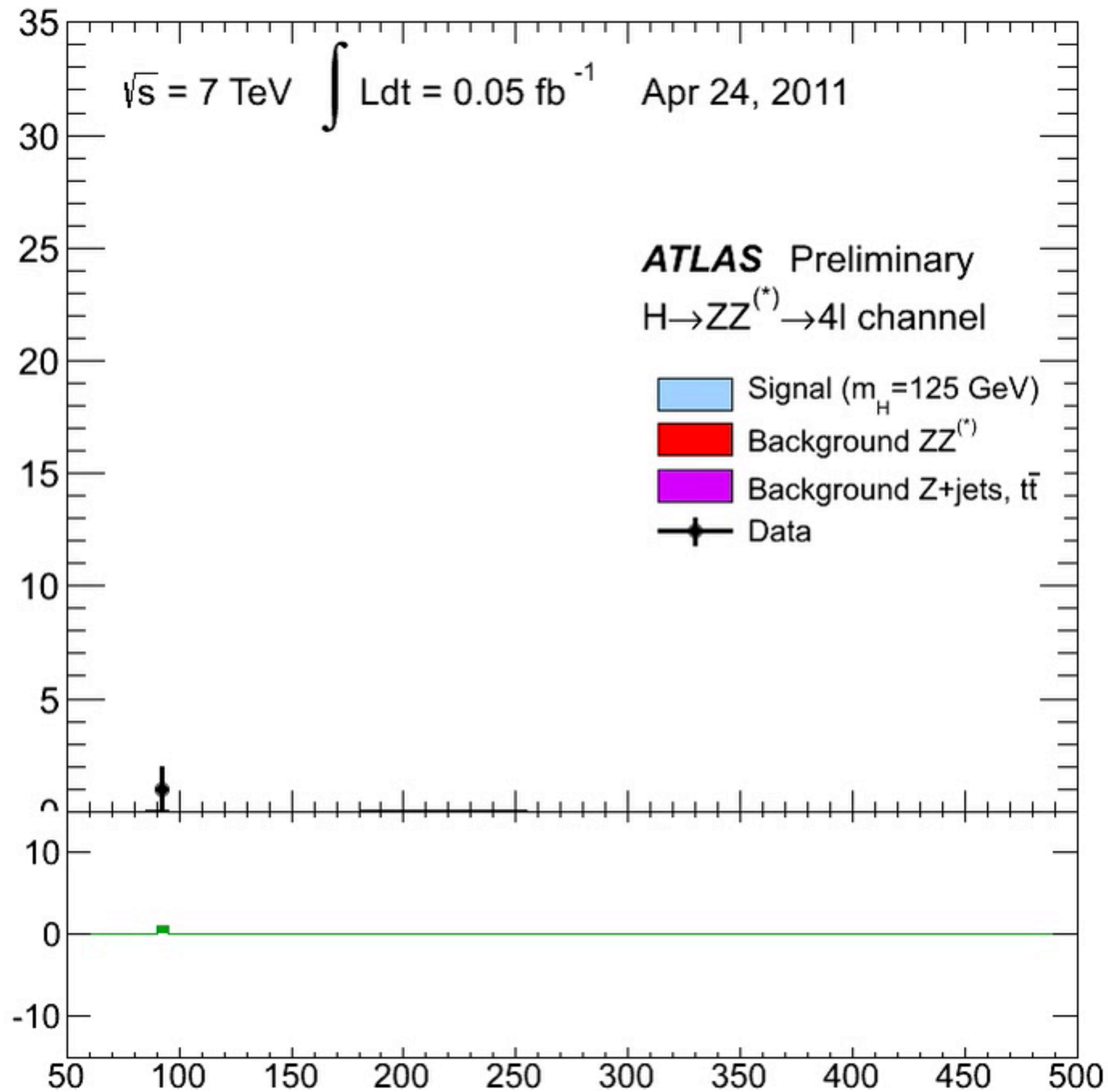

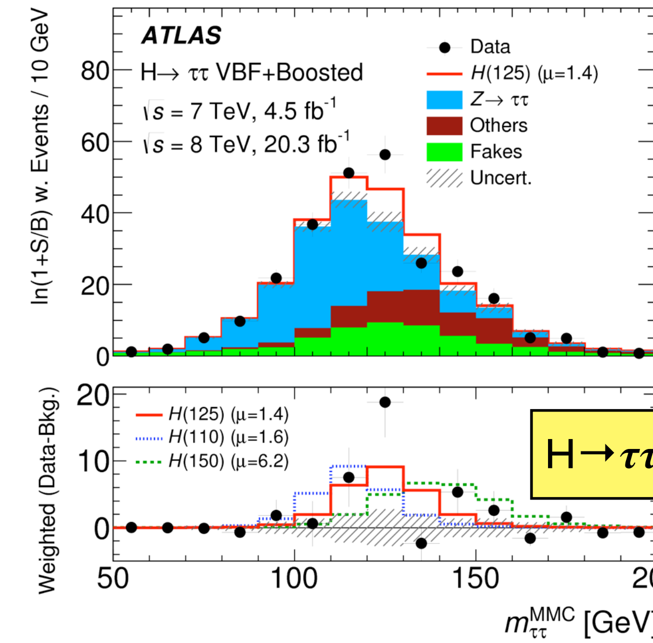

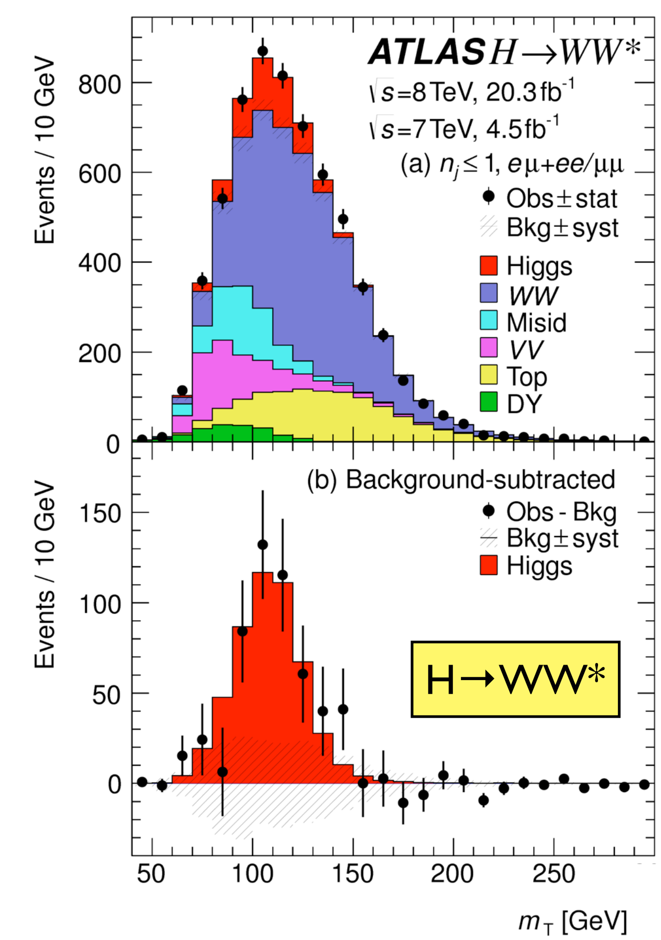

Where can we test the SM?

- Particle physics experiment designed to test specific aspects of SM.
- Major historical experiments:
  - @LEP (ALEPH, DELPHI, L3, OPAL): electroweak, QCD
  - @Tevatron (CDF, D0): electroweak, QCD, quark mixing
- Major running experiments:
  - ATLAS, CMS @LHC

THE STANDARD MODEL
HOW SIMPLE, REALLY?

- The Standard Model does not predict:

Determine experimentally

3 Couplings	g_s, e, $\sin\theta_W$
4 CKM parameters	ϑ_1, ϑ_2, ϑ_3, δ
2 Boson masses	m_Z, m_H
3 Lepton masses	m_e, m_μ, m_τ
6 Quark masses	m_u, m_d, m_s, m_c, m_t, m_b .
1 QCD vacuum angle θ	

19 free SM parameters

Plus 7 from neutrino mass

$$m_W^2 = \tfrac{1}{2}\, g^2\, \rho_0^2$$
$$m_Z^2 = \tfrac{1}{2}\,(g^2 + g'^2)\,\rho_0^2$$
$$m_H^2 = 4\,\lambda\rho_0^2$$

$$g = e/\sin\theta_W$$
$$g' = e/\cos\theta_W$$

$$m_f = c_f\rho_0$$

Summary

- SM unites electromagnetic, weak, strong forces.
- SM predicts cross-sections, couplings.
- Latest success: the discovery of the Higgs boson!

- However, SM is not completely satisfactory…:
  - 26 free parameters
  - Relations between some free parameters are predicted, which allow for experimental tests.

Standard model limitation

THE STANDARD MODEL
PREDICTS RELATIONSHIPS

- All observables can be predicted in terms of 26 free parameters (including neutrino masses, mixing parameters).
- If we have > 26 measurements of those observables, we overconstrain the SM.
  - Overconstrain: we don't have any more ad hoc inputs AND we can test the consistency of the model.
- In practice:
  - Pick well measured set of observables.
  - Calculate other observables in terms of these well known quantities.
  - Test predictions, measure observables, and compare to theory.

STANDARD MODEL LIMITATIONS

PHILOSOPHICAL NUISANCES ("WHY" RATHER THAN "WHAT/HOW")

- Why are there three generations of matter?
  - "Because" this is required for CP violation? And CP violation is required for a matter-dominated universe: us?
    - Anthropic principle – perhaps not great Science…
- Why are some of the Standard Model parameters "unnaturally" large / small?
  - Parameters that look like "1" give theorists a warm and fuzzy feeling.
- Why are the masses what they are?
  - And why are they so different?! $m_t/m_\nu \sim 10^{14}$!

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FACING THE STANDARD MODEL LIMITATIONS

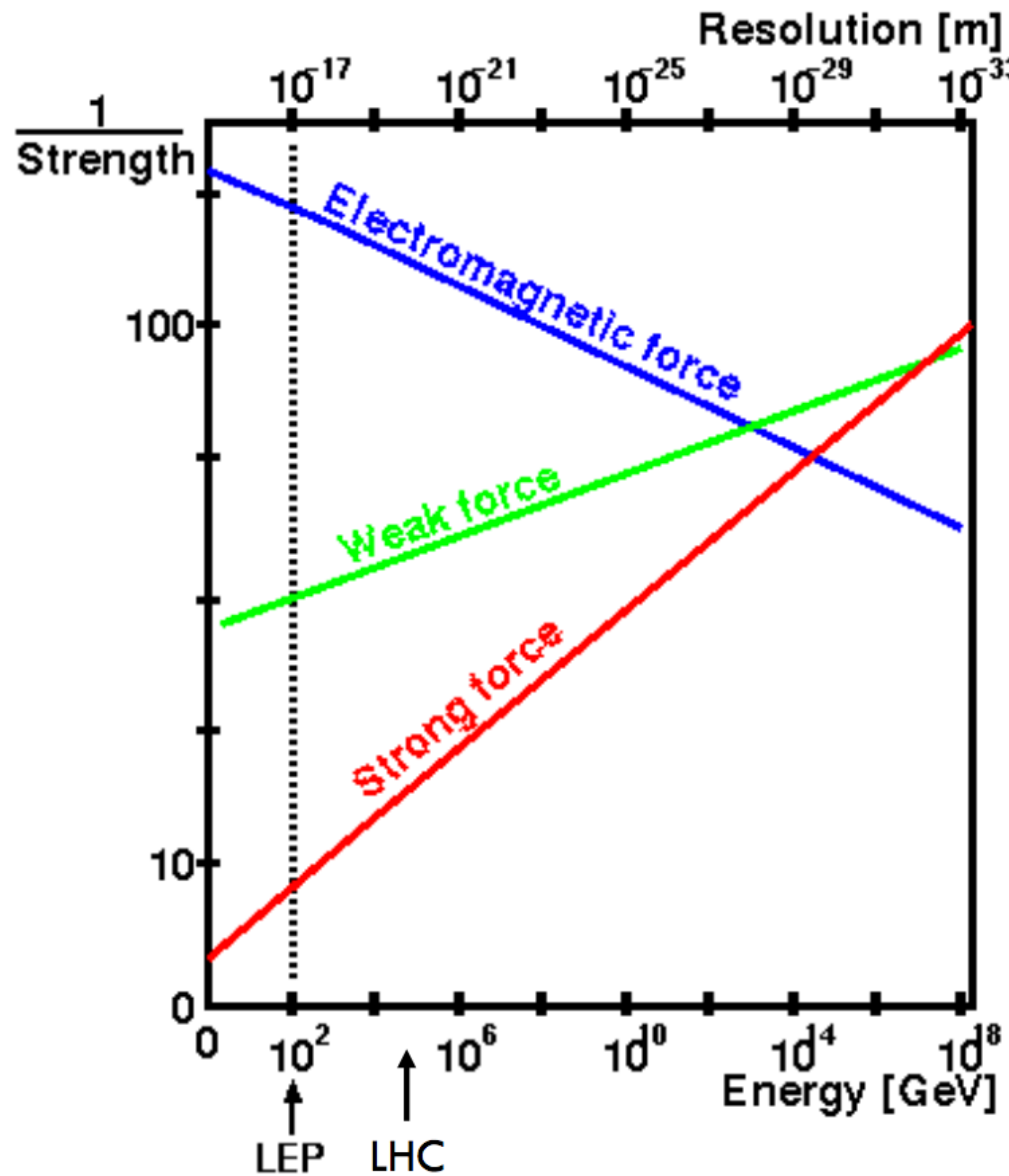
- Over the history of particle physics, a great deal of time has been spent by both theorists and experimentalists to try to resolve the limitations of the Standard Model.
- Generally, the strategy has been to extend the Standard Model in a way or another:
  - Grand Unified Theories.
    - Predicts proton decay.
  - Supersymmetry.
    - Might explain dark matter.
- A longer term, and more ambitious project has been to formulate a Theory of Everything, that includes gravity.
  - So far these theories have lacked predictive power…

GUT SCALE

- Grand unification scale:

  10^{16} GeV

- The Standard Model provides no explanation for what may happen beyond this unification scale, nor why the forces have such different strengths at low energies.



GRAND UNIFIED THEORIES

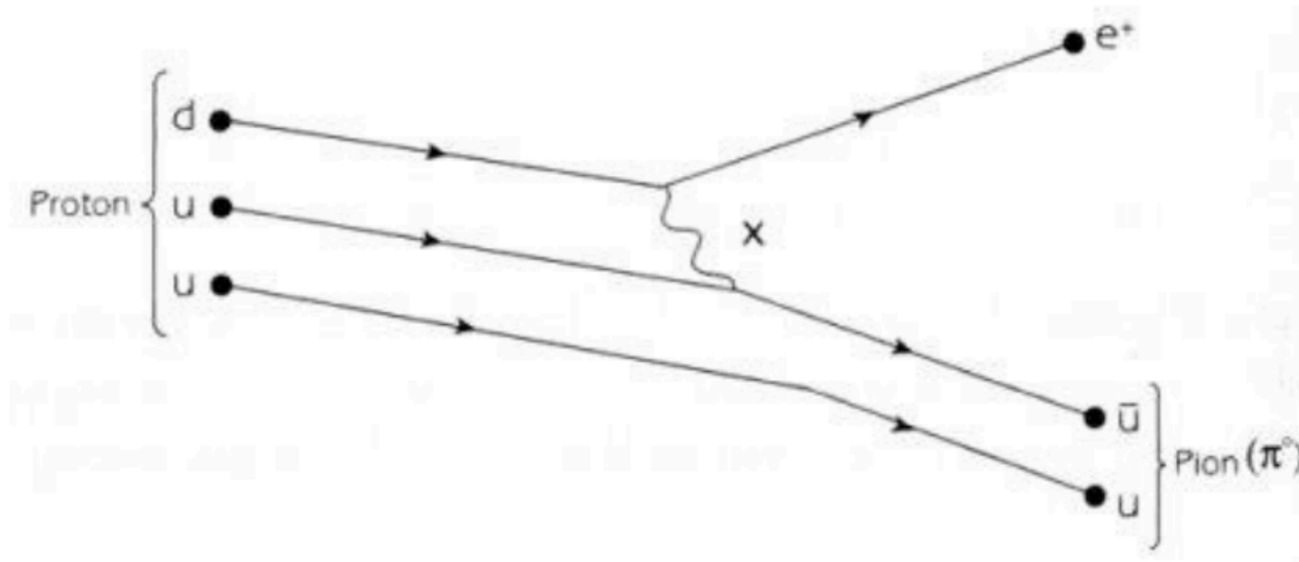
- In the 1970's, people started to think a lot about how to combine the SU(3), SU(2), and U(1) gauge symmetries of the Standard Model into a more fundamental, global symmetry.
- The first such Grand Unified Theory is a 1974 model based on SU(5) symmetry.
- This model groups all of the known fermions – i.e., the leptons and quarks – into multiplets.
- Inside the multiplets, quarks and leptons can couple to each other and transform into one another.
- In essence, this theory imposes a grand symmetry: all fermions, whether quarks or leptons, are fundamentally the same.

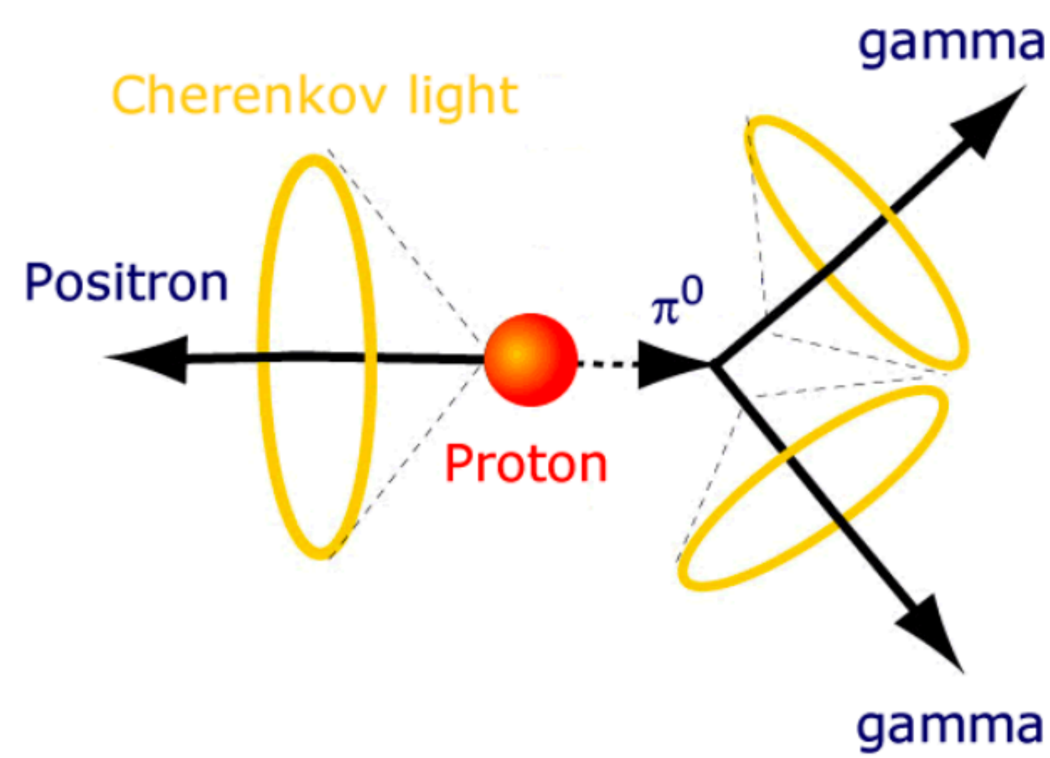
THE SU(5) GUT

- In this early model, interactions between the quarks and leptons are mediated by two new massive bosons, called the **X** and **Y**.
- To conserve electric and color charge, the X and Y have electric charges of -4e/3 and -e/3, and one of three possible colors. They are also incredibly massive, close to the grand unification scale of 10^{16} GeV.
- Hence, including both particles and antiparticles, the model predicts 12 types of X and Y.
- In addition to these 12, there are also 8 gluons, 3 weak bosons, and 1 photon, for a total of 24. This makes sense, for recall that a theory exhibiting SU(n) gauge symmetry requires the existence of n^2-1 gauge bosons.

SU(5) GUT OBSERVATIONAL CONSEQUENCES

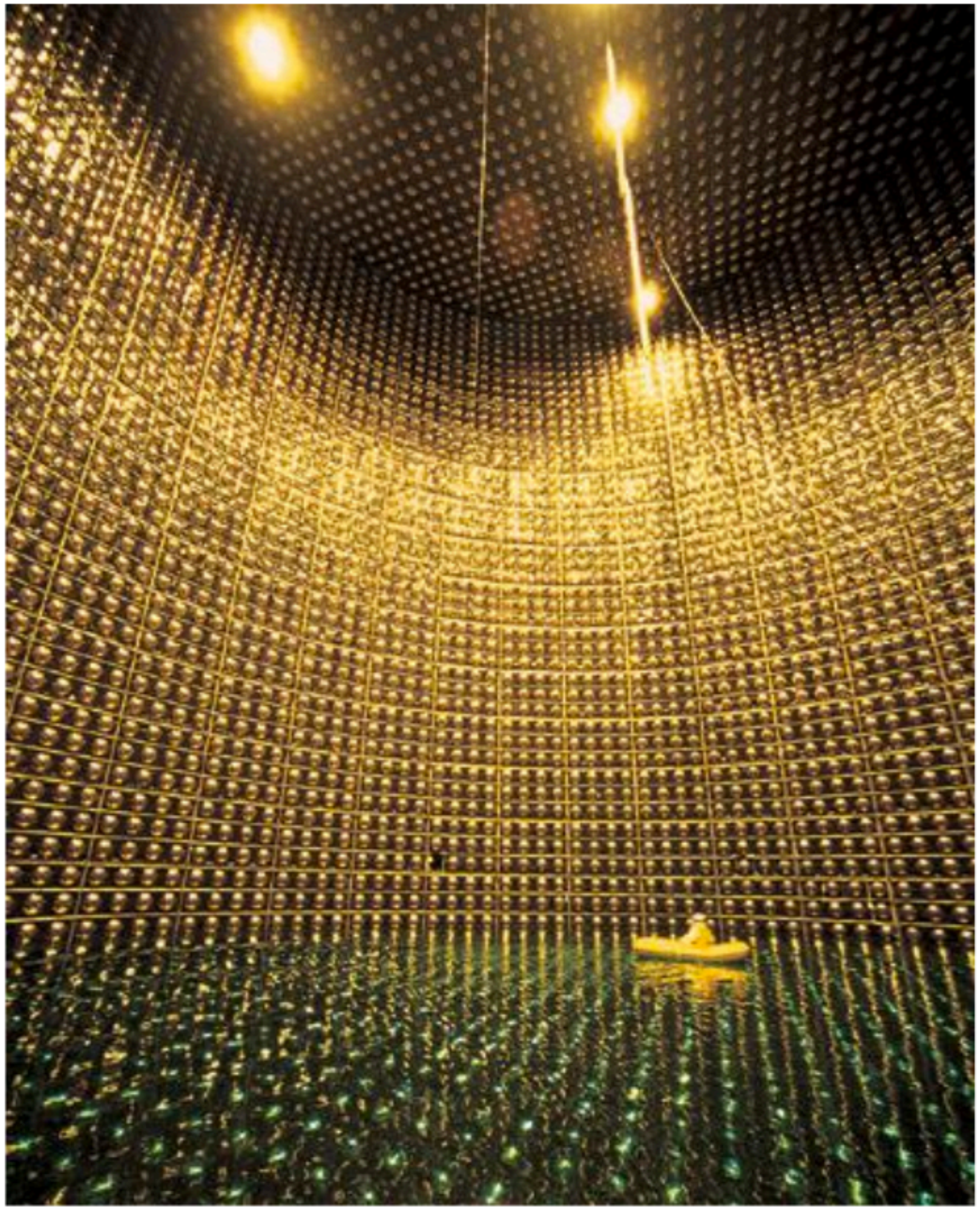
- The SU(5) GUT implies that it would take huge energies to even hope to see an X or Y particle "in the wild".
- However, even at "normal" (i.e. low) energies, virtual X and Y exchanges can take place.
- This is major: if quarks can decay into leptons via virtual X and Y exchange, then "stable" particles might actually be unstable!
- Example: the proton could possibly decay via exchange of a virtual X.





PROTON DECAY

- The instability of the proton is one of the few tests of GUT physics that would be manifest at everyday energies.
- Computations show that relative to most elementary particles, the proton is very stable; its lifetime according to the SU(5) GUT is 10^{30} years!
- How can we detect such an effect?
- Put many protons together – e.g., in a huge tank of water – and wait for some to decay...
- The Super-Kamiokande water Cherenkov detector, was built to search for proton decay, although it has made major contributions to understanding neutrino oscillations!



PROTON DECAY

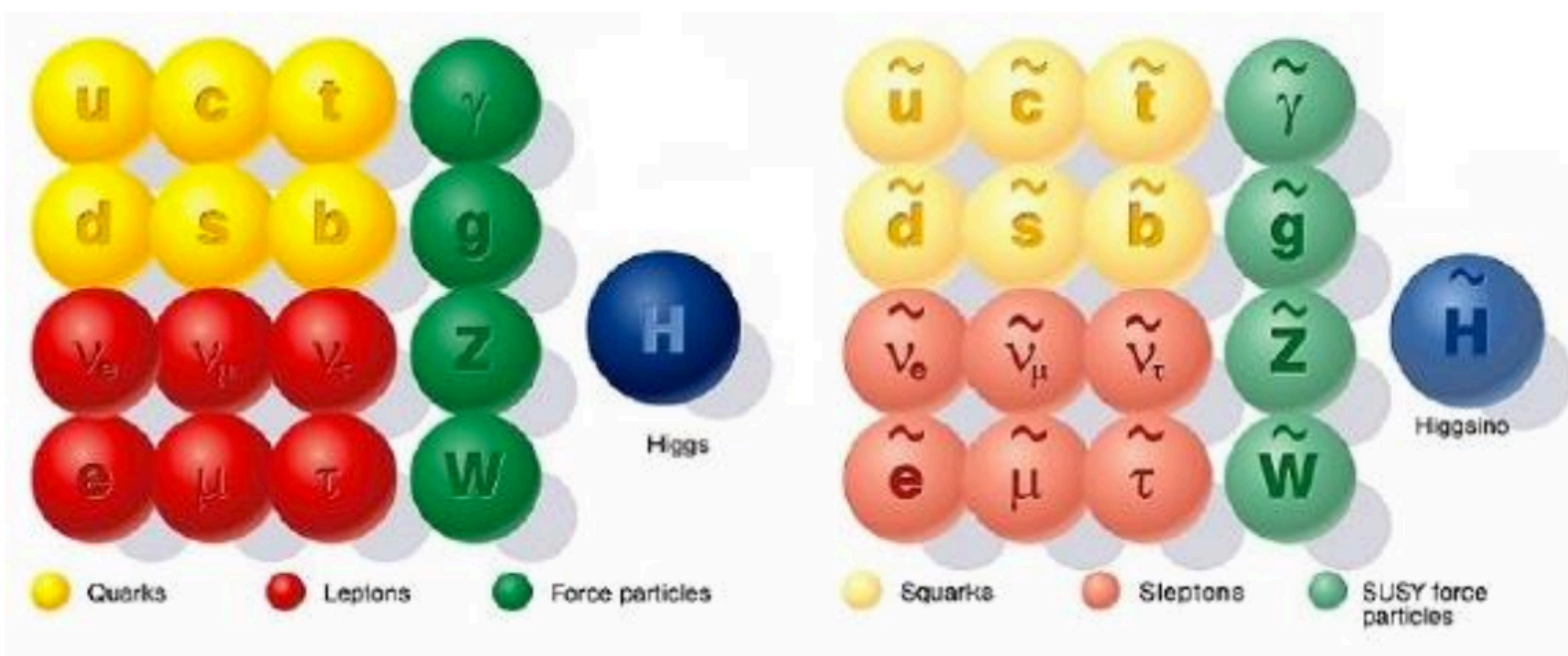
- Even though the proton lifetime is very long, a kiloton of material contains about 10^{32} protons, so about one decay per day should occur in such a sample.
- The Super-Kamiokande detector holds 50 kilotons of water viewed by 11000 photomultiplier tubes. It is located underground, shielded from background noise due to cosmic radiation at the Earth's surface.
- **No** proton decays have been observed despite more than two decades of searching, leading to lower limits on the proton lifetime of about 10^{34} years.
- Is this a disaster for the theory? Not necessarily. GUT physics can be saved if we introduce supersymmetry...

SUPERSYMMETRY

- The idea behind GUTs like SU(5) symmetry is to present different particles as transformed versions of each other.
- The SU(5) GUT treats quarks and leptons as the same underlying "something", unifying the fermions in the Standard Model.
- However, these early approaches to grand unification failed to incorporate the gauge bosons and Higgs scalars of the Standard Model into a unified scheme.
- Hence, some eventually suggested that the bosons and fermions should be united somehow by invoking a new kind of symmetry: supersymmetry (or SUSY for short).
- Aside: theorists did not develop supersymmetry explicitly for the Standard Model; the field started obscurely in the early 1970's during investigations of the spacetime symmetries in quantum field theory.

SUPERSYMMETRY

- So what, exactly, does supersymmetry predict?
- It says that for each known boson, there should be a fermion of identical mass and charge, and similarly for each known fermion, there exists a boson of identical mass and charge.



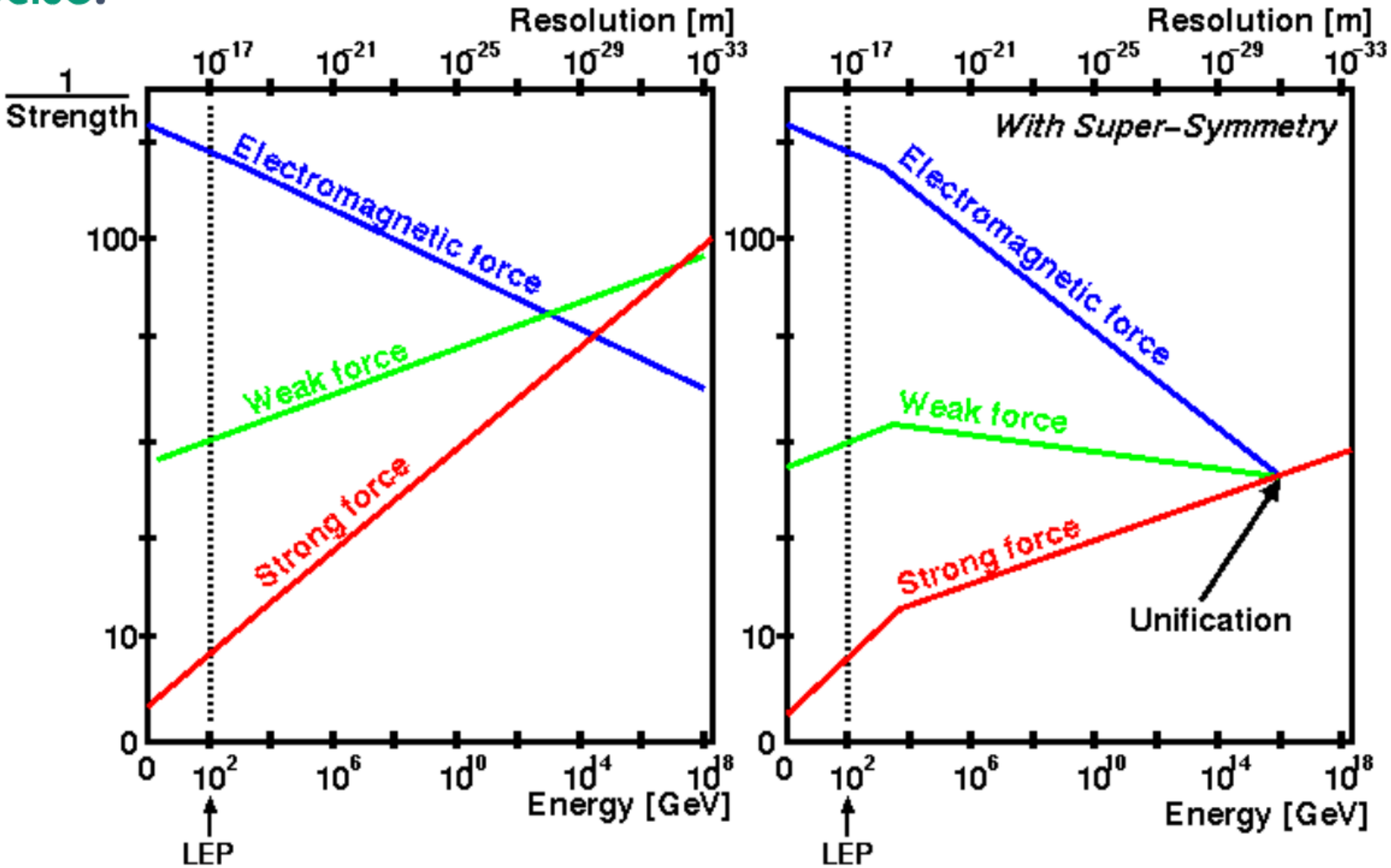
Standard particles SUSY particles

SUPERSYMMETRY

- So what, exactly, does supersymmetry predict?
- It says that for each known boson, there should be a fermion of identical mass and charge, and similarly for each known fermion, there exists a boson of identical mass and charge.
- If this is the case, why haven't we observed a selectron of mass 0.511 MeV?
  - Supersymmetry is a broken symmetry at our energy scale.
  - All sparticles are expected to be more massive than their Standard Model partners.

SUPERSYMMETRY AND THE GUT SCALE

- If instead of the Standard Model, we start from the Minimal Supersymmetric Standard Model, the alignment of the three coupling constants at the GUT scale becomes much more precise.



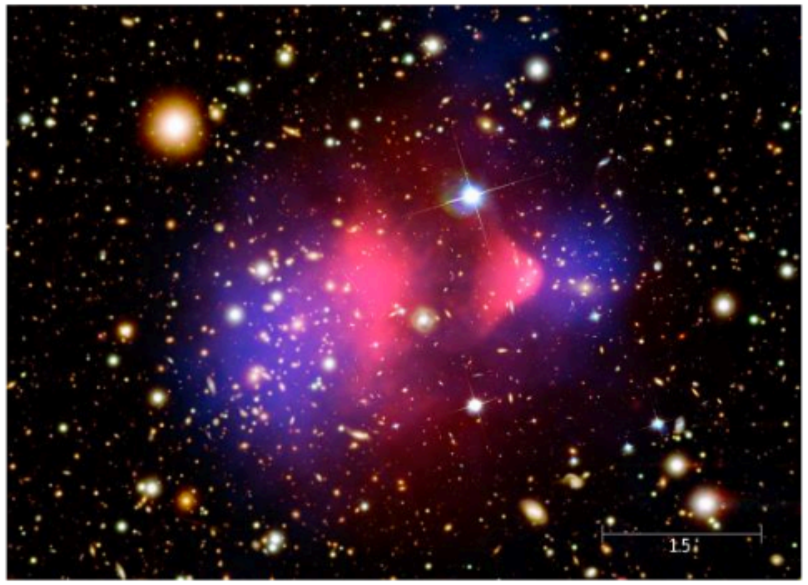
SUPERSYMMETRY AT THE LHC

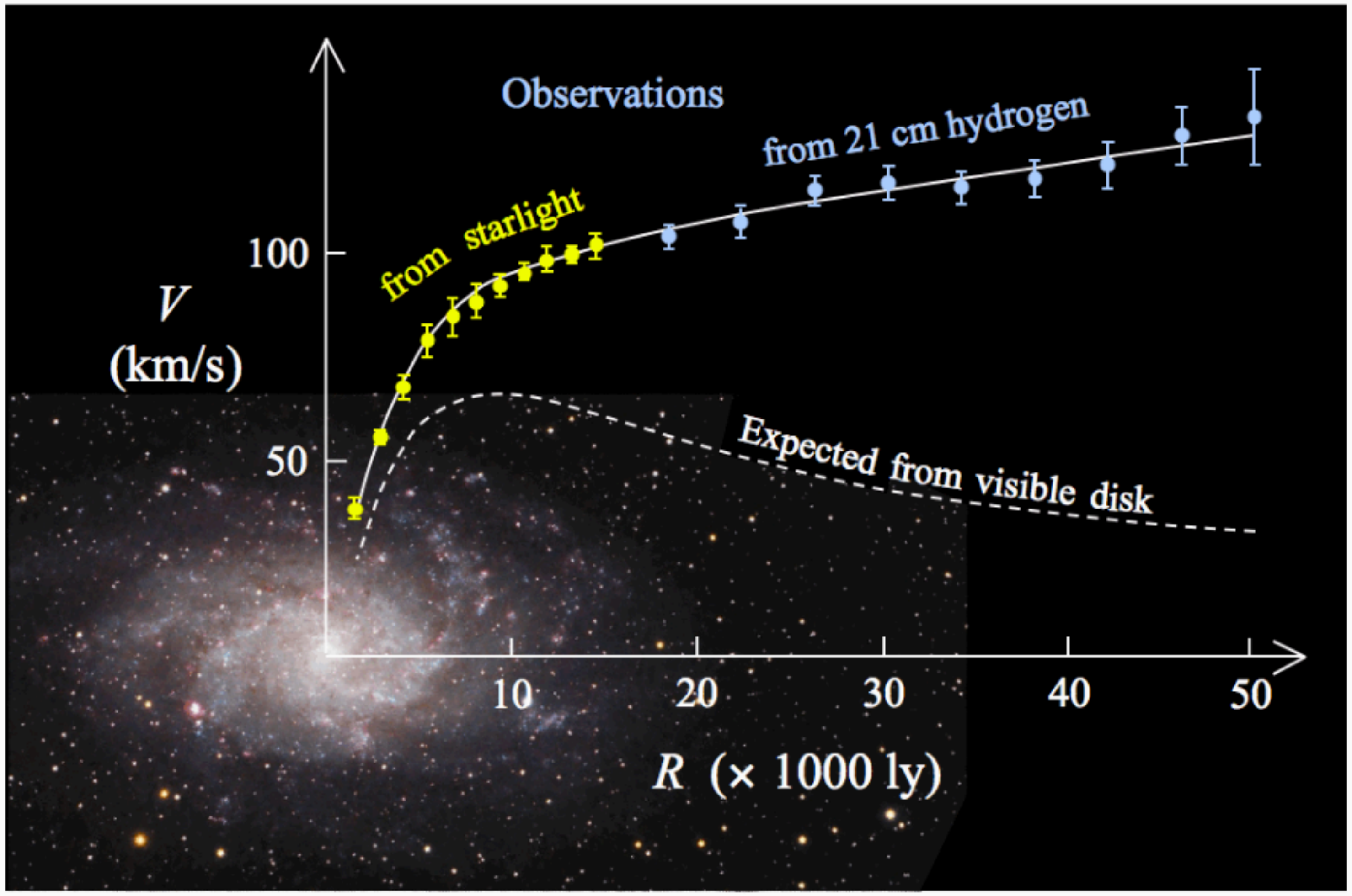
- If SUSY is a symmetry of nature, some of the sparticles might have a mass around 1000 GeV.
- We hope to see evidence for SUSY at the LHC.
  - But so far no candidates have been found…

Excited LHC physicists still haven't found SUSY.

DARK MATTER

- Galaxies rotate faster than expected, implying more mass than what is seen.
- A host of other observations point in the same direction…
- Alternatively, general relativity might need modification.
  - Probably not the solution…
- Scientific consensus leaning towards the weakly interacting massive particle hypothesis.
  - We need to observe it in the lab!



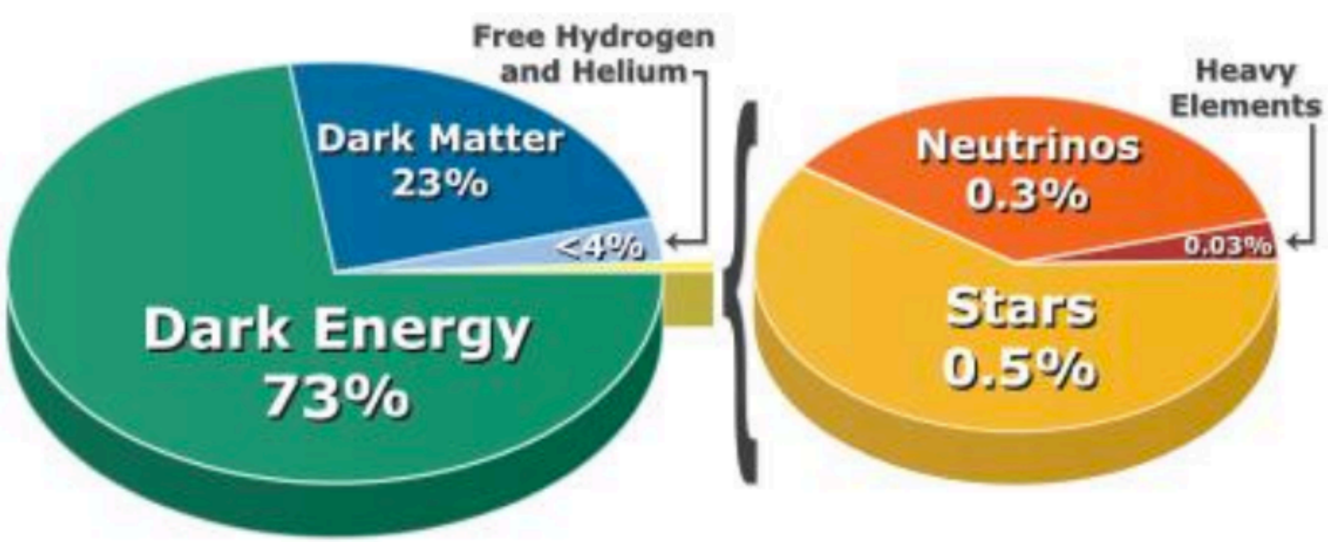


SUPERSYMMETRY AND DARK MATTER

- The neutrinos that we know are not good candidates to explain dark matter.
  - They are “dark” but also too light – moving too fast to cluster around galaxies.
- To account for dark matter, we need a particle that looks a lot like a neutrino, but more massive.
  - No electric charge or color.
- Lightest Supersymmetric Particle
  - We haven’t seen proton decay so, if SUSY exists, decays of sparticles into particles must be highly supressed.
    - We call this R-parity.
  - Hence the LSP is stable, much heavier than neutrinos, and in several SUSY models, charge neutral.
    - A dark matter candidate!

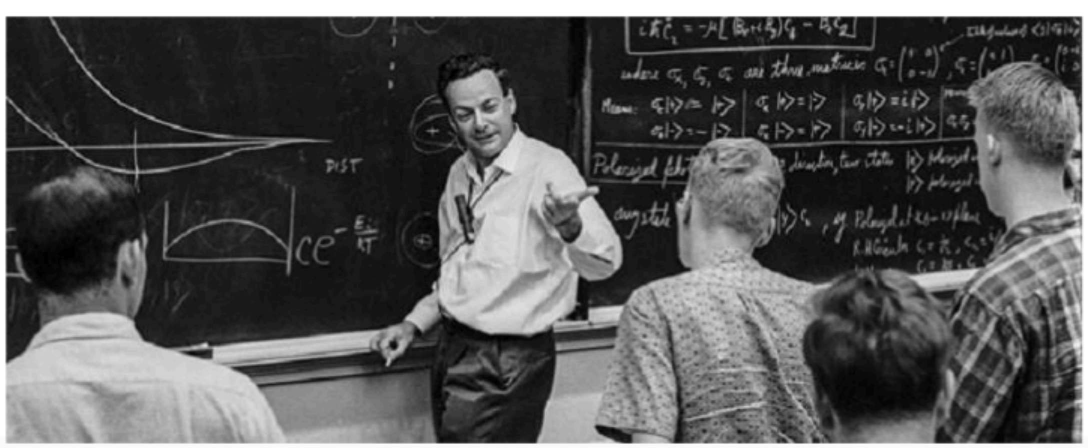
…AND THERE'S ALSO DARK ENERGY

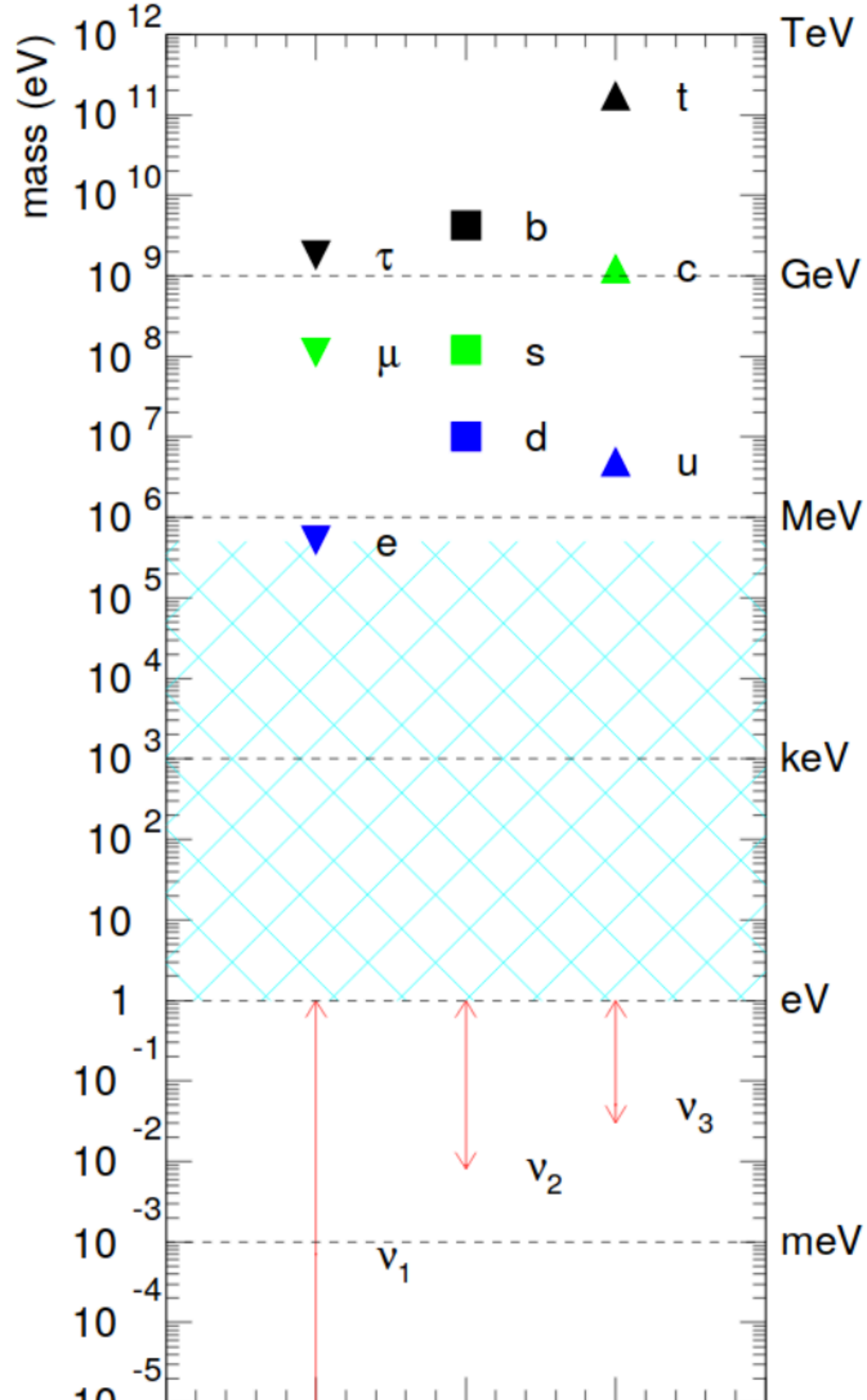
- The universe is expanding at an accelerating rate!
  - This can be accommodated in general relativity by having a non-zero cosmological constant (Λ), also called vacuum energy.
- What is the origin of this vacuum energy? Quantum field theories tend to predict vacuum energies that are far too large…



PARTICLE MASSES

- "There remains one especially unsatisfactory feature [of the Standard Model of particle physics]: the observed masses of the particles, m. There is no theory that adequately explains these numbers. We use the numbers in all our theories, but we do not understand them – what they are, or where they come from. I believe that from a fundamental point of view, this is a very interesting and serious problem."
  - R. P. Feynman



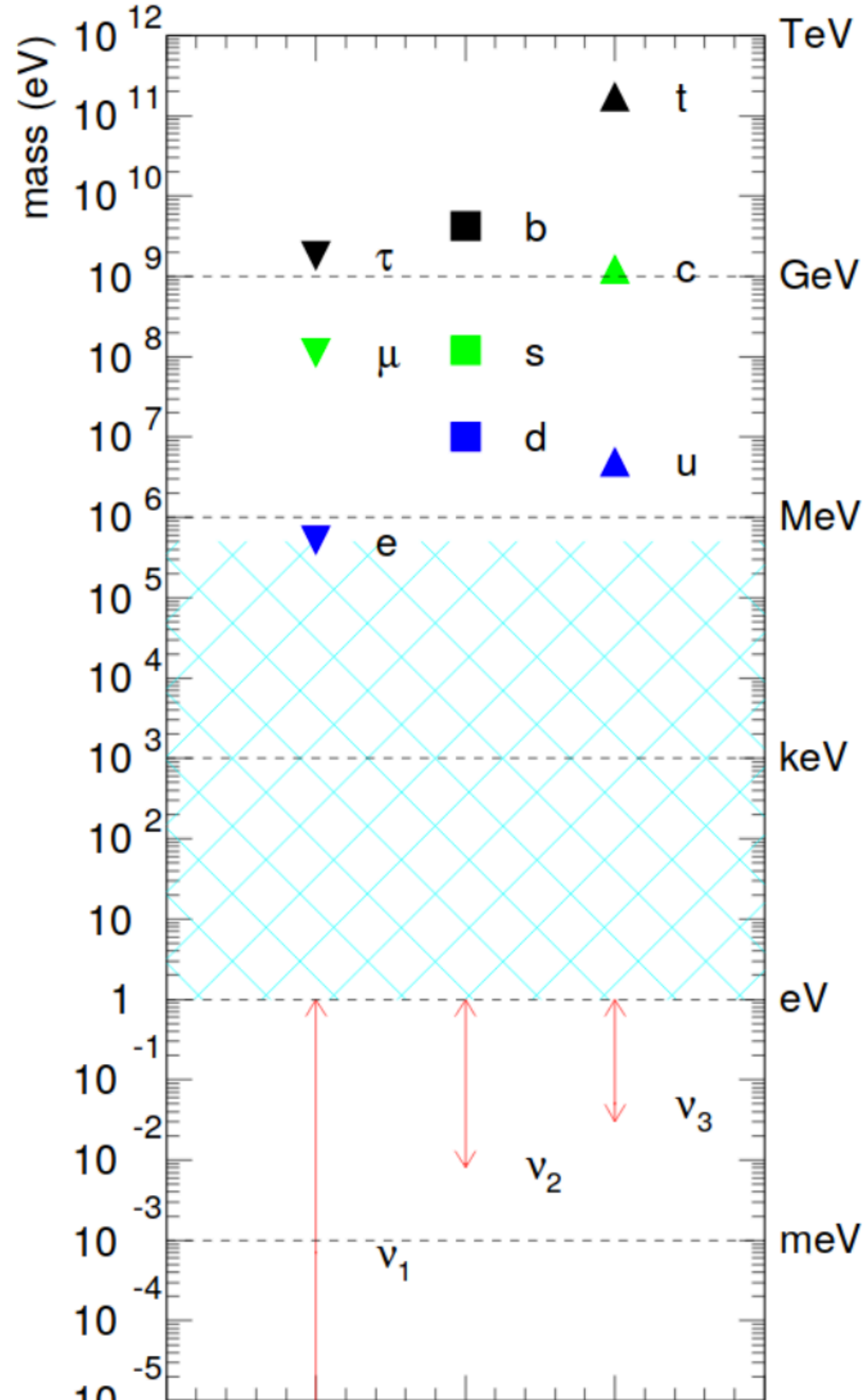


de Gouvêa, Neutrino 2016

PARTICLE MASSES

- Is there a theory that can accurately predict the masses of the fermions?
  - None so far…
  - Would it elucidate the nature of the three generations?

- Why are neutrino masses so much smaller than the other fermion masses?
  - Is the mechanism that generates neutrino masses qualitatively different from the mechanism that generates other fermion masses?



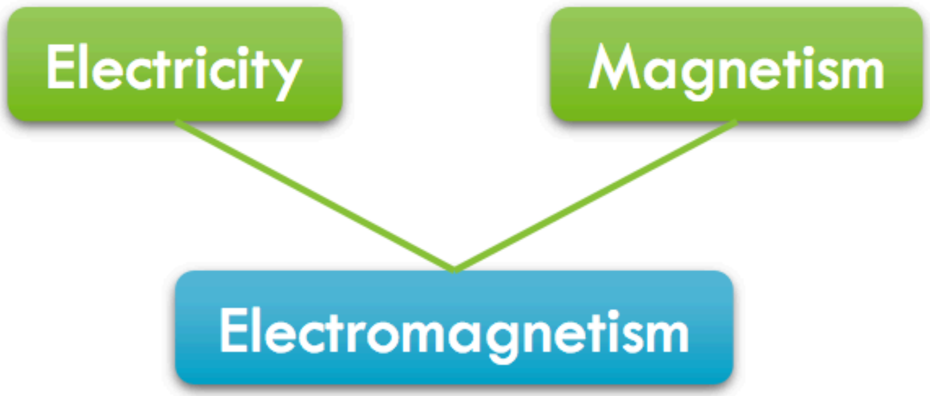
de Gouvêa, Neutrino 2016

A THEORY OF EVERYTHING?

Electricity

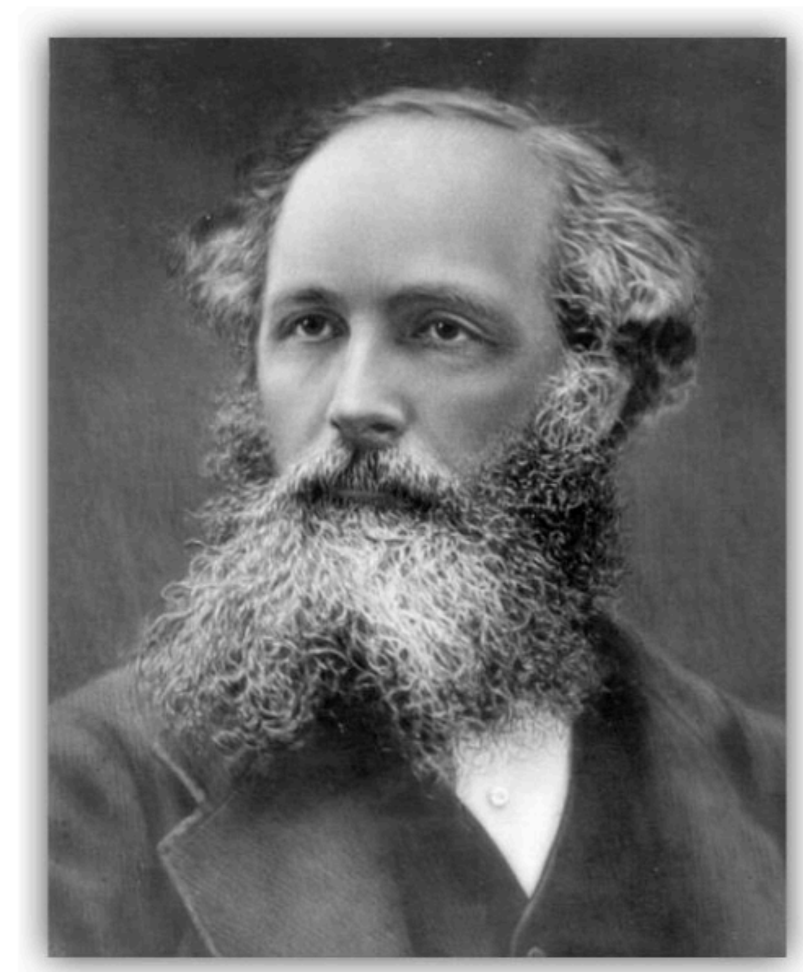
Magnetism

A THEORY OF EVERYTHING?



$$\nabla \cdot \mathbf{D} = \rho$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

James Clerk Maxwell
1800s



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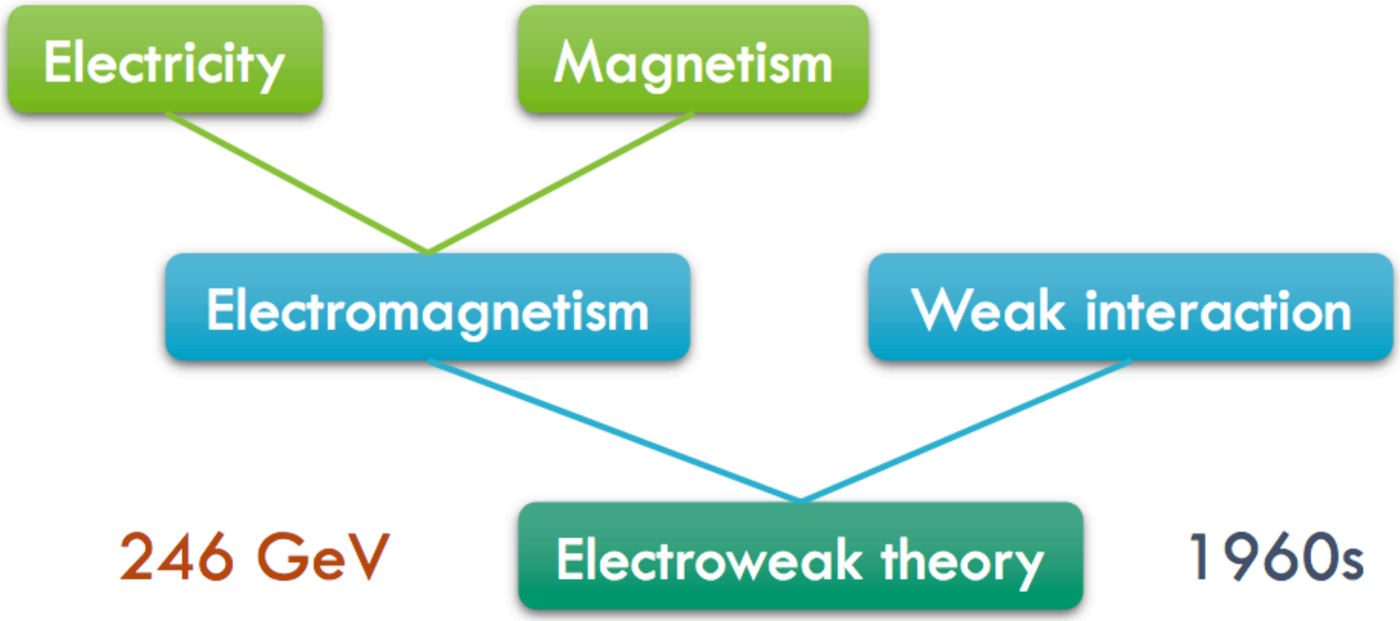
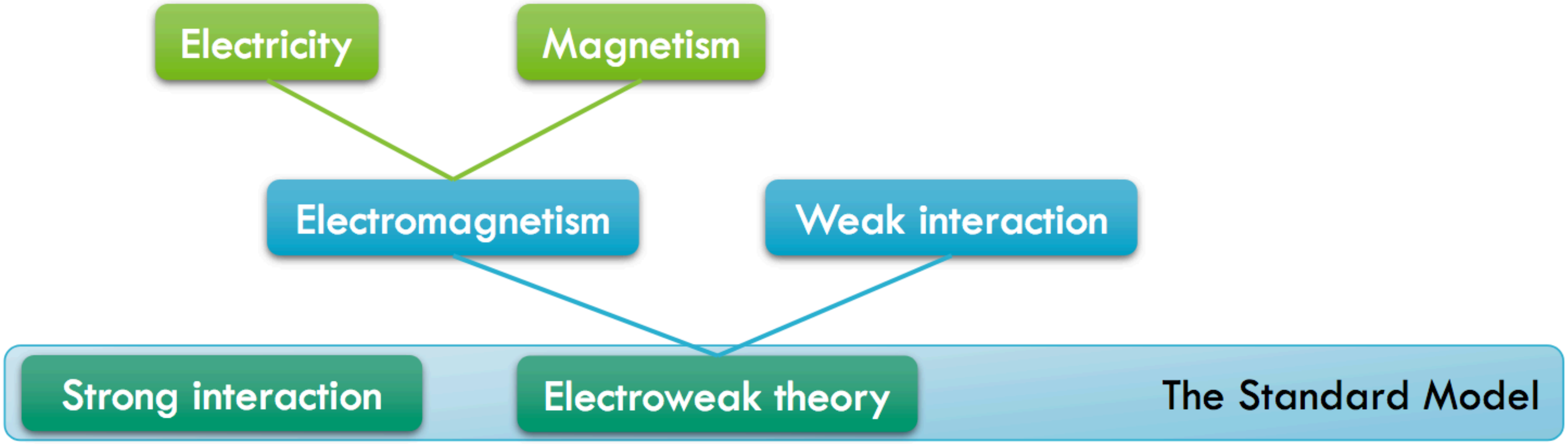


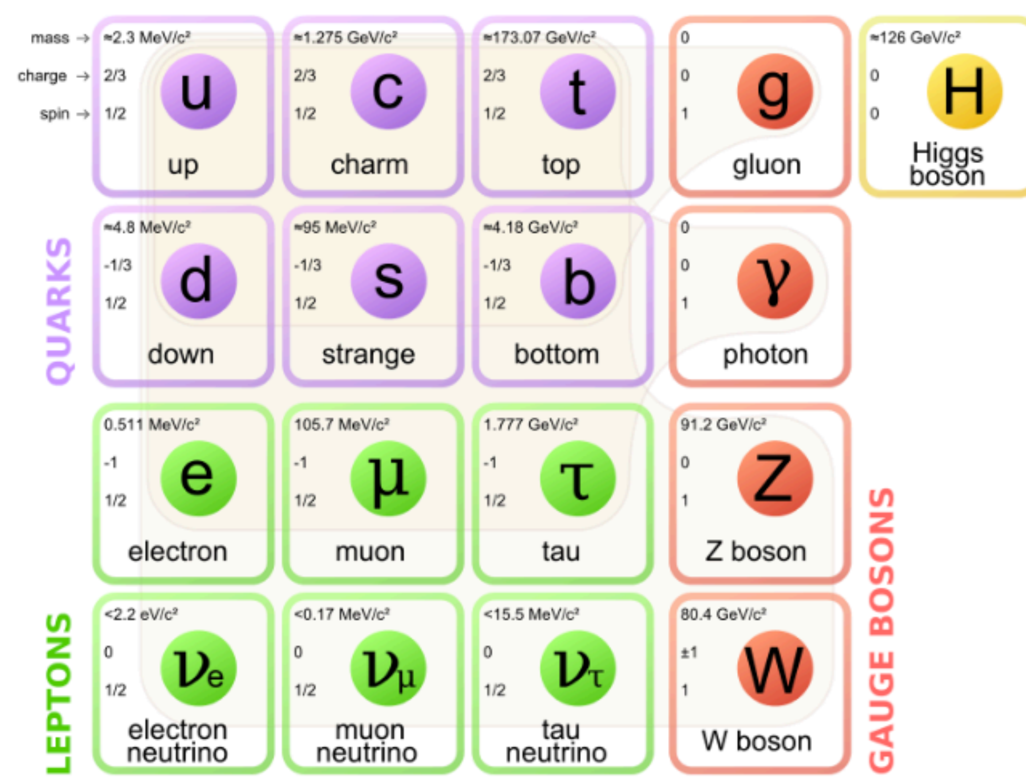

Sheldon Lee Glashow


Abdus Salam


Steven Weinberg

A THEORY OF EVERYTHING?





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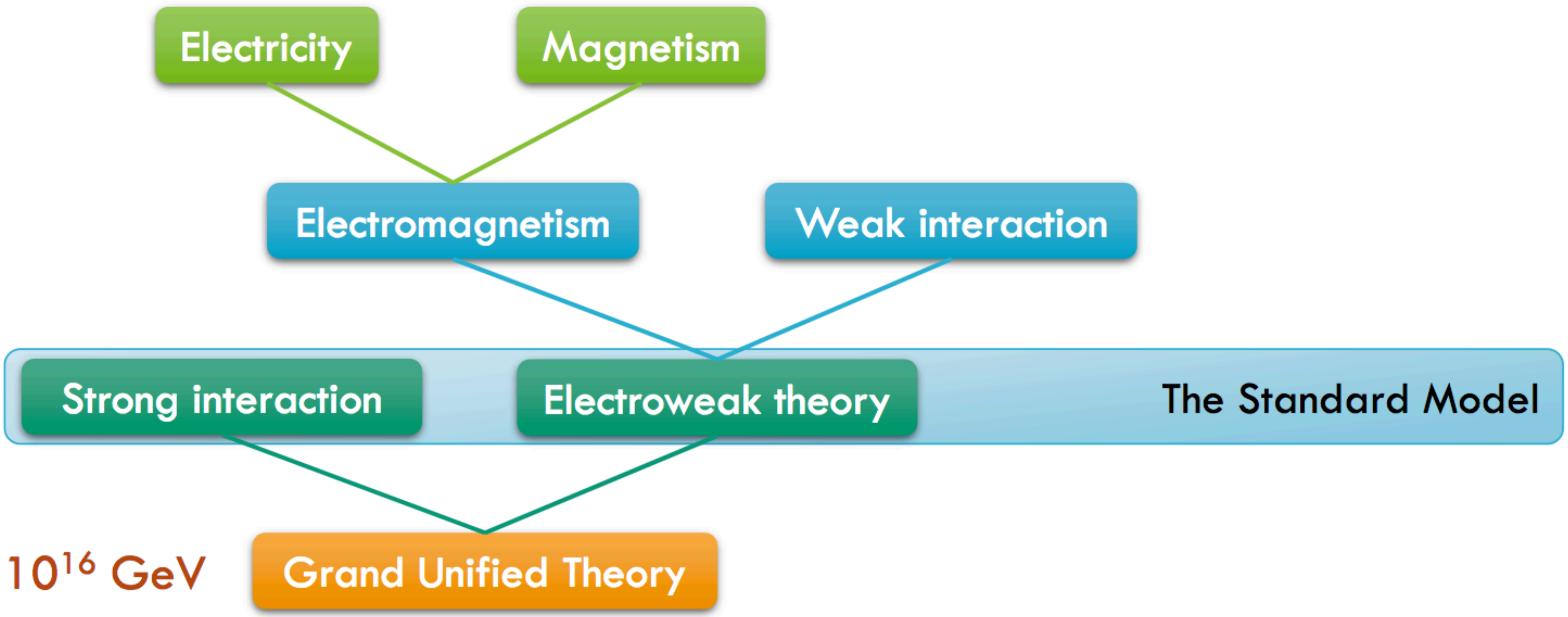
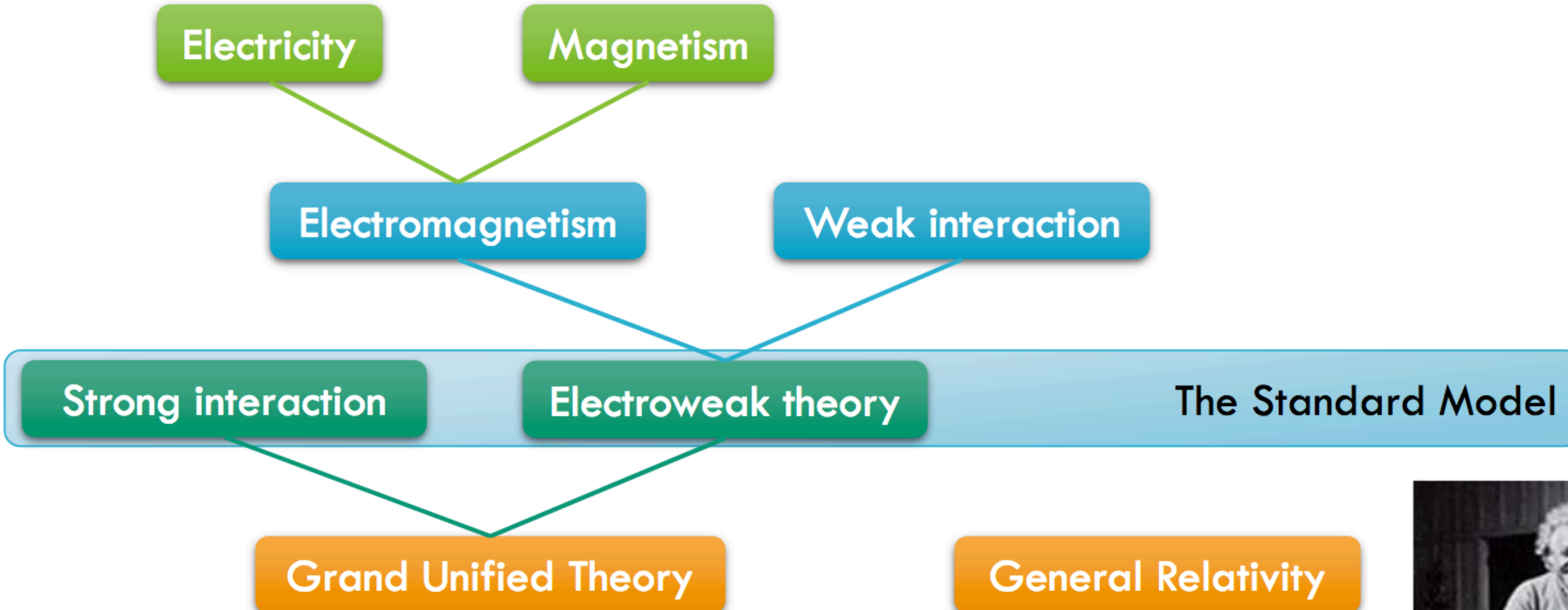


SU(5)? SO(10)? SU(8)? O(16)?
1970s – today…

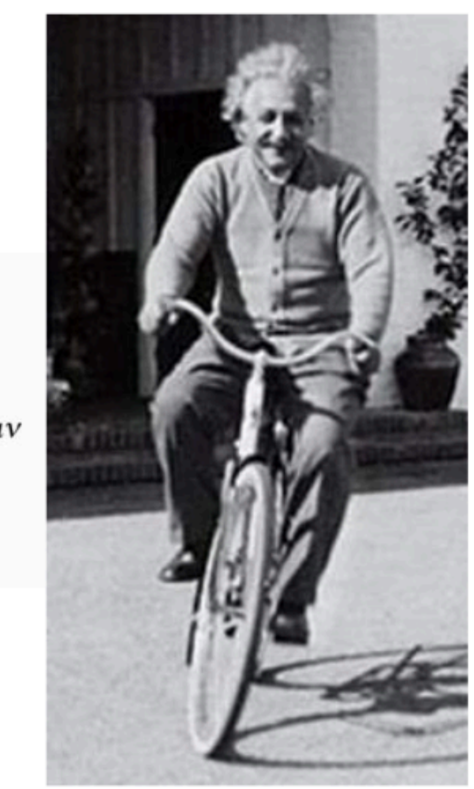
A THEORY OF EVERYTHING?



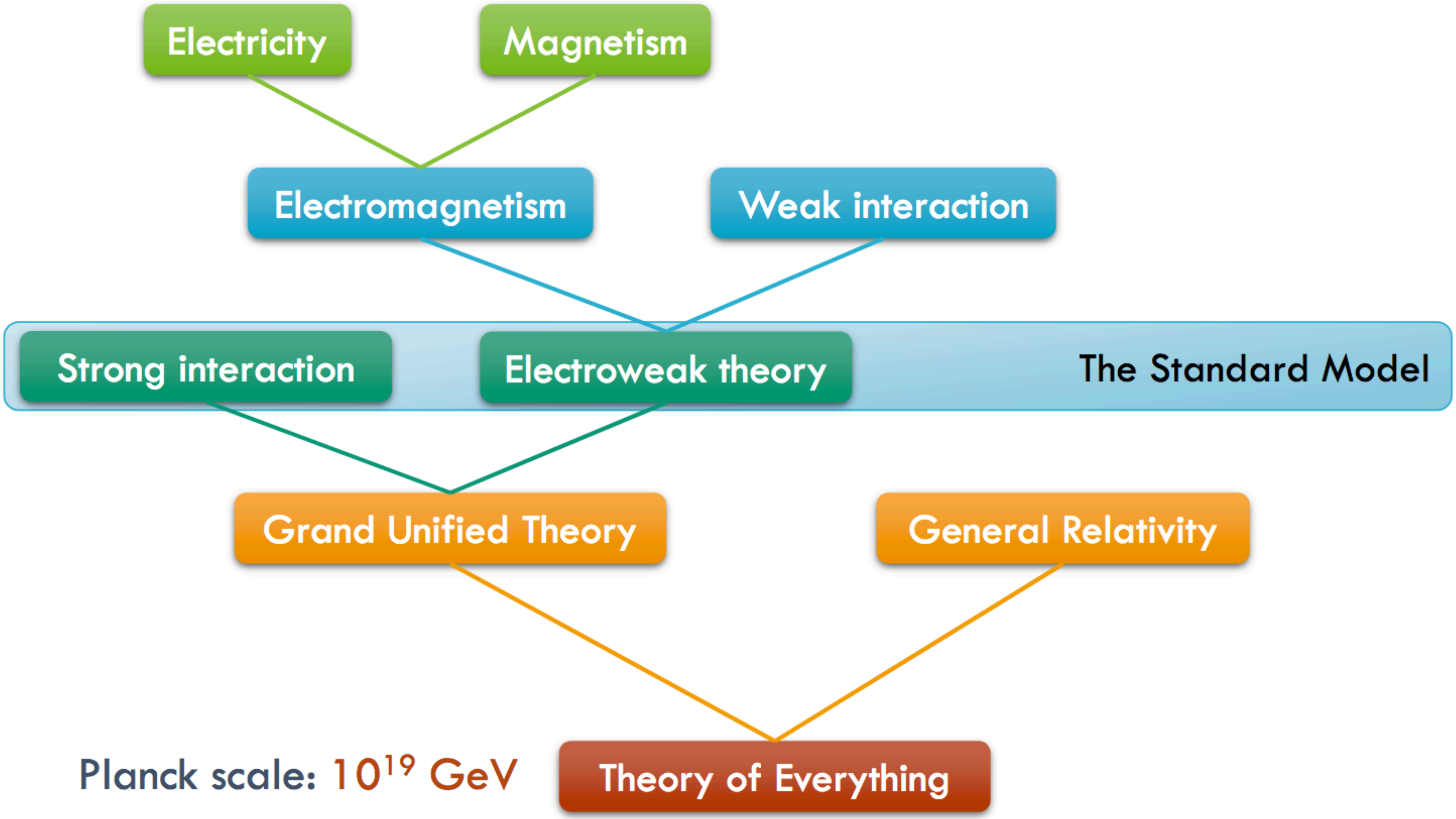
SU(5)? SO(10)? SU(8)? O(16)?
1970s – today…

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Albert Einstein
1925

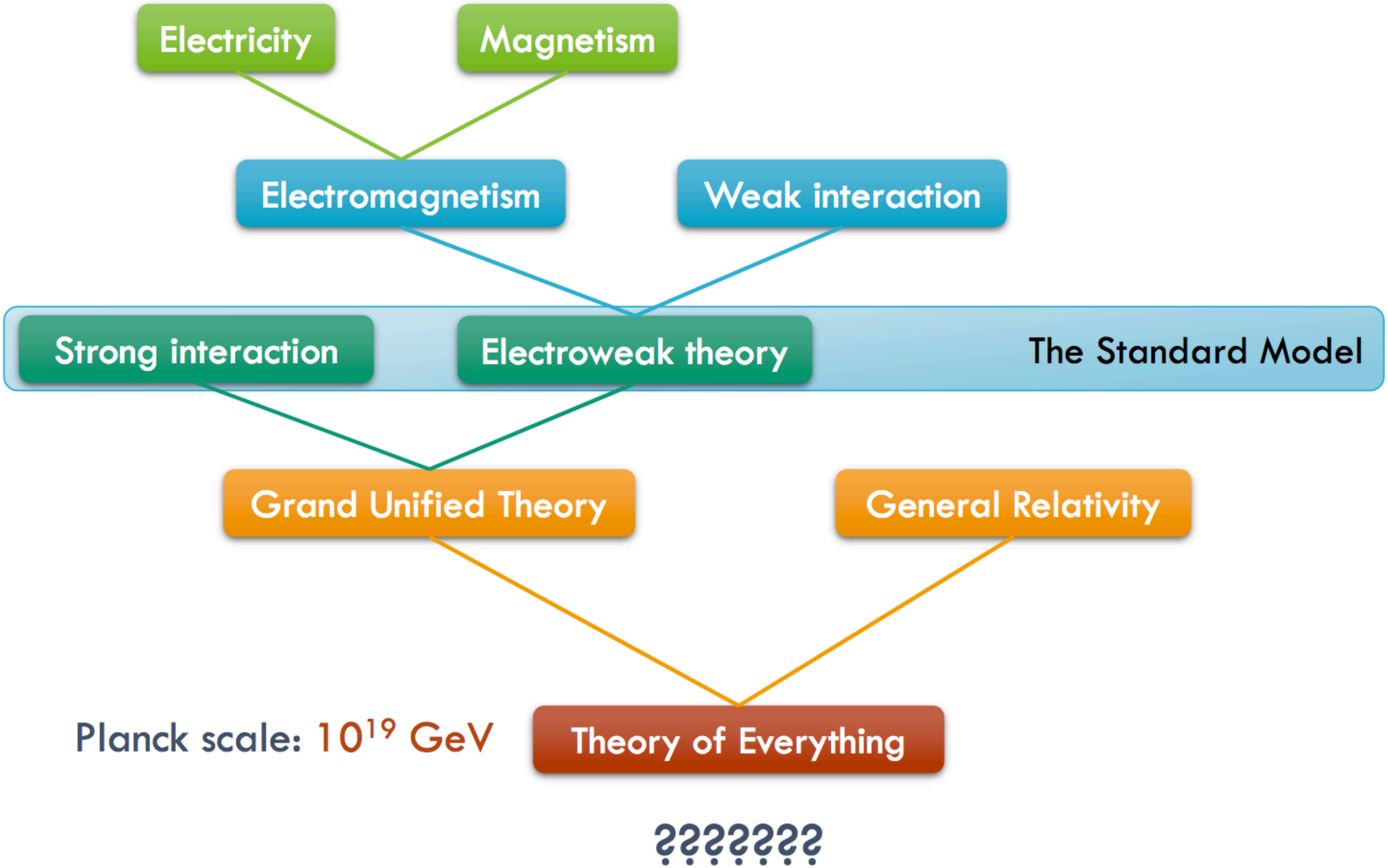


A THEORY OF EVERYTHING?



Planck scale: 10^{19} GeV

A THEORY OF EVERYTHING?



A THEORY OF EVERYTHING?

- No one knows how to build a theory of everything.
  - General relativity and quantum mechanics are fundamentally different:
    - GR predicts smooth spacetime; QM predicts vacuum fluctuations.
  - A Grand Unified Theory is probably an intermediate step.
- String theory / M-theory
  - Particles are replaced by one-dimensional "strings".
    - Different vibration modes give rise to different particle properties.
  - Needs extra dimensions to work! (9+1; 10+1)
  - Looks like supersymmetry at low energies.
- Loop quantum gravity
  - Granular description of space – quantization!
  - Needs only three space dimensions…

STANDARD MODEL LIMITATIONS SUMMARY

- We know that the Standard Model is incomplete: it does not fully account for experimental observations.
  - It doesn't describe gravity – and it seems very difficult to extend it in order to do so.
  - There isn't a unique way of describing neutrino mass in the SM context.
  - It doesn't provide a candidate for dark matter.

- Offers unsatisfactory answers to some fundamental questions:
  - Doesn't predict particle masses.
  - Or why there are three generations of matter.
  - Very large differences in the scale of some parameters remain unexplained.
    - Including some "lucky" cancelations.

https://youtu.be/sw4_9xhGzjo