Week6
Standard Model-1

Class Schedule

Date	Topic	
Week 1 (9/22/18)	Introduction	YJ
Week 2 (9/29/18)	History of Particle Physics	YJ
Week 3 (10/6/18)	Special Relativity	Ed
Week 4 (10/13/18)	Quantum Mechanics	Ed
Week 5 (10/20/18)	Experimental Methods	Ed
Week 6 (10/27/18)	The Standard Model - Overview	YJ
Week 7 (11/3/18)	The Standard Model - Limitations	YJ
Week 8 (11/10/18)	Neutrino Theory	Ed
Week 9 (11/17/18)	Neutrino Experiment	Ed
Week 10 (12/1/18)	LHC and Experiments	YJ
Week 11 (12/8/18)	The Higgs Boson and Beyond	YJ
Week 12 (12/15/18)	Particle Cosmology	Ed

Class Policy

- Classes from 10:00 AM to 12:30 PM (10 min break at ~ 11:10 AM).
- Attendance record counts.
- Up to four absences
- Lateness or leaving early counts as half-absence.
- Send email notifications of all absences to shpattendance@columbia.edu.

Class Policy

- No cell phone uses during the class.
- Feel free to step outside to the hall way in case of emergencies, bathrooms, starvations.
- Feel free to stop me and ask questions / ask for clarifications.
- Resources for class materials, Research Opportunities + Resources to become a particle physicist

https://twiki.nevis.columbia.edu/twiki/pub/Main/ScienceHonorsProgram

Quantum Field Theory

- QFT; Foundation of particle theroy
- Small and fast moving; Quantum and relativistic.
- QFT is meant to be dealt with by incorporating relativistic and quantum principles.
- QFT depicts subatomic behaviors.
- One of the QFTs, the Standard Model has incredible accuracy.

The Standard Model

- The theory which attempts to fully describe the weak, electromagnetic, and strong interactions within a **common framework**:
  - A "common ground" that would unite all of laws and theories which describe particle dynamics into one integrated theory of everything, of which all the other known laws would be special cases, and from which the behavior of all matter and energy can be derived.
- A theory of "almost everything": does not accommodate gravity, dark matter, dark energy, others…

The Standard Model

- **The Standard Model (SM):** good model, but "old news".
- The SM was solidified in the 1970's, with the discovery of quarks (confirmation of theory of strong interactions).
- Under scrutiny for the last 40 years, has managed to survive many experimental tests (!):
  - All particles predicted by this theory have been found experimentally!
- We already know it is incomplete ("almost everything", plus it has some "unnatural properties").
  - More discussion on this next week!

Today's Agenda

- Historical background (see lecture 2)
- SM particle content
- SM particle dynamics
  - Quantum Electrodynamics (QED)
  - Quantum Chromodynamics (QCD)
  - Weak Interactions
  - Force Unification
- Lagrangian / Field formulation
- Tests and predictions
- Higgs mechanism and Higgs boson discovery

- Historical background (see lecture 2)
- **SM particle content**
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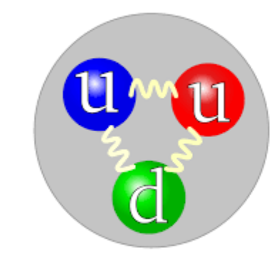
SM particle content

- **Fermions:** quarks and leptons
  - Spin-1/2 particles

- **Bosons:** force mediators and Higgs field
  - Integer-spin particles

SM particle content : Quarks

- Quarks:
  - There are no free quarks.
  - They form colorless composite objects, **hadrons**:
    - baryons (qqq)
    - mesons (qq)



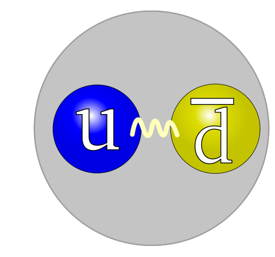


Table 1.1 The twelve fundamental fermions divided into quarks and leptons. The masses of the quarks are the current masses.

	Leptons				Quarks			
	Particle		Q	mass/GeV	Particle		Q	mass/GeV
First generation	electron	(e^-)	−1	0.0005	down	(d)	−1/3	0.003
	neutrino	(ν_e)	0	$< 10^{-9}$	up	(u)	+2/3	0.005
Second generation	muon	(μ^-)	−1	0.106	strange	(s)	−1/3	0.1
	neutrino	(ν_μ)	0	$< 10^{-9}$	charm	(c)	+2/3	1.3
Third generation	tau	(τ^-)	−1	1.78	bottom	(b)	−1/3	4.5
	neutrino	(ν_τ)	0	$< 10^{-9}$	top	(t)	+2/3	174

	Color	Electric	Weak
Quarks	☑	☑	☑

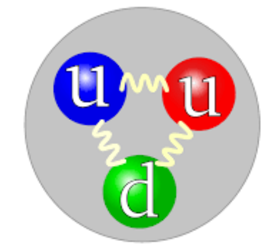
The forces experienced by different particles.

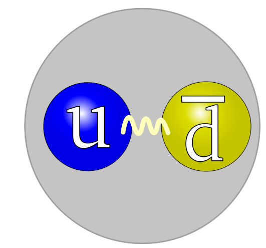
					strong	electromagnetic	weak
Quarks	down-type	d	s	b	✓	✓	✓
	up-type	u	c	t			
Leptons	charged	e^-	μ^-	τ^-		✓	✓
	neutrinos	ν_e	ν_μ	ν_τ			✓

SM particle content : Quarks

- Quarks:
  - There are no free quarks.
  - They form colorless
    - baryons (qqq)
    - mesons (qq)

Possessing charge of an interaction makes a particle to participate in the interaction.
~Sensory system





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	Color	Electric	Weak
Quarks	☑	☑	☑

The forces experienced by different particles.

					strong	electromagnetic	weak
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	up-type	u	c	t			
Leptons	charged	e^-	μ^-	τ^-		✓	✓
	neutrinos	ν_e	ν_μ	ν_τ			✓

SM particle content : Quarks

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    - mesons ($q\bar{q}$)

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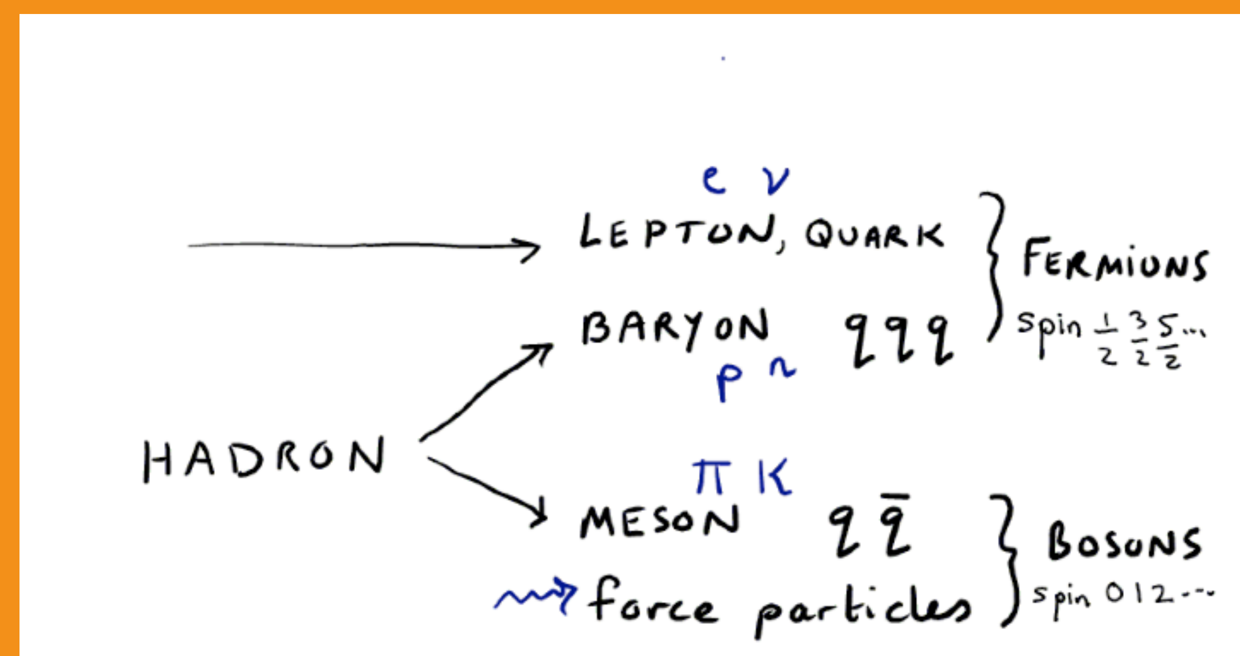
Pop Quiz !!!
Are hadrons bosons or fermions?



They can be either fermions or bosons.

					weak
Quark					✓
Leptons					✓
	neutrinos	ν_e	ν_μ	ν_τ	✓

SM particle content : Leptons

- Leptons: (can exist as free particles)

	Color	Electric	Weak
Quarks	☑	☑	☑
Leptons (charged)	☐	☑	☑
(neutrinos)	☐	☐	☑

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Possessing charge of an interaction makes a particle to participate in the interaction.
~Sensory system

Table 1.2 The forces experienced by different particles.

					strong	electromagnetic	weak
Quarks	down-type	d	s	b	✓	✓	✓
	up-type	u	c	t			
Leptons	charged	e^-	μ^-	τ^-		✓	✓
	neutrinos	ν_e	ν_μ	ν_τ			✓

- Historical background (see lecture 2)
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- **SM particle dynamics**
    - Quantum Electrodynamics (QED)
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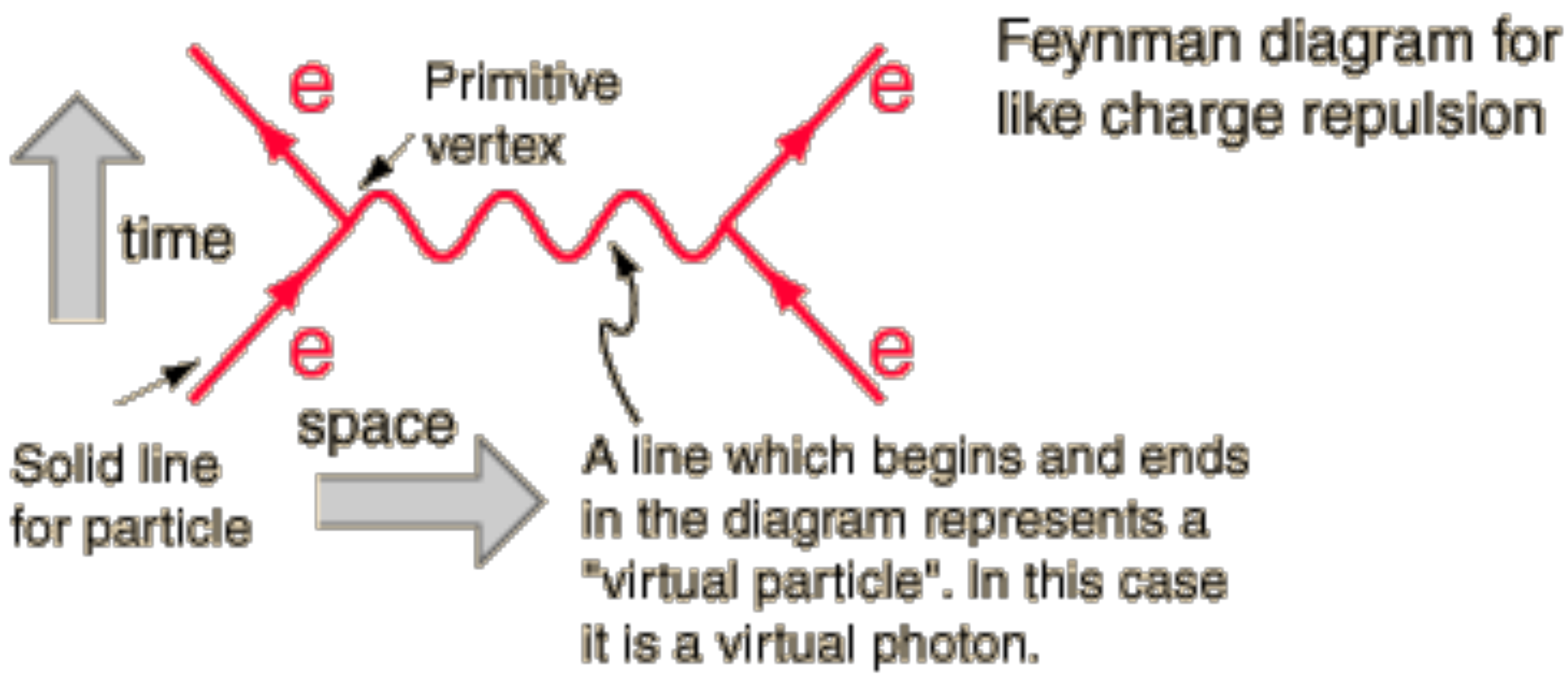
Particle dynamics : Pictorial way

Richard P. Feynman

- Feynman Rules!
- 1948: introduced pictorial representation scheme for the mathematical expressions governing the behavior of subatomic particles.
  - Can be used to easily calculate "probability amplitudes".
  - Other options: cumbersome mathematical derivations.

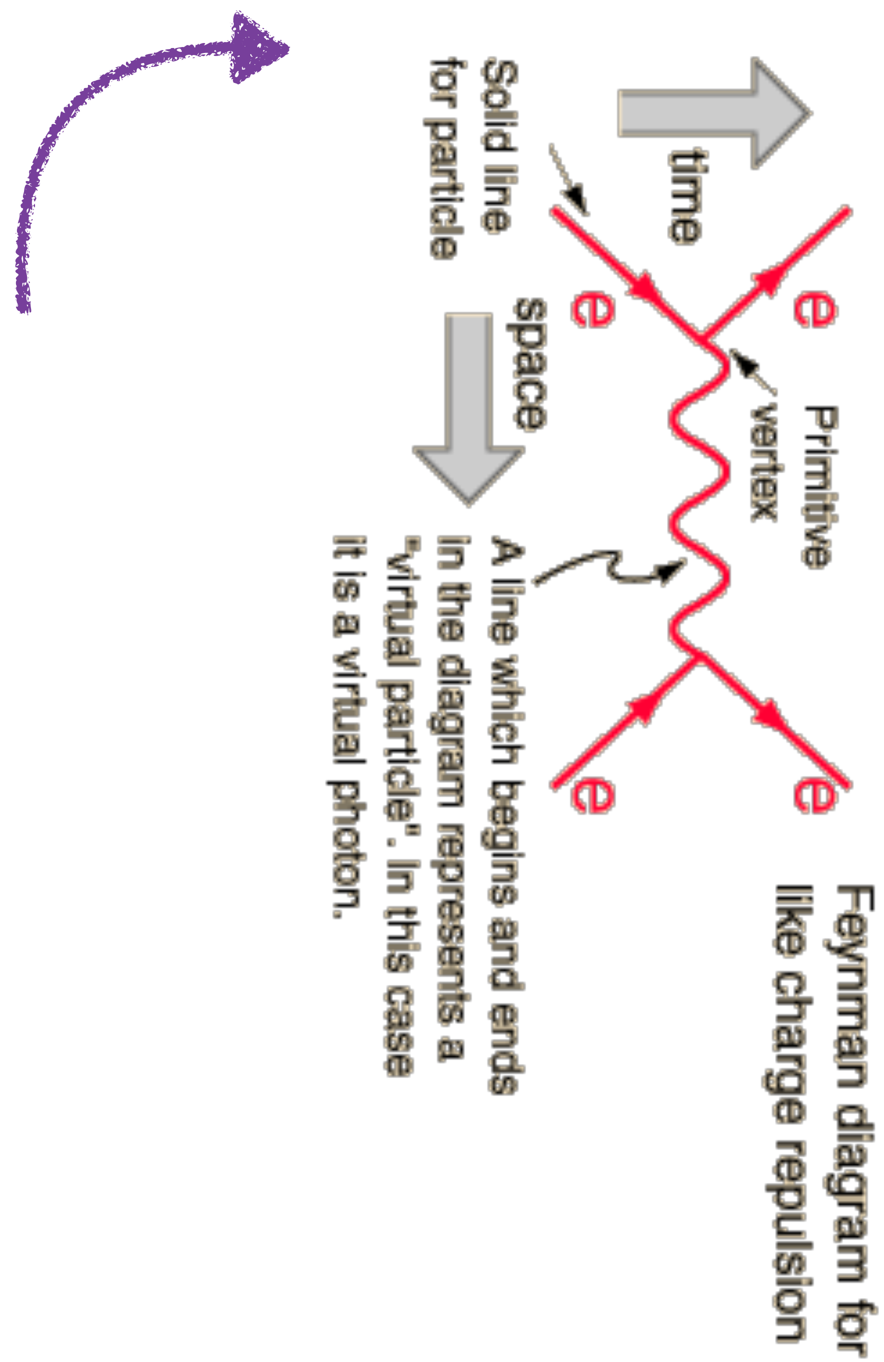
Particle dynamics : Pictorial way

- Feynman diagrams deciphered:



Particle dynamics (pictorially)

- Feynman diagrams deciphered:

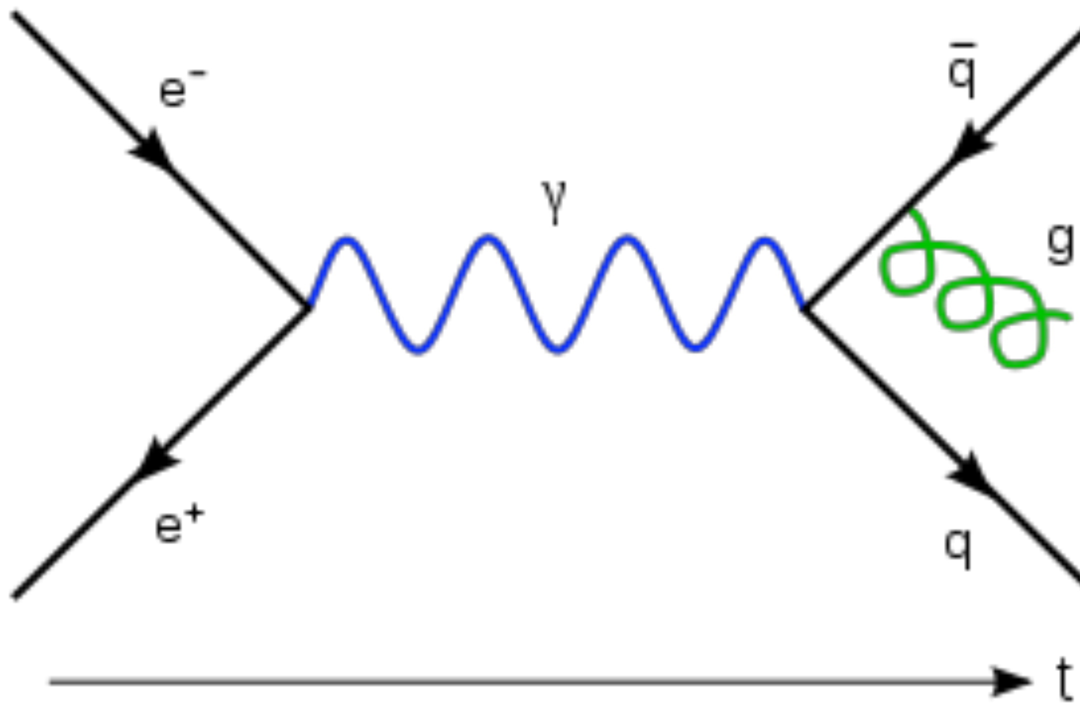


Beware of the time direction!
(You'll see it used in either way.)

If t was on y-axis, this would be a different process.

Particle dynamics (pictorially)

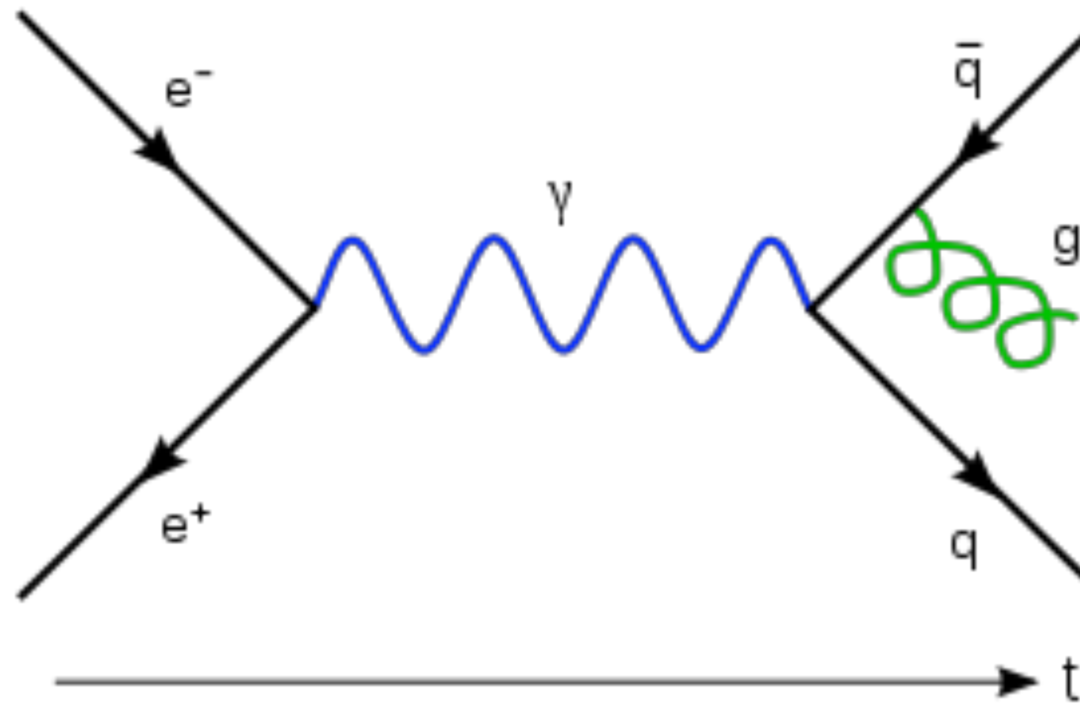
- Example:



An electron and a positron annihilate, producing a virtual photon (represented by the blue wavy line) that becomes a quark-antiquark pair. Then one radiates a gluon (represented by the green spiral).

Particle dynamics (pictorially)

- Example:



Note, at every vertex:
Q conservation
L conservation
L_e conservation
B conservation

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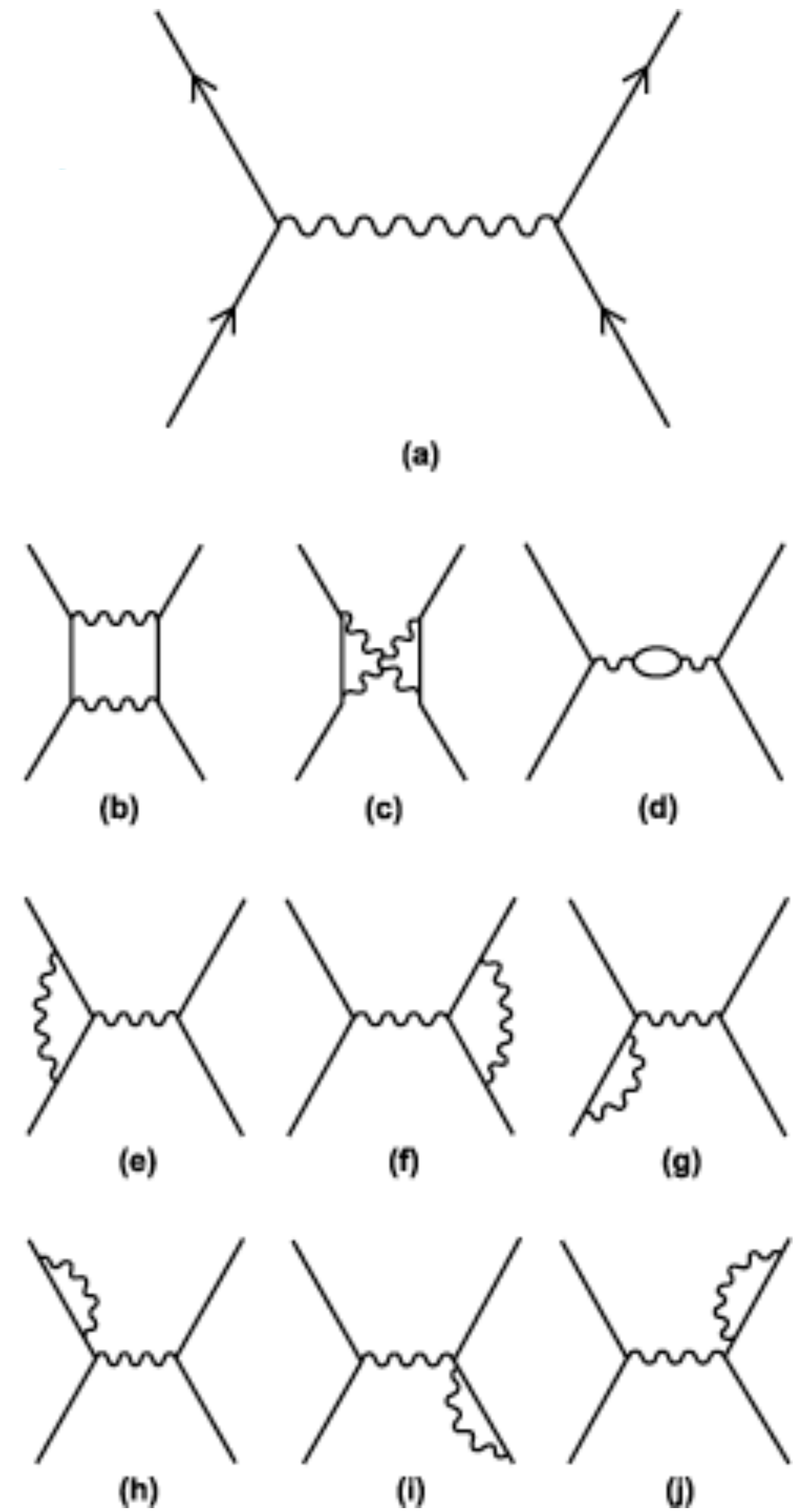
Quantum Electrodynamics (QED)

- Quantum electrodynamics, or QED, is the theory of the interactions between light and matter.
- As you already know, electromagnetism is the dominant physical force in your life. All of your daily interactions - besides your attraction to the Earth - are electromagnetic in nature.
- As a theory of electromagnetism, QED is primarily interested in the behavior and interactions of charged particles with each other and with light.
- As a quantum theory, QED works in the submicroscopic world, where particles follow all possible paths and can blink in and out of existence (more later).

Quantum Electrodynamics (QED)

- The **vertices** are interactions with the electromagnetic field.
- The straight lines are electrons and the wiggly ones are photons.
  - Between interactions (vertices), they propagate under the free Hamiltonian (free particles).
- The higher the number of vertices, the less likely for the interaction to happen.



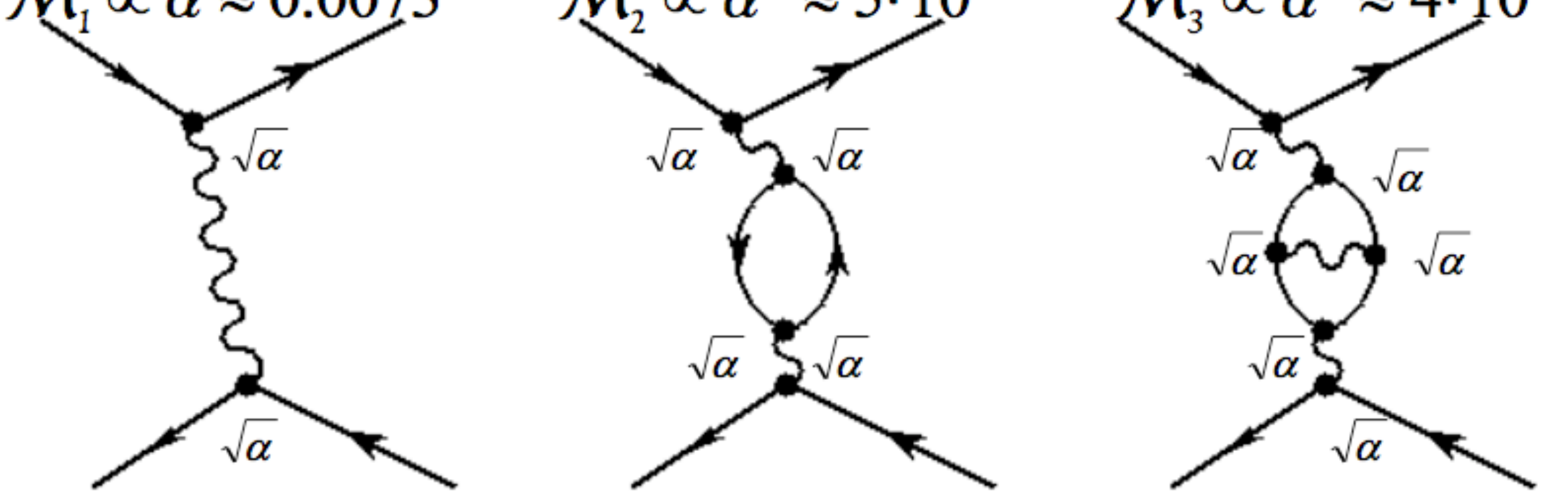
Quantum electrodynamics (QED)

- Each vertex contributes a coupling constant $\sqrt{\alpha}$, where α is a small dimensionless number:

$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

- Hence, higher-order diagrams (diagrams with more vertices) get suppressed relative to diagrams with less vertices because α is so small.

$\mathcal{M}_1 \propto \alpha \approx 0.0073$ $\quad$ $\mathcal{M}_2 \propto \alpha^2 \approx 5 \cdot 10^{-5}$ $\quad$ $\mathcal{M}_3 \propto \alpha^3 \approx 4 \cdot 10^{-7}$



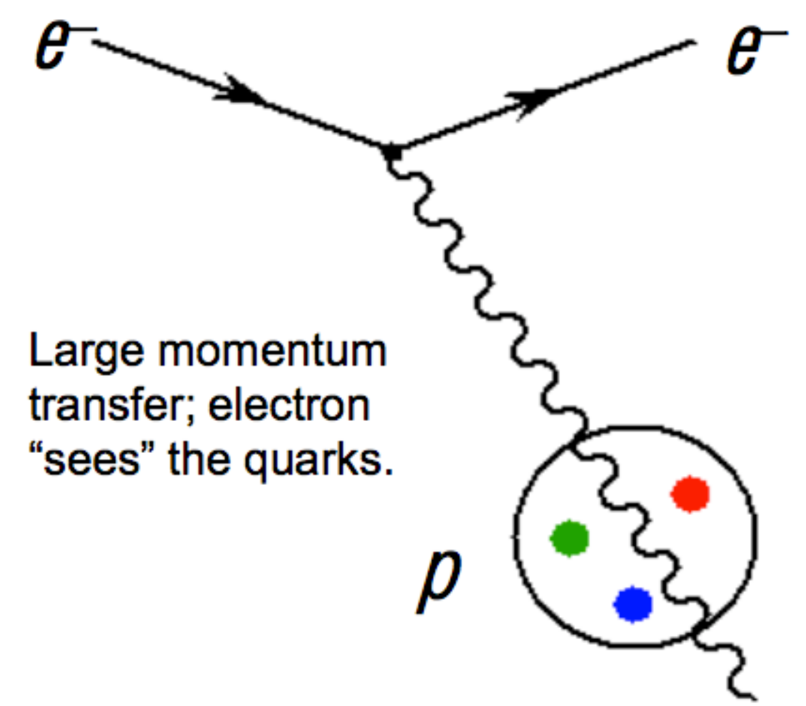
amplitude

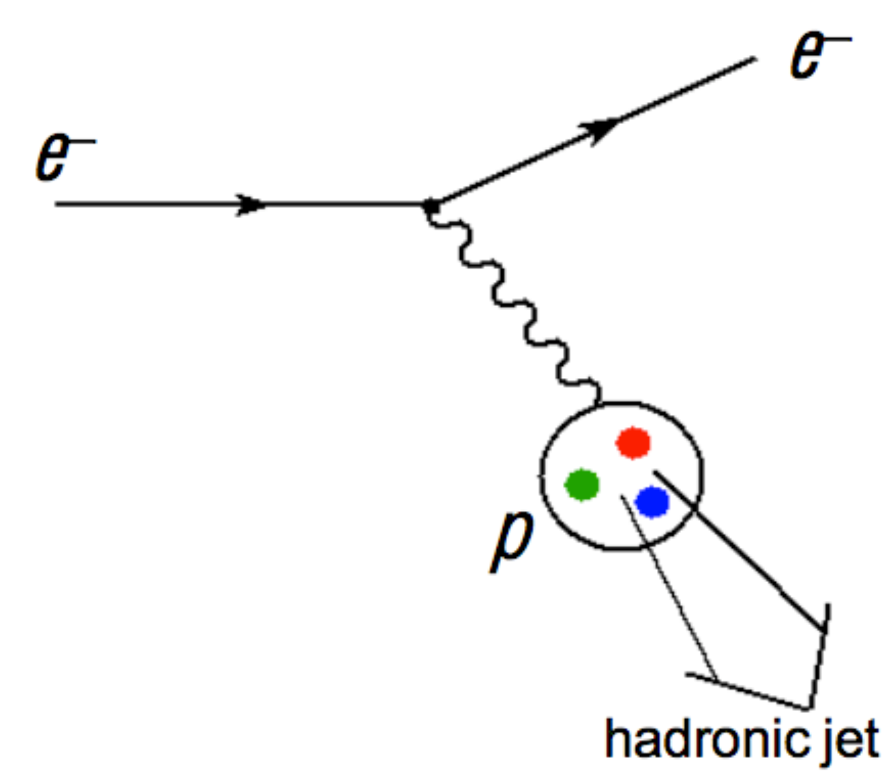
↓

$$\mathcal{M}_1 : \mathcal{M}_2 : \mathcal{M}_3$$
$$18769 : 137 : 1$$

When QED is not adequate

- When we go to higher energy e-p collisions, experiment shows that new particles start to form.
- This is so-called *inelastic scattering*, in which two colliding particles can form (hundreds of) new hadrons.
- QED cannot explain phenomena like inelastic scattering.
  - We need an additional theory of subnuclear interactions.





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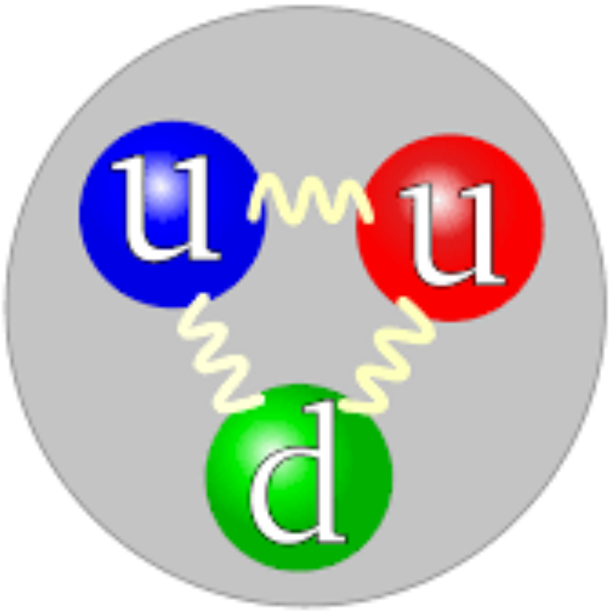
Quantum chromodynamics (QCD)

- QCD can explain many phenomena that QED fails to explain.
  - The binding of nucleons in atoms and the phenomena of inelastic scattering are both explained by a single field theory of quarks and gluons: QCD.
  - QCD describes the interactions between quarks (particles that carry "color" charge) via the exchange of massless gluons.
  - Note: the quark-gluon interactions are also responsible for the binding of quarks into the bound states that make up the hadron zoo (ρ's, η's, Λ, Ξ, Σ's, …).
  - Problem: QCD is conceptually similar to QED, but its calculations are even more complicated. We'll discuss why…

Quantum chromodynamics (QCD)

- Quarks and bound states:
  - Since quarks are spin-1/2 particles (fermions), they must obey the Pauli Exclusion Principle.
- Pauli Exclusion Principle: fermions in a bound state (e.g., the quarks inside a hadron) cannot share the same quantum numbers.

- **Question:** then, how can we squeeze three quarks into a baryon?
- **Answer:** let's give them an additional "charge", called color.
  - This removes the quantum numbers degeneracy.



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- Pauli Exclusion Principle: fermions in a bound state (e.g., the quarks inside a hadron) cannot share the same quantum numbers.

Proposal: quark color comes in three types: **red**, **green**, and **blue**;

- All free, observed particles are colorless.

Red, blue, and green combine to give white (color-neutral).

Quantum chromodynamics (QCD)

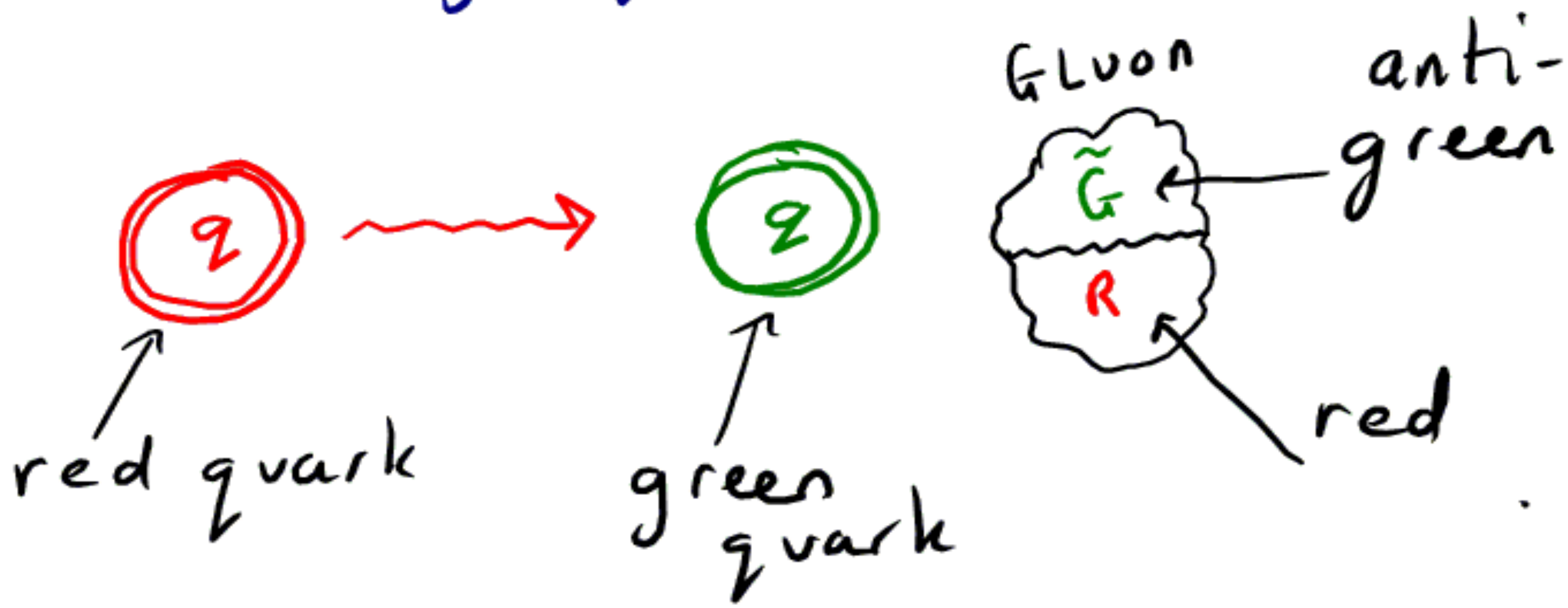
- Quarks and bound states:
  - Since quarks are spin-1/2 particles (fermions), they must obey the Pauli Exclusion Principle.
- Pauli Exclusion Principle: fermions in a bound state (e.g., the quarks inside a hadron) cannot share the same quantum numbers.

- What do the anti-colors look like?
- Red plus anti-red gives white, but combining red with blue and green gives white.
- Hence, anti-red is blue+green; similarly, anti-blue is red+green, and anti-green is red+blue.

Quantum chromodynamics (QCD)

- Gluons have both color and anti-color.
  - There are 9 possible combinations, but 1 is white, which is not allowed.
  - This leaves 8 types of gluon.

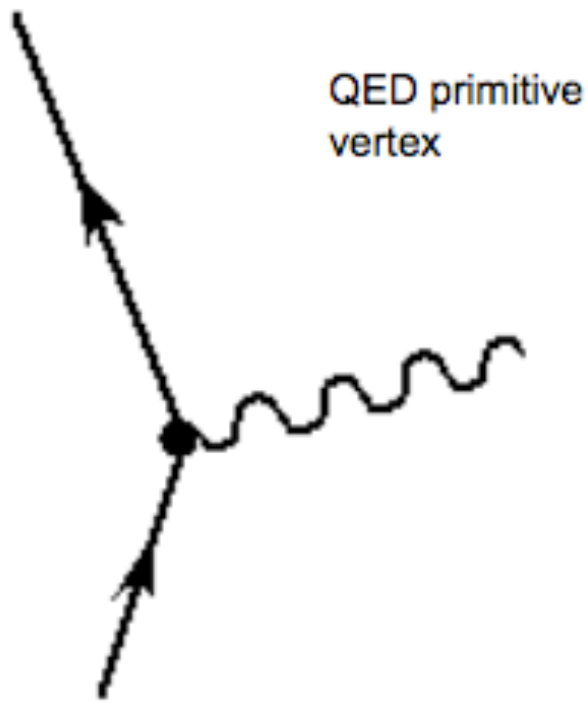
A quark changes color by emitting or absorbing gluons.

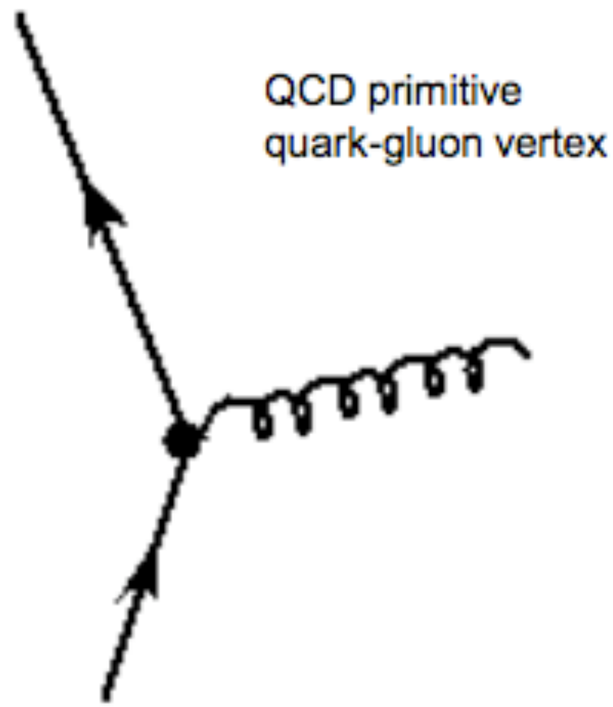


H. Fearn CSUF Physics

Quantum chromodynamics (QCD)

- Quarks are electrically charged, so they also interact via the EM force, exchanging photons.
- The strong interaction is gluon-mediated, but the Feynman diagram for the *quark-gluon vertex* looks just like the primitive QED vertex.

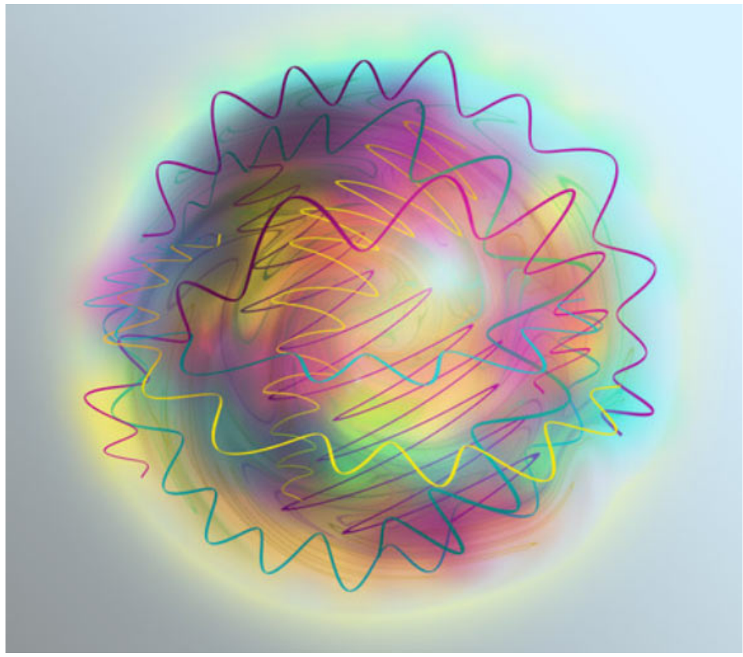




Quantum chromodynamics (QCD)

- Glueballs!

- Hypothetical composite particles, consisting solely of gluons, no quarks. Such a state is possible because gluons carry color charge and experience (strong) self-interaction.



TU Wien

QED vs QCD

- QCD is much harder to handle than QED.
- What makes it so difficult? Let's start with *perturbation theory*.

- Recall: In QED, each vertex contributes a coupling constant $\sqrt{\alpha}$, where α is a small dimensionless number:

$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

- Hence, we saw that higher-order diagrams (diagrams with more vertices) get suppressed relative to diagrams with less vertices, if α is small.

Aside: perturbation theory

- Use a power series in a parameter ε (such that ε << 1) - known as perturbation series - as an approximation to the full solution.
- For example:

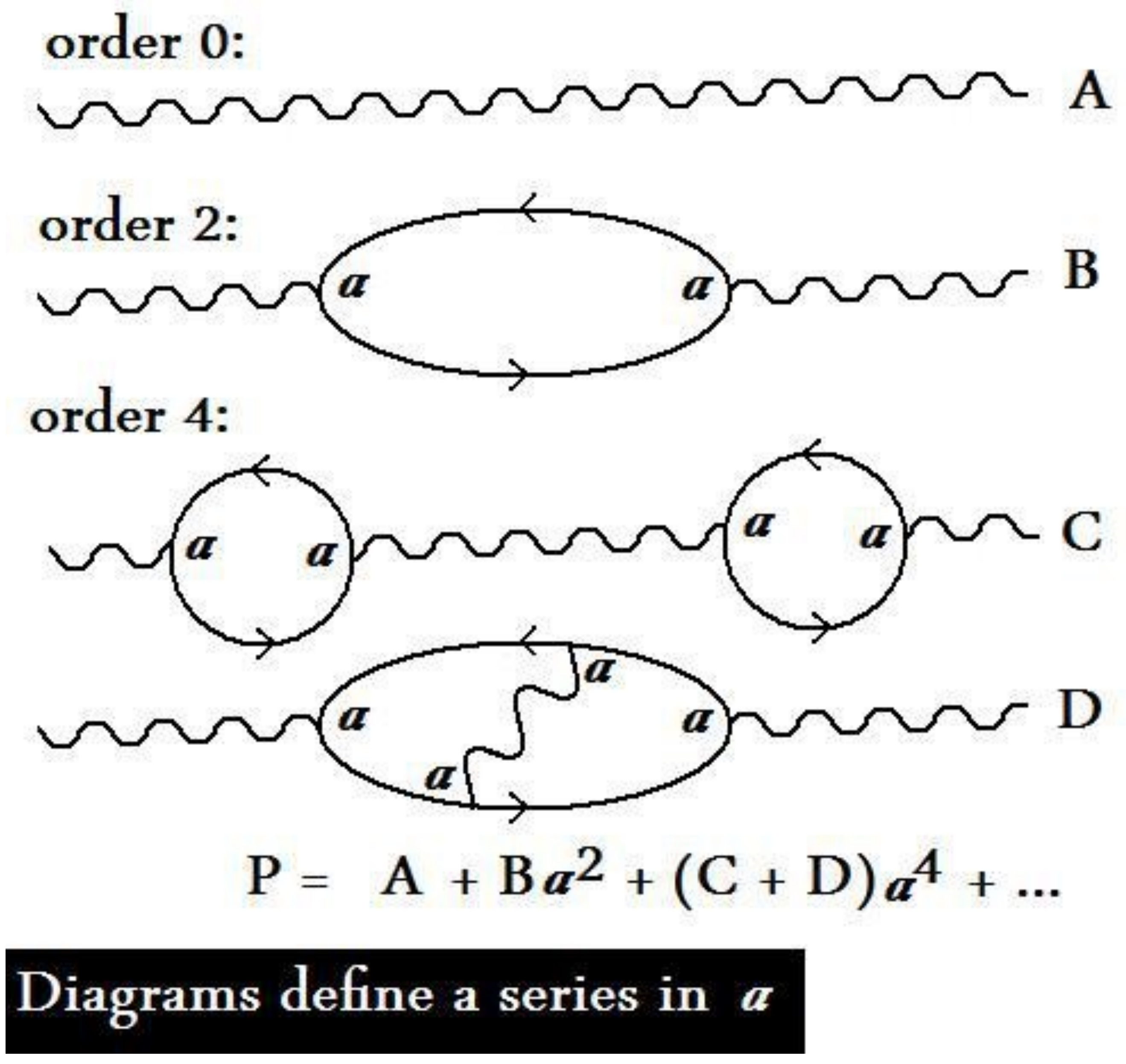
$$A = A_0 + \epsilon A_1 + \epsilon^2 A_2 + \ldots$$

- In this example, A_0 is the "leading order" solution, while A_1, A_2, … represent higher-order terms.
- **Note:** if ε is less than 1, the higher-order terms in the series become successively smaller.
- Approximation:

$$A \approx A_0 + \epsilon A_1$$

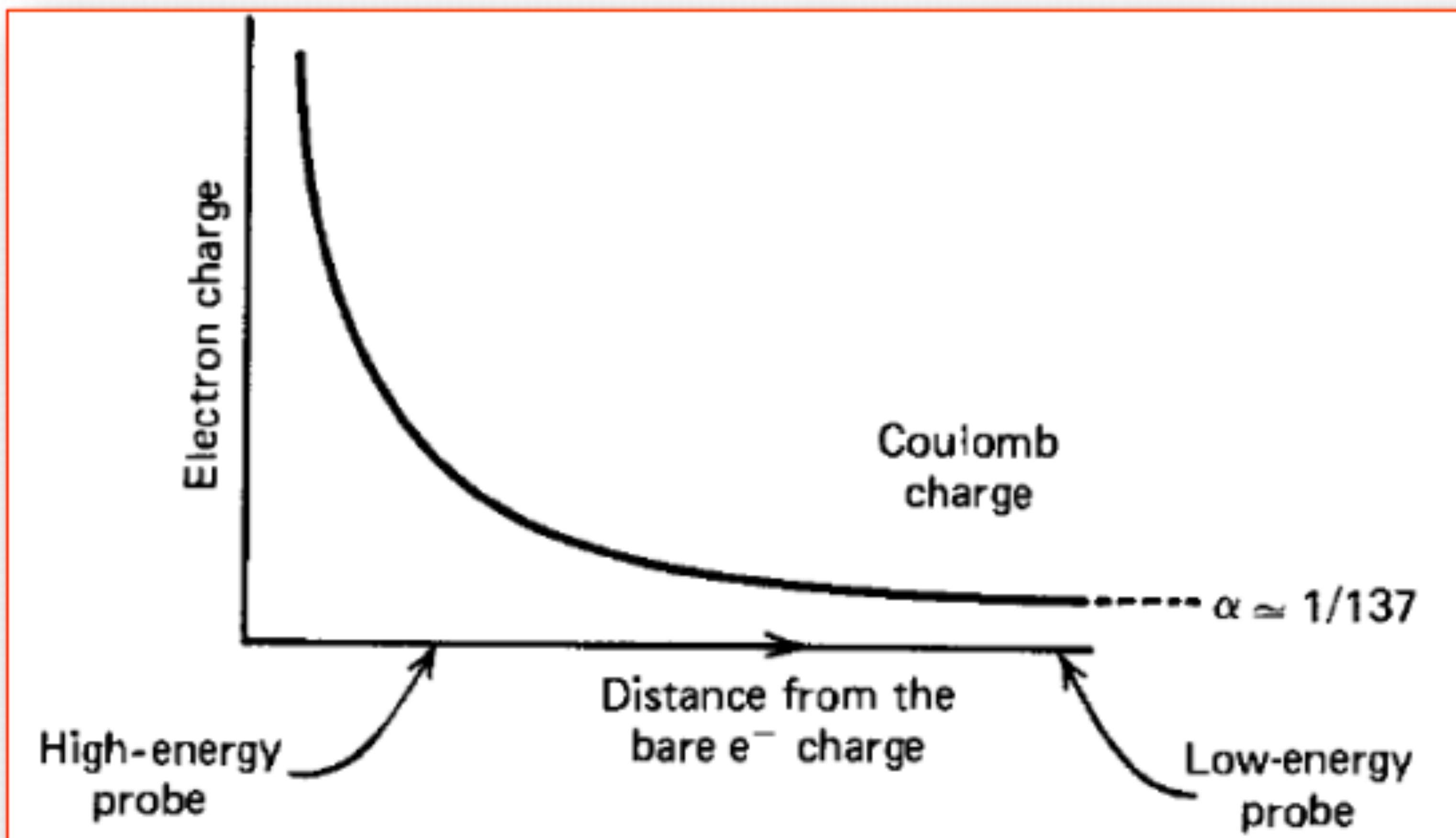
Aside: perturbation theory in QFT

- Perturbation theory allows for well-defined predictions in quantum field theories (as long as they obey certain requirements).
- QED is one of those theories.
- Feynman diagrams correspond to the terms in the perturbation series!



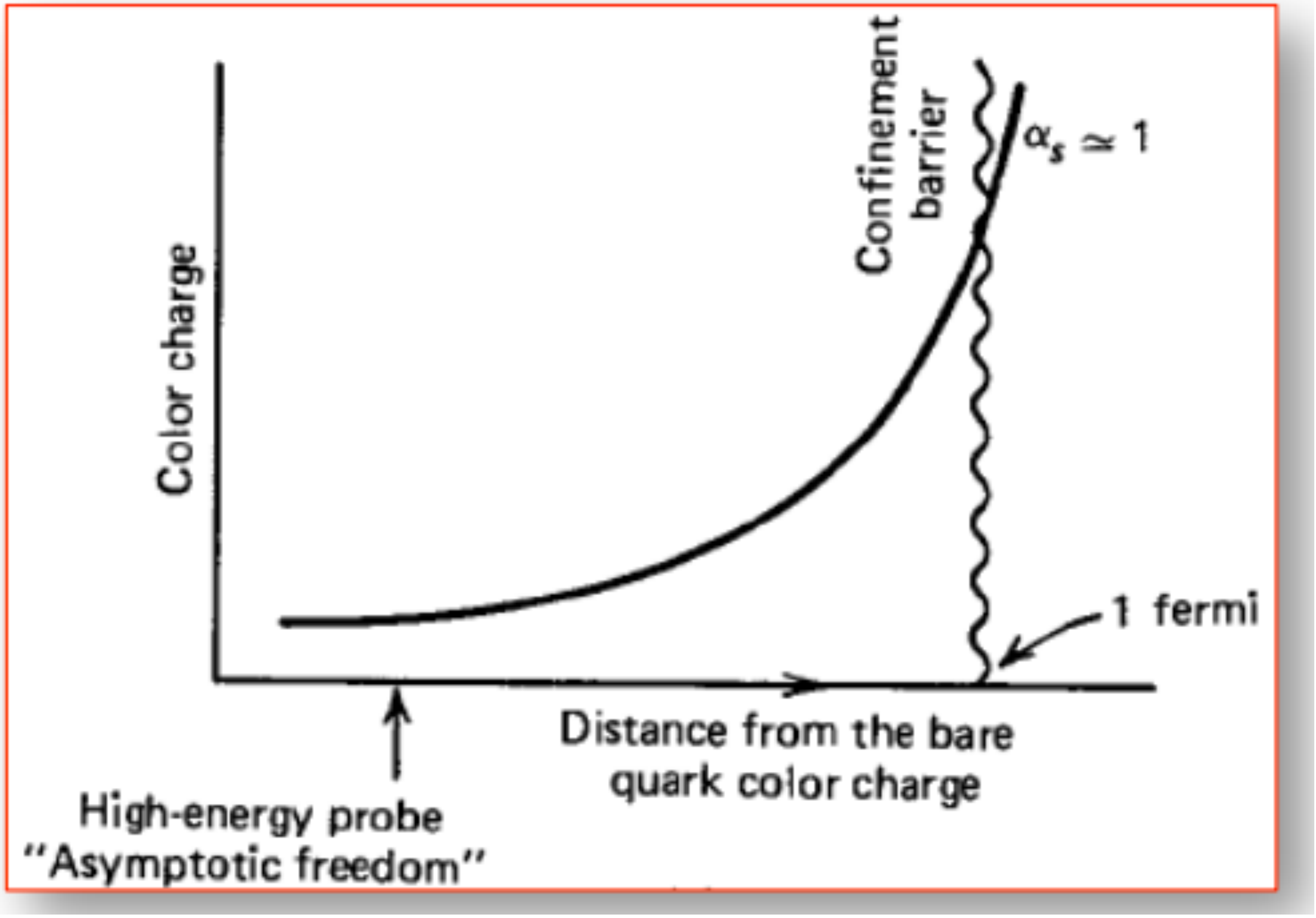
QED vs QCD

- Recall: In QED, each vertex contributes a coupling constant $\sqrt{\alpha}$.
- α is not exactly a constant though… it "runs" with the scale of the interaction.



QED vs QCD

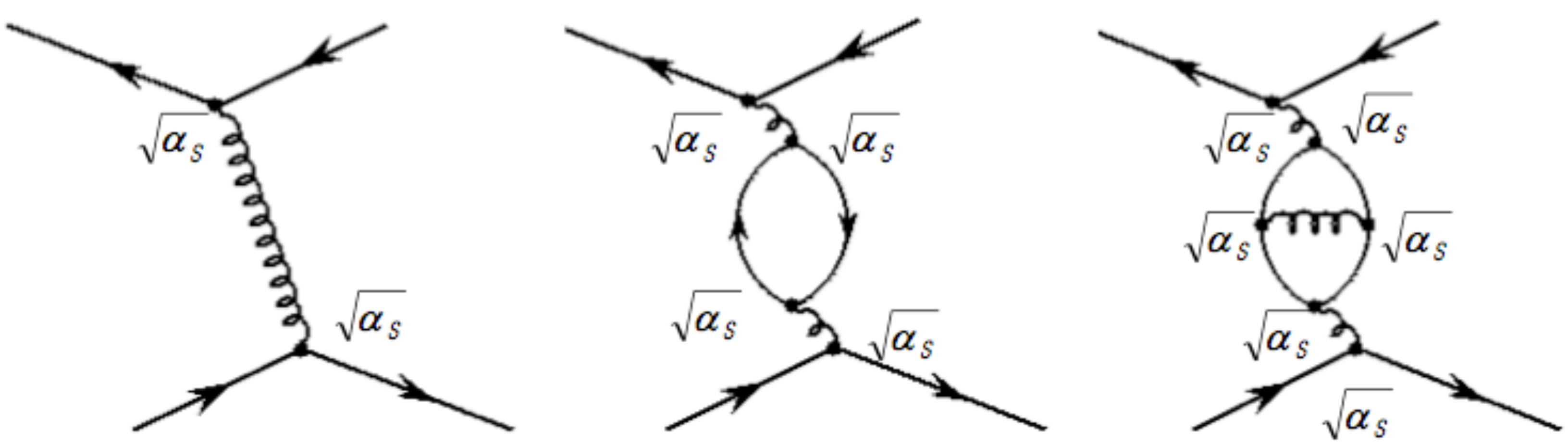
- The coupling constant for QCD, α_s, "runs" in a different way with energy.



QED vs QCD

- In QCD, the coupling between quarks and gluons, given by the number α_s, is **much larger** than 1/137 at low energies.
- In fact, at low energies, $\alpha_s >> 1$, making higher-order diagrams just as important as those with fewer vertices!
- This means we can't truncate the sum over diagrams.
  - Calculations quickly become complex!

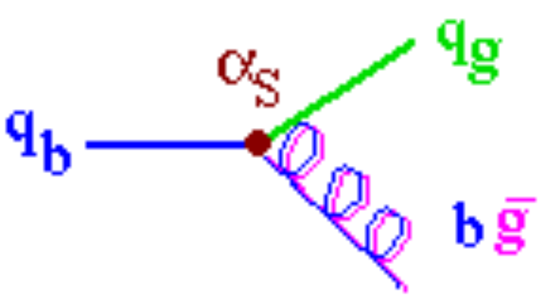
$\mathcal{M}_1 \propto \alpha_S \approx 1$ $\mathcal{M}_2 \propto \alpha_S{}^2 \approx 1$ $\mathcal{M}_3 \propto \alpha_S{}^3 \approx 1$

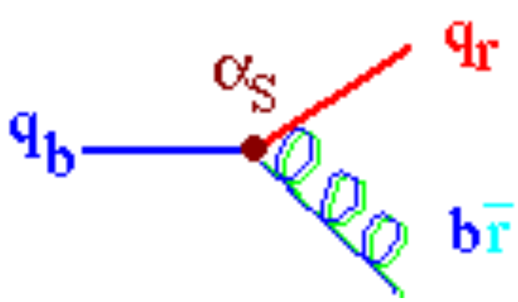


$$\mathcal{M}_1 : \mathcal{M}_2 : \mathcal{M}_3$$
$$1 : 1 : 1$$

Another complication: gluon color

- Typically, quark color changes at a quark-gluon vertex.
- In order to allow this, the gluons have to carry off "excess" color.
- Color is conserved at the vertex, like electric charge is conserved in QED.





Color, like electric charge, must be conserved at every vertex. This means that the gluons cannot be color-neutral, but in fact carry some color charge. It turns out that there are 8 distinct color combinations!

- Gluons themselves are <u>not</u> color-neutral. That's why we don't usually observe them outside the nucleus, where only colorless particles exist.
- Hence, the **strong force is short-range.**

Gluon confinement

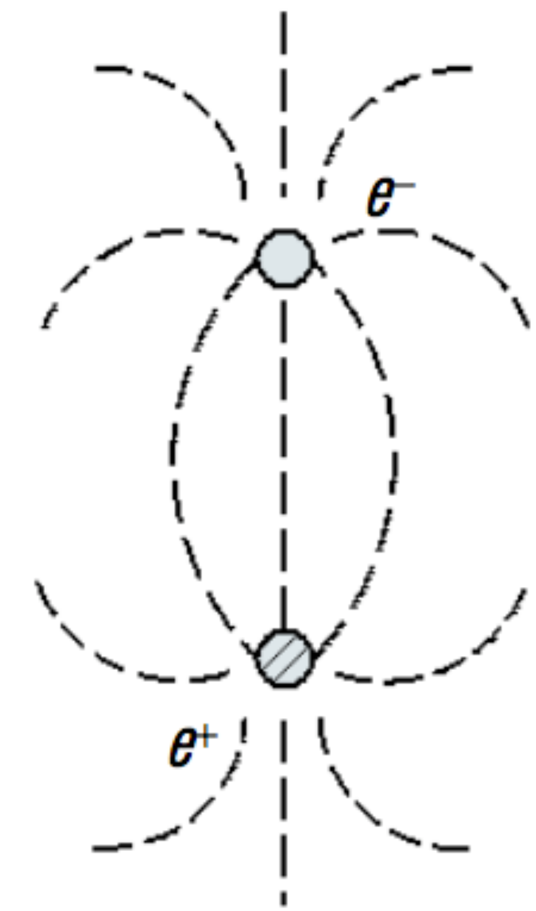
- ***Confinement*** is the formal name for what we just discussed.
- The long-range interactions between gluons are theoretically unmanageable. The math is very complicated and riddled throughout with infinities.
- If we assume the massless gluons have infinite range, we find that an infinite amount of energy would be associated with these self-interacting long-range fields.
- Solution:
  - Assume that any physical particle must be colorless: there can be no long-range gluons.

Quark confinement

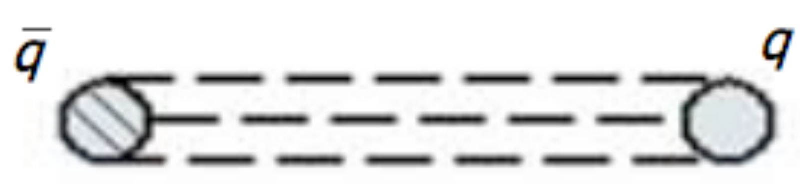
- Confinement also applies to quarks. All bound states of quarks must have a color combination such that they are *white*, or *colorless*.
  - Protons, neutrons, and other baryons are bound states of three quarks of different colors.
  - The mesons are composed of a quark-antiquark pair; these should have opposite colors (e.g. red and anti-red).
- As a consequence of confinement, one cannot remove just one quark from a proton, as that would create two "colorful" systems.
  - We would need an infinite amount of energy to effect such separation!
- Hence, the quarks are confined to a small region (<1 fm) near one another.

Understanding confinement

- The mathematics of confinement are complex, but we can understand them in terms of a very simple picture.
- Recall, the Coulomb field between a e^+e^- pair looks like $V(r) \sim 1/r$.
  - As we pull the pair apart, the attraction weakens.
- Imagine the color field between a quark-antiquark pair like Hooke's Law: $V(r) \sim r$.
  - As we pull the pair apart, the attraction between them increases.
- So, separating two quarks by a large r puts a huge amount of energy into the color field: $V(r) \rightarrow \infty$



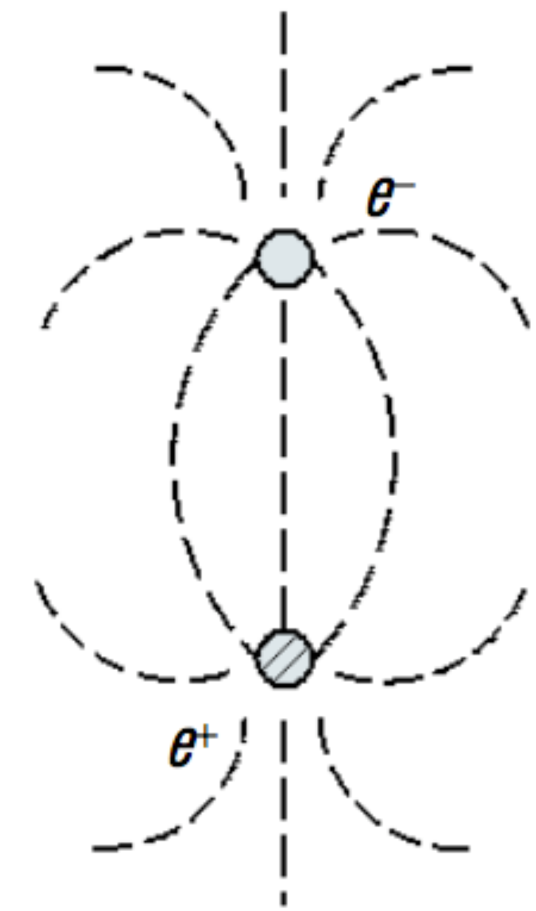
Dipole field for the Coulomb force between opposite electrical charges.



Dipole field between opposite-color quarks.

Understanding confinement

- How do we understand this picture?
- When a quark and anti-quark separate, their color interaction strengthens (more gluons appear in the color field).
- Through the interaction of the gluons with each other, the color lines of force are squeezed into a tube-like region.
- Contrast this with the Coulomb field: nothing prevents the lines of force from spreading out.
  - <u>There is no self-coupling of photons to contain them</u>.
- If the color tube has constant energy density per unit length k, the potential energy between quark and antiquark will increase with separation, V(r)~kr.



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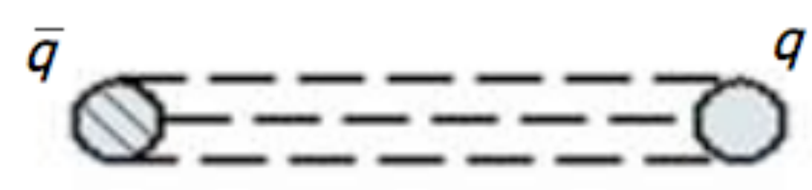
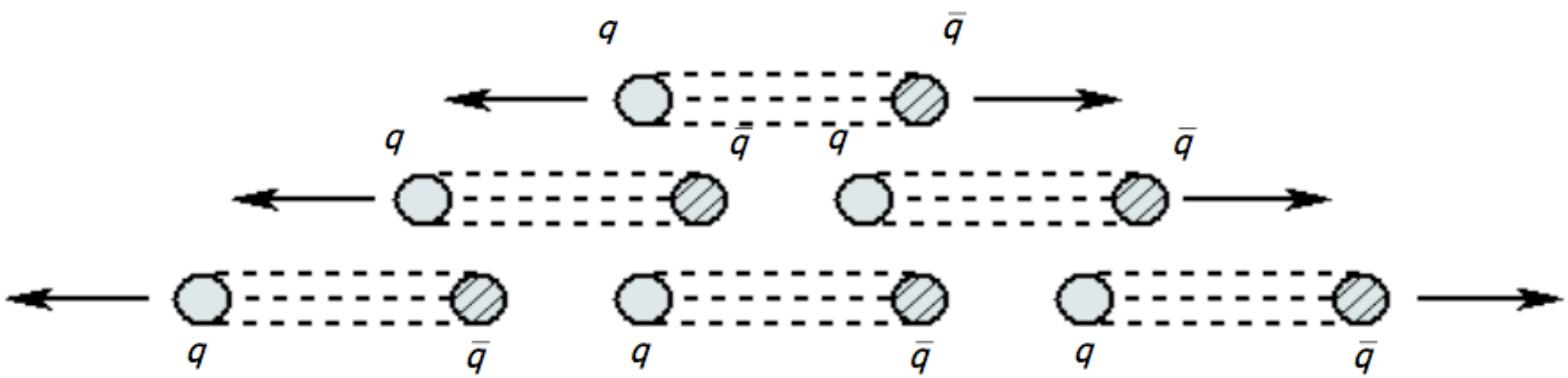


Dipole field between opposite-color quarks.

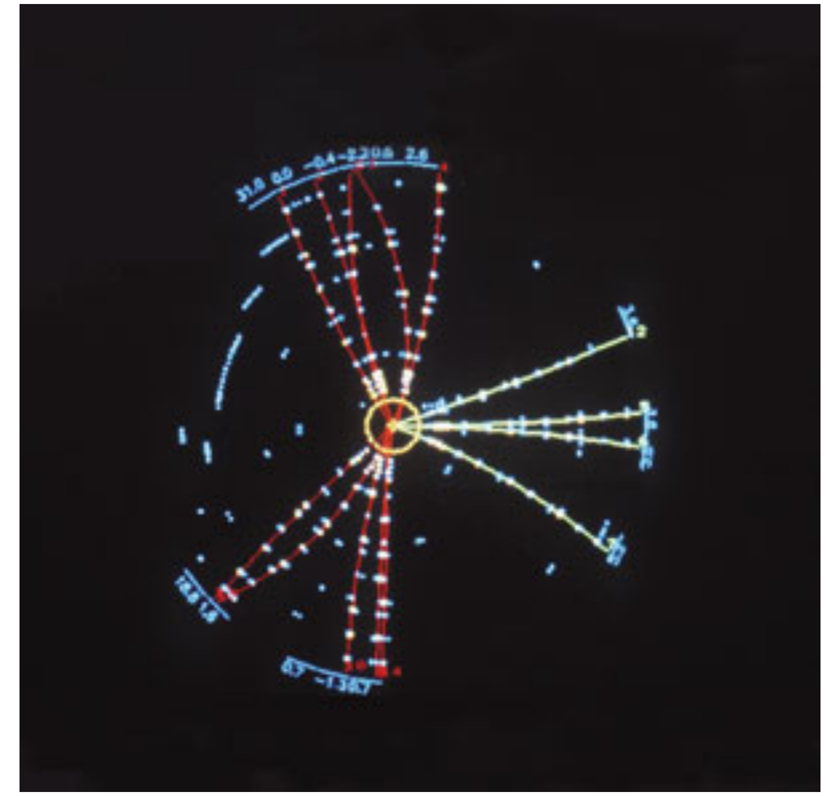
Color lines and hadron production

- Why you can't get free quarks:
  - Suppose we have a meson and we try to pull it apart. The potential energy in the quark-antiquark color field starts to increase.
  - Eventually, the energy in the gluon field gets big enough that the gluons can pair-produce another quark-antiquark pair.
    - The new quarks pair up with the original quarks to form mesons, and thus our four quarks remain confined in colorless states.
  - Experimentally, we see two particles!

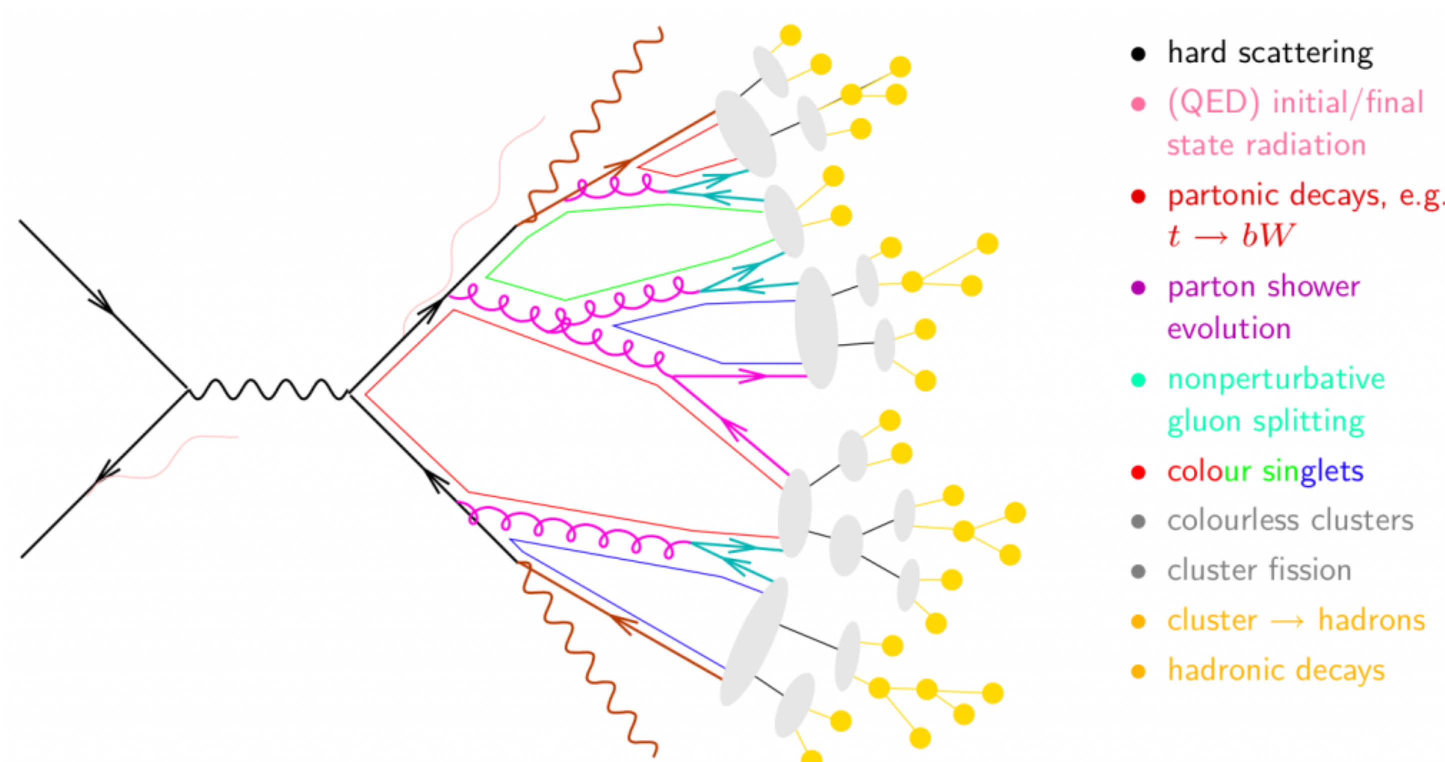


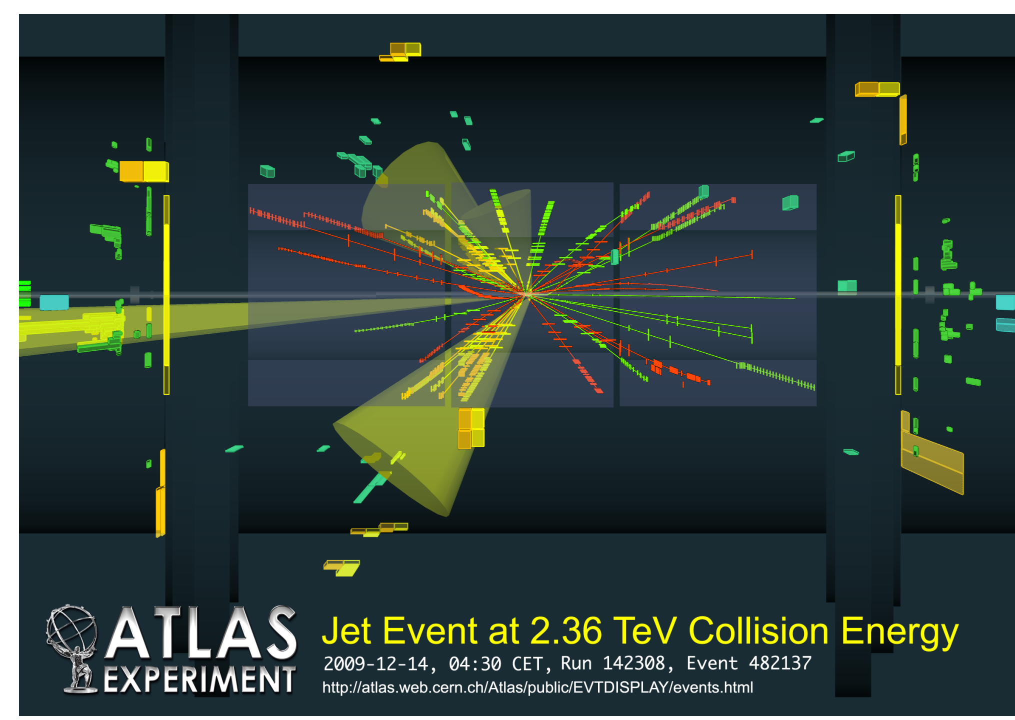
Hadronic jets

- The process just described is observed experimentally in the form of hadron jets.
- In a collider experiment, two particles annihilate and form a quark-antiquark pair.
- As the quarks move apart, the color lines of force are stretched until the potential energy can create another quark-antiquark pair.



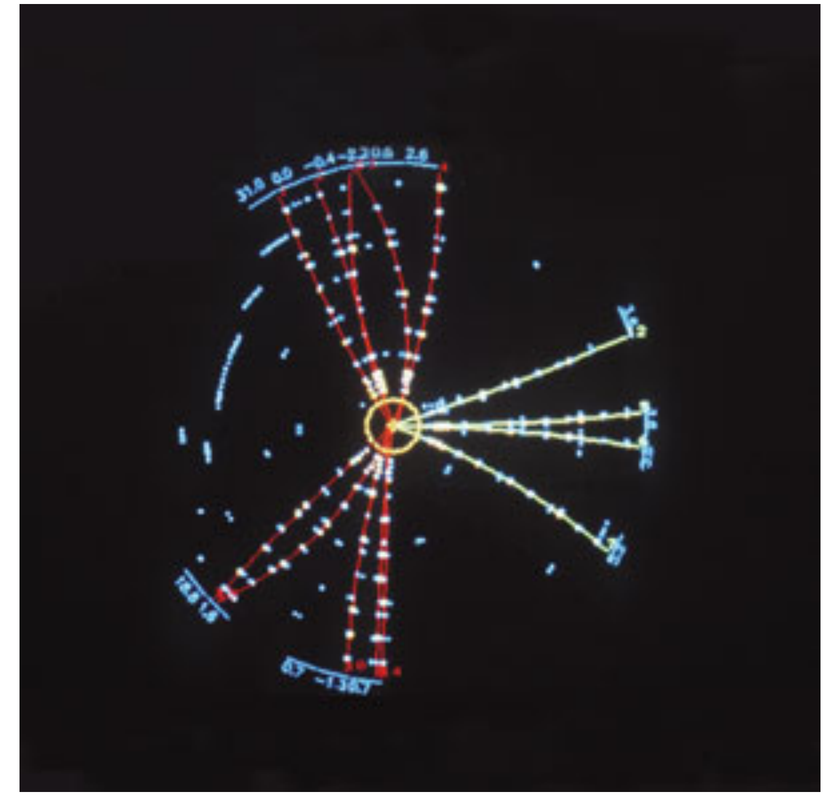
Jet formation at TASSO detector at PETRA



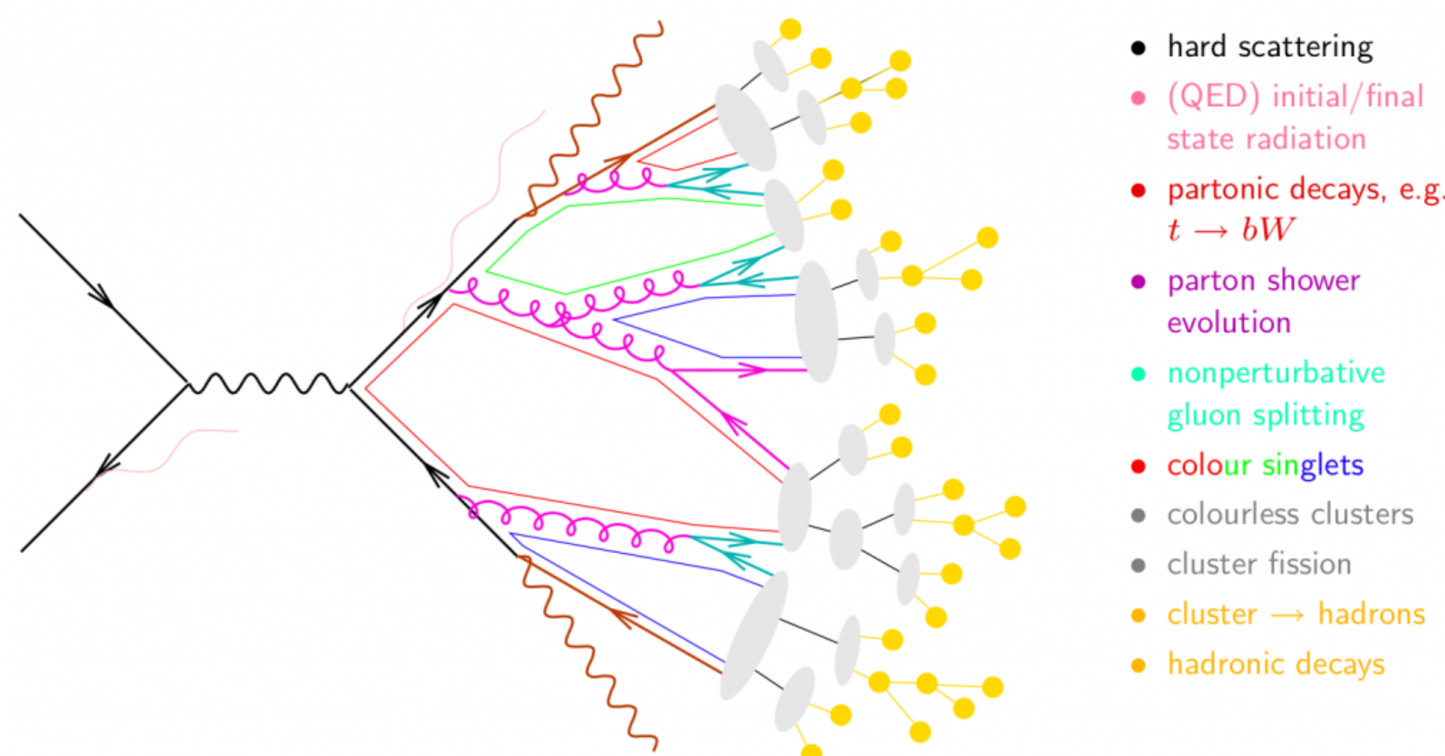


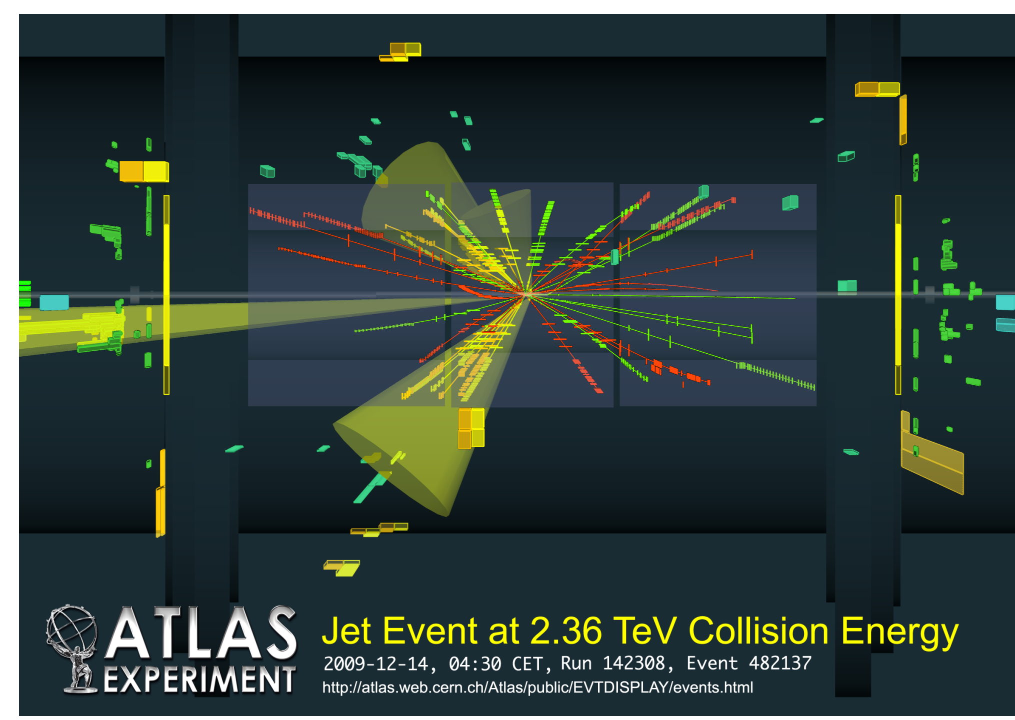
Hadronic jets

- This process continues until the quarks' kinetic energy have totally degraded into cluster of quarks and gluons with zero net color.
- The experimentalist then detects several "jets" of hadrons, but never sees free quarks or gluons.
- https://youtu.be/FMH3T05G_to



Jet formation at TASSO detector at PETRA





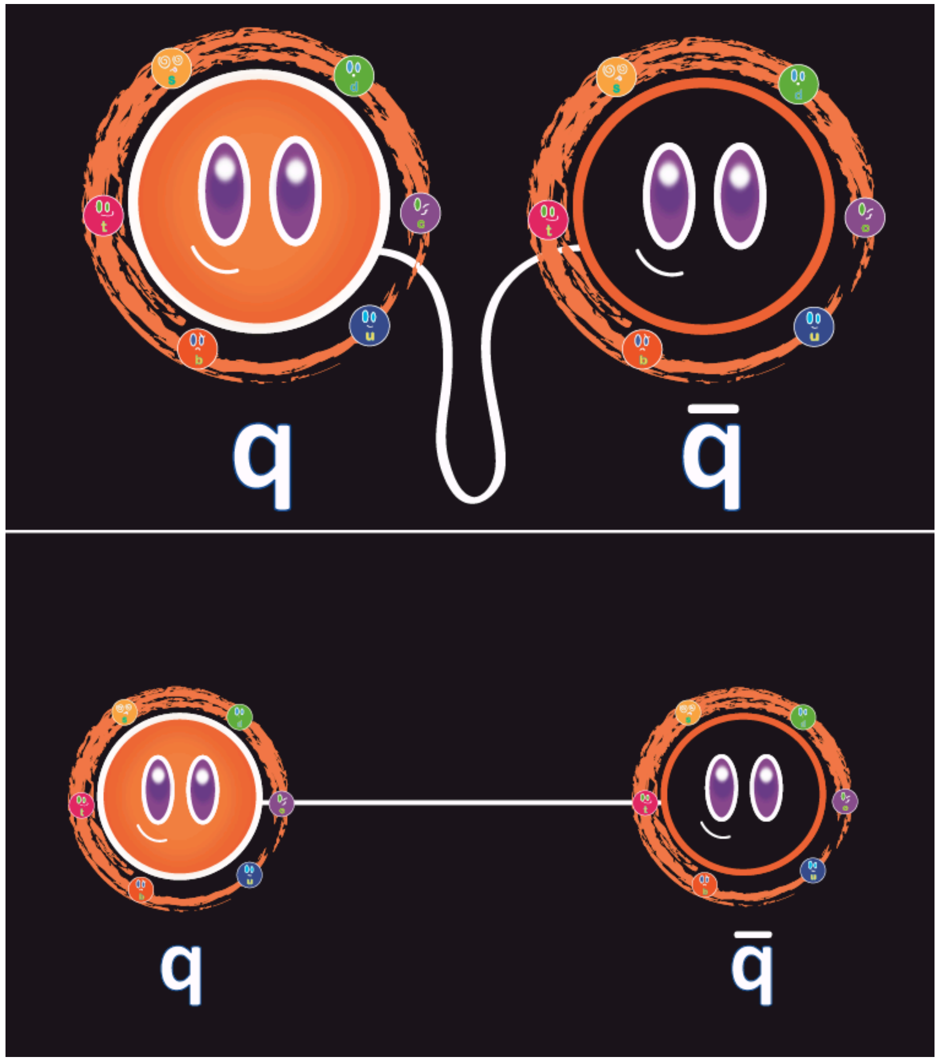
Asymptotic freedom

- As mentioned earlier, perturbation theory can only be applied when the coupling constant α is small.
- At these lower energy regimes of jet formation, α_s is of the order of unity, and that means we can’t ignore the many-vertex Feynman diagrams as we do in QED (we can’t treat QCD perturbatively!).
- However, as we already saw, the coupling “constant" is actually not a constant at all, and depends on the energy of the interaction.
- As the energy increases, the coupling constant becomes smaller.
  - In fact, at high enough energies, α_s gets so small, that QCD can be dealt with as a perturbative theory (e.g. LHC high-energy collisions!)

Asymptotic freedom

- **Asymptotic freedom:** as the energy of interactions goes up, QCD asymptotically approaches a regime in which quarks act like free particles.
  - Looking at quarks with a very high energy probe.

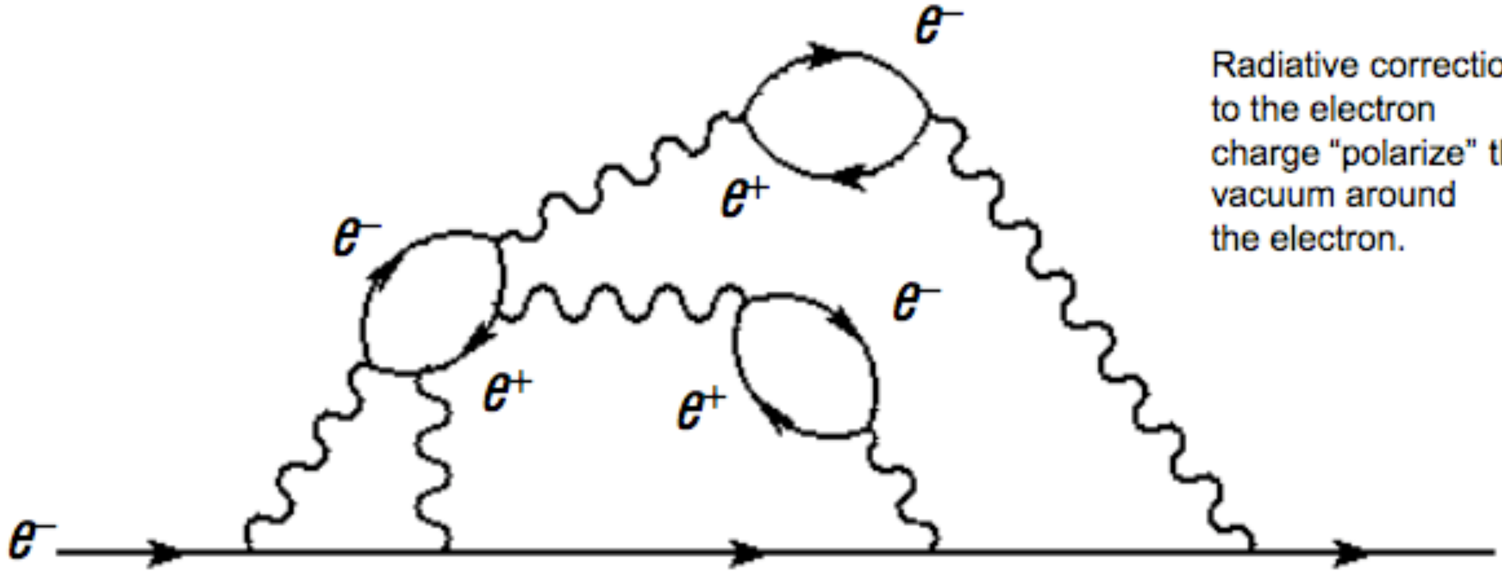
- D. Gross, H. Politzer, F. Wilczek (1970's): asymptotic freedom suggests that QCD can be a valid theory of the strong force.
- Nobel Prize (2004): "for the discovery of asymptotic freedom in the theory of strong interactions."



PhysicsAndMor

Back to QED: Polarization of the vacuum

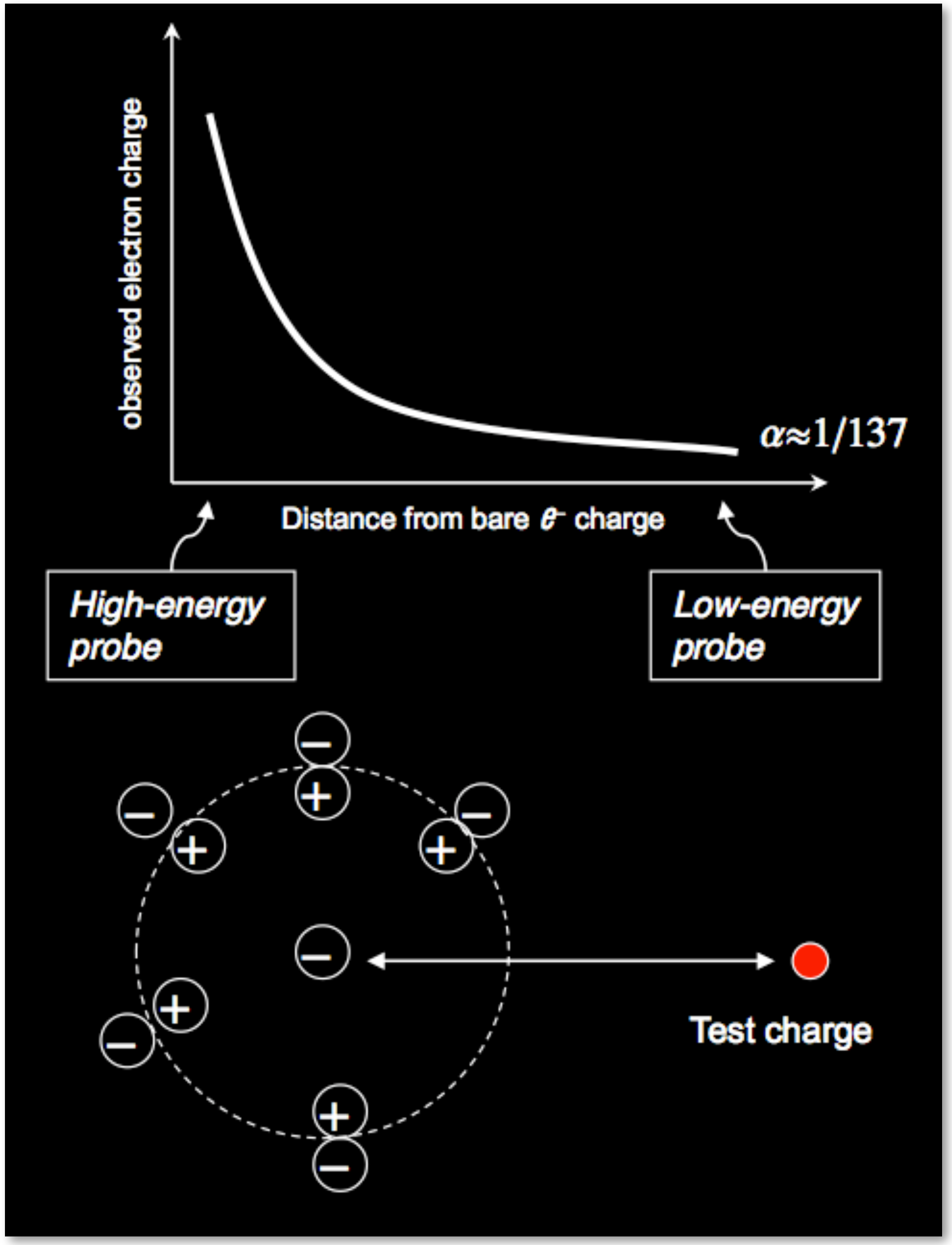
- The vacuum around a moving electron can fill up with virtual e^+e^- pairs.
  - This is a purely a quantum effect, and is allowed by Heisenberg's Uncertainty Principle.
- Because opposite charges attract, the virtual positrons in the e^+e^- loops will move closer to the electron.
- Therefore, the vacuum around the electron becomes polarized (a net electric dipole develops), just like a dielectric inside a capacitor can become polarized.



Radiative corrections to the electron charge "polarize" the vacuum around the electron.

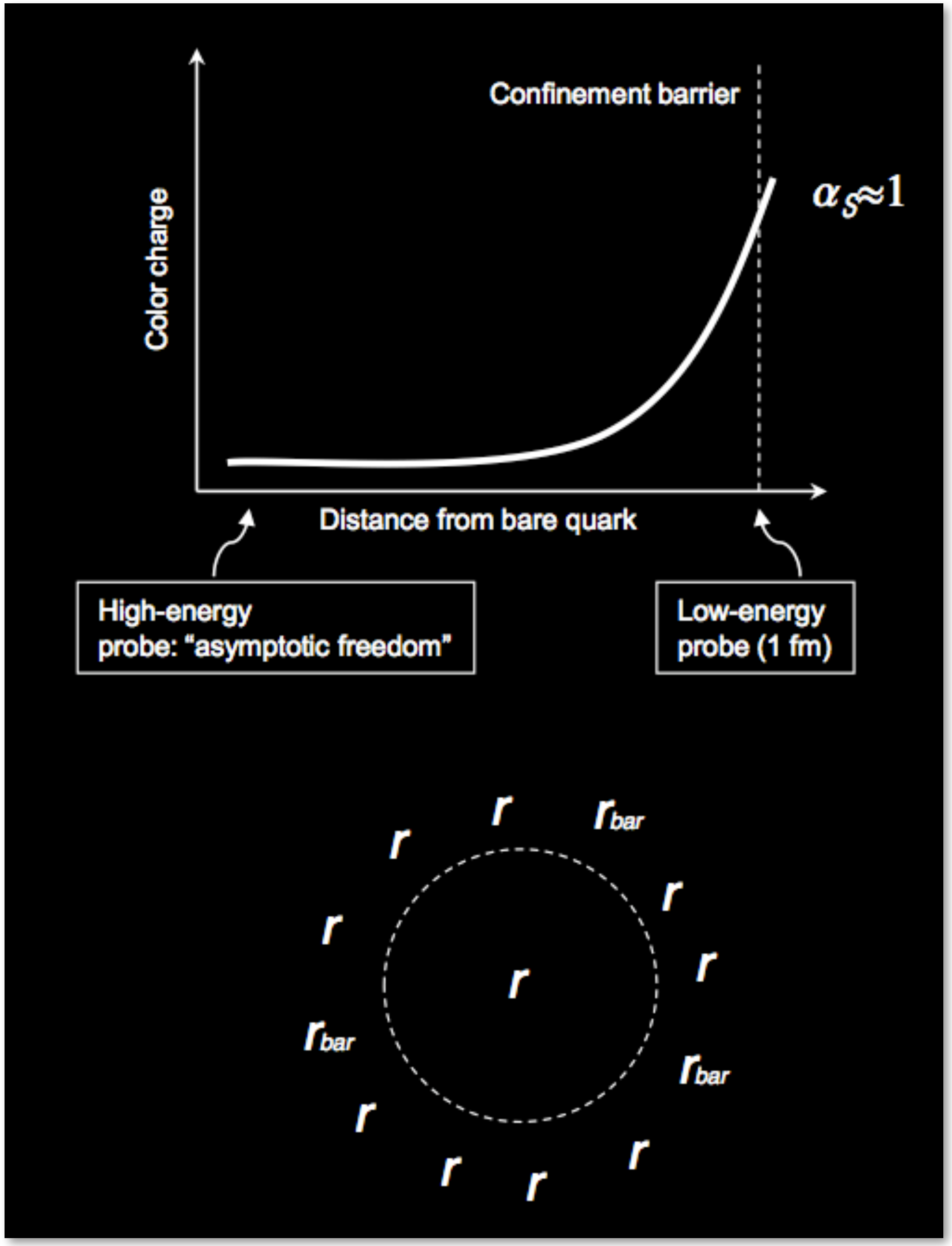
QED: Charge screening

- Now, suppose we want to measure the charge of the electron by observing the Coulomb force experienced by a test charge.
- Far away from the electron, its charge is screened by a cloud of virtual positrons, so the effective (observed) charge is smaller than its bare charge.
- As we move closer in, fewer positrons are blocking our line of sight to the electron.
- Hence, with decreasing distance, the effective charge of the electron increases.
- We can think of this as α increasing with energy.



QCD: Color antiscreening

- In QCD, the additional gluon loop diagrams reverse the result of QED:
  - A red charge is preferentially surrounded by other red charges.
- By moving the test probe closer to the original quark, the probe penetrates a sphere of mostly red charge, and the measured red charge decreases.
- This is "antiscreening".
- We can think of this as α_s decreasing with energy.



Running constants

- As we probe an electron at increasingly higher energies, its effective charge appears to increase.
- This can be rephrased in the following way: as interactions increase in energy, the QED coupling strength α between charges and photons also increases.
  - This should not really be a surprise; after all, the coupling strength of EM depends directly on the electron charge.
- Since α is not a constant, but a (slowly-varying) function of energy, it is called a **running coupling constant**.
- In QCD, the net effect is that the quark color charge and α_s decrease as the interaction energy goes up.

Underlying source: self-interactions of mediators

- Gluon self-interaction!
- W and Z (weak force mediators) also self-interact.
  - Similar behavior.
  - The weak coupling constant also decreases as the energy scale goes up.

- Historical background (see lecture 2)
- SM particle content
- SM particle dynamics
  - Quantum Electrodynamics (QED)
  - Quantum Chromodynamics (QCD)
  - **Weak Interactions**
  - Force Unification
- Lagrangian / Field formulation
- Tests and predictions
- Higgs mechanism and Higgs boson discovery

Weak interactions

- We can evaluate the strength of the weak interaction in comparison to the strong and EM coupling constants.

Interaction	Mediator	Strength
Strong	gluon	1
Electromagnetic	photon	10^{-2}
Weak	$W^{\pm}$, Z^0	10^{-7}
Gravity	graviton (?)	10^{-39}

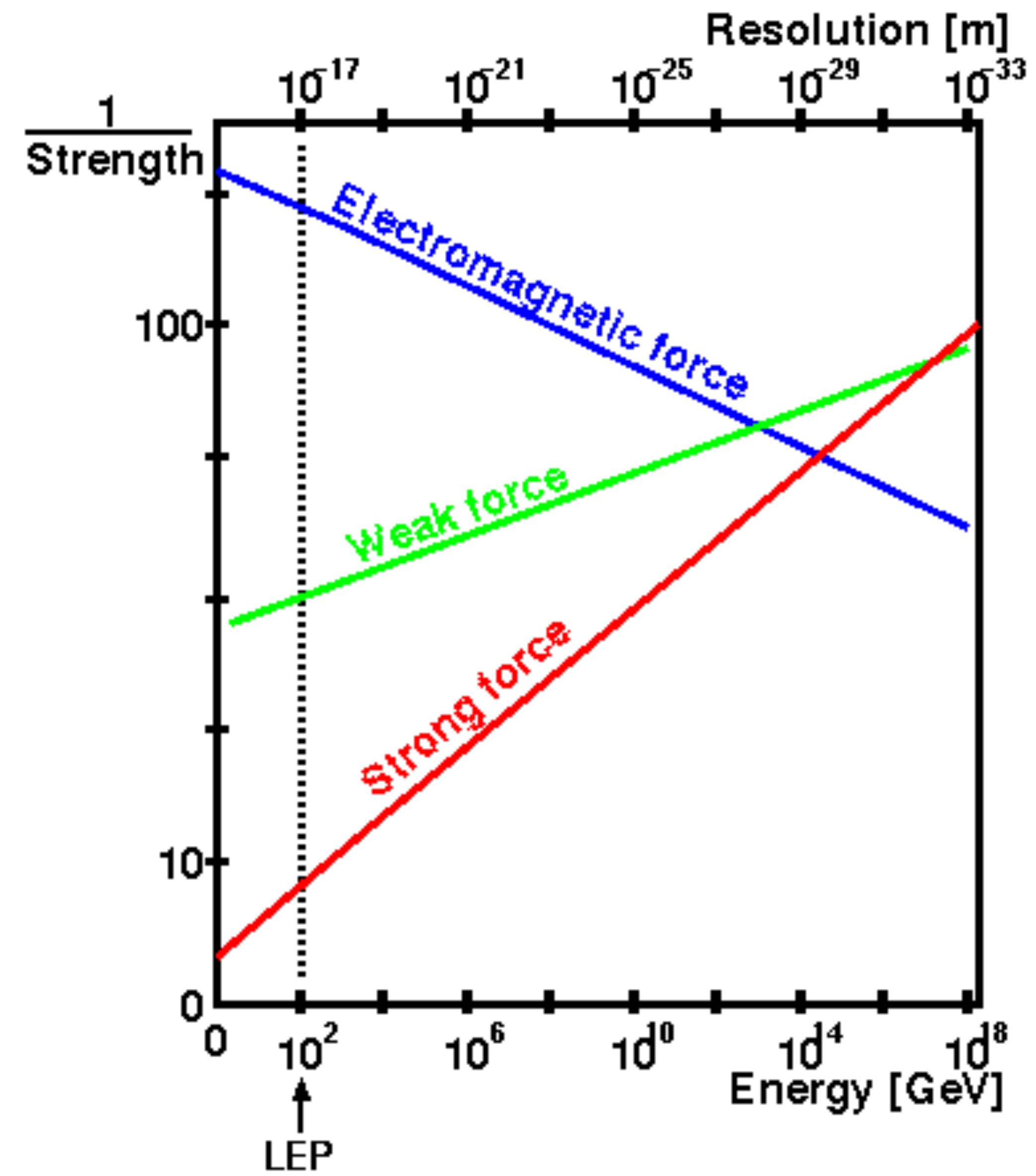
- It has an incredibly short range, due to the large mass of their mediators (80-100 GeV).

Weak interactions

- At low energies, the effective weak coupling strength is 1000 times smaller than the EM force.
- As interaction energies start to approach the mass-energy of the W and Z particles (~100 GeV), the effective coupling rapidly approaches the intrinsic strength of the weak interaction α_W.
    - At these energies, the weak interaction actually dominates EM.
- Beyond that, the effective weak coupling starts to decrease.

Aside: Force Unification

- At laboratory energies near M_W ~ O(100) GeV, the measured values of $1/\alpha$ are rather different.
- However, their energy dependences suggest that they approach a common value near 10^{16} GeV.
- This is an insanely high energy!
- The SM provides no explanation for what may happen beyond this unification scale, nor why the forces have such different strengths at low energies.



- Historical background (see lecture 2)
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    - Quantum Electrodynamics (QED)
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Particle/Field formulation

- In particle physics, we define fields like $\phi(x,t)$ at every point in spacetime.
- These fields don't just sit there; they fluctuate harmonically about some minimum energy state.
- The oscillations combine to form wave packets.
- The wave packets move around in the field and interact with each other. We interpret them as elementary particles.
- Terminology: the wave packets are called the quanta of the field $\phi(x,t)$.

Particle/Field formulation

- The Higgs mechanism is described in terms of the Lagrangian of the Standard Model. In quantum mechanics, single particles are described by wavefunctions that satisfy the appropriate wave equation.
- In Quantum Field Theory (QFT), *particles* are described by excitations of a quantum field that satisfies the appropriate quantum mechanical field equations.
- The dynamics of a quantum field theory can be expressed in terms of the Lagrangian density. Lagrangian formalism is necessary for the discussion of the Higgs mechanism.

Lagrangian Mechanics

- Developed by Euler, Lagrange, and others during the mid-1700's.
- This is an energy-based theory that is equivalent to Newtonian Mechanics (a force-based theory, if you like).

- **Lagrangian:** equation which allows us to infer the dynamics of a system.

Lagrangian Mechanics; Euler-Lagrange Equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0. \qquad (17.2)$$

For example, consider a particle moving in one dimension where the Lagrangian is a function of the coordinate x and its time derivative $\dot{x}$, with

$$L = T - V = \tfrac{1}{2}m\dot{x}^2 - V(x).$$

The derivatives of the Lagrangian with respect to x and $\dot{x}$ are

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} \quad \text{and} \quad \frac{\partial L}{\partial x} = -\frac{\partial V}{\partial x},$$

and the Euler–Lagrange equation (17.2) for the coordinate $q_i = x$ is simply

$$m\ddot{x} = -\frac{\partial V(x)}{\partial x}.$$

Since the derivative of the potential gives the force, this is equivalent to $F = m\ddot{x}$ and Newton's second law of motion is recovered.

Lagrangian Mechanics

$$L\left(q_i, \frac{\mathrm{d}q_i}{\mathrm{d}t}\right) \rightarrow \mathcal{L}\left(\phi_i, \partial_\mu \phi_i\right).$$

In the Lagrangian density, the generalised coordinates q_i are replaced by the *fields* $\phi_i(t, x, y, z)$, and the time derivatives of the generalised coordinates $\dot{q}_i$ are replaced by the derivatives of the fields with respect to each of the four space-time coordinates,

$$\partial_\mu \phi_i \equiv \frac{\partial \phi_i}{\partial x^\mu}.$$

The fields are continuous functions of the space-time coordinates x^μ and the Lagrangian L itself is given by

$$L = \int \mathcal{L}\, \mathrm{d}^3\mathbf{x}.$$

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)}\right) - \frac{\partial \mathcal{L}}{\partial \phi_i} = 0.$$

Particle/Field formulation

- How do we describe interactions and fields mathematically?
- Classically,

Lagrangian L = kinetic energy - potential energy

L = T - V

- Particle physics:
  - Same concept, using Dirac equation to describe free spin-1/2 particles:

$$L = \bar{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi$$

field

Ψ = wavefunction

m = mass

γ^{μ} = μ^{th} gamma matrix

∂_{μ} = partial derivative

Standard Model Lagrangian

Free Fields

Interaction

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

H.-C. Schultz-Coulon

Standard Model Lagrangian

Free Fields

Interaction

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

$L = \bar{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi$

from previous slide,
but with m=0
(massless fermions)

Gauge Bosons

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi$$

Fermions

$$\mathcal{L}' = e\bar{\psi}\gamma^{\mu}A_{\mu}\psi$$

Fermion-Boson
Coupling

$$eA_{\mu} = \frac{g_s}{2}\lambda_{\nu}G^{\nu}_{\mu} + \frac{g}{2}\vec{\tau}\,\vec{W}_{\mu} + \frac{g'}{2}YB_{\mu}$$

$$F_{\mu\nu}F^{\mu\nu} = G_{\mu\nu}G^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}$$

H.-C. Schultz-Coulon

Standard Model Lagrangian

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Fermions

$$\mathcal{L}' = e\bar{\psi}\gamma^\mu A_\mu \psi$$

Fermion-Boson Coupling

strong interaction

electro-weak interactions

$$eA_\mu = \frac{g_s}{2}\lambda_\nu G_\mu^\nu + \frac{g}{2}\vec{\tau}\,\vec{W}_\mu + \frac{g'}{2} Y B_\mu$$

$$F_{\mu\nu}F^{\mu\nu} = G_{\mu\nu}G^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}$$

gluons

combinations give W, Z, γ…

H.-C. Schultz-Coulon

Understanding the phase

- You may not have seen numbers like $\mathbf{e^{i\theta}}$, so let's review.
- Basically, $e^{i\theta}$ is just a fancy way of writing sinusoidal functions; from Euler's famous formula:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

- Note: those of you familiar with complex numbers (of the form $z=x+iy$) know that $e^{i\theta}$ is the phase of the so-called polar form of z, in which $z=re^{i\theta}$, with:

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Symmetries / Invariance

- In physics, exhibition of symmetry under some operation implies some conservation law:

symmetry	invariant
Space translation	momentum
Time translation	energy
Rotation	Angular momentum
Global phase; $\Psi \to e^{i\theta}\Psi$	Electric charge
Local phase; $\Psi \to e^{i\theta(x,t)}\Psi$	Lagrangian + gauge field ($\to$ QED)

Symmetries / Invariance

- There are carefully chosen sets of transformations for Ψ which give rise to the observable gauge fields:
  - That is how we get electric, color, weak charge conservation!

QED from local gauge invariance

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix},$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

- Apply local gauge symmetry to Dirac equation:

$$\bar{\psi} \equiv \psi^\dagger \gamma^0$$

$$\mathcal{L} = i\overline{\psi}\gamma^\mu\partial_\mu\psi - m\overline{\psi}\psi, \qquad \psi(x) \to \psi'(x) = e^{iq\chi(x)}\psi(x).$$

This type of transformation leaves quantum mechanical amplitudes invariant.

- The effect on the Lagrangian is:

$$\begin{aligned}\mathcal{L} \to \mathcal{L}' &= ie^{-iq\chi}\overline{\psi}\gamma^\mu\left[e^{iq\chi}\partial_\mu\psi + iq\left(\partial_\mu\chi\right)e^{iq\chi}\psi\right] - me^{-iq\chi}\overline{\psi}e^{iq\chi}\psi \\ &= \mathcal{L} - q\overline{\psi}\gamma^\mu\left(\partial_\mu\chi\right)\psi.\end{aligned}$$

If Lagrangian is invariant, then δL=0.

QED from local gauge invariance

- To satisfy $\delta L=0$, we "engineer" a mathematical "trick":

1. Introduce a gauge field A_μ to interact with fermions, and A_μ transform as: $A_\mu \to A'_\mu = A_\mu - \partial_\mu \chi.$
2. In resulting Lagrangian, replace $\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu$

- In that case, L is redefined:

$$L = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

The new Lagrangian is invariant under local gauge transformations.

Not the whole story…

- Need to add kinetic term for field (field strength):

Define $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
Add term $-1/4 F_{\mu\nu}F^{\mu\nu}$ (Lorentz invariant, matches Maxwell's equations)

QED Lagrangian

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Final lagrangian (for QED!):

$$L = -1/4 F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

- Note: No mass term for A_μ allowed; otherwise L is not invariant.
  - The gauge field is massless!

QED Lagrangian

We have mathematically engineered a quantum field which couples to fermions, obeys Maxwell's equations, and is massless!

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PHOTON

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QCD and Weak Lagrangians

- Follow similar reasoning, but allow for self-interaction of gauge bosons.
  - Jargon: QCD and weak interactions based on non-abelian theories.
- In non-abelian theories gauge invariance is achieved by adding n^2-1 massless gauge bosons for SU(n).
  - SU(n): gauge group.
  - SU(3): 8 massless gluons for QCD ✓
  - SU(2): 3 massless gauge bosons (W_1, W_2, W_3) for weak force
  - If mixing with U(1), we get (W_1, W_2, W_3) and B: electroweak force.

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By the same mechanism as the **massless photon** arises from QED, a set of **massless bosons** arise when the theory is extended to include the weak nuclear force.

But Nature tells us they have mass!

Higgs mechanism

- A theoretically proposed mechanism which gives rise to elementary particle masses: W^+, W^- and Z bosons (and solve other problems…).

- It actually predicts the mass of W^+, W^- and Z^0 bosons:
  - The W^+, W^- bosons should have a mass of 80.390 ± 0.018 GeV
  - The Z^0 boson should have a mass of 91.1874 ± 0.0021 GeV
  - Measurements:

✓ 80.387 ± 0.019 GeV
✓ 91.1876 ± 0.0021 GeV

- Beware: it ends up also providing a mechanism for fermion masses, but it doesn't make any prediction for those (…).

Particle masses

$$\mathcal{L}_{\text{QED}} \rightarrow \overline{\psi}(i\gamma^\mu\partial_\mu - m_\text{e})\psi + e\overline{\psi}\gamma^\mu A_\mu\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_\gamma^2 A_\mu A^\mu.$$

For the U(1) local gauge transformation of (17.11), the photon field transforms as

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu\chi,$$

and the new mass term becomes

$$\tfrac{1}{2}m_\gamma^2 A_\mu A^\mu \rightarrow \tfrac{1}{2}m_\gamma^2\left(A_\mu - \partial_\mu\chi\right)(A^\mu - \partial^\mu\chi) \neq \tfrac{1}{2}m_\gamma^2 A_\mu A^\mu,$$

Mass function is not gauge invariant, same for weak interaction and QCD.

Particle masses

$$\mathcal{L}_{\rm QED} \rightarrow \overline{\psi}(i\gamma^\mu\partial_\mu - m_{\rm e})\psi + e\overline{\psi}\gamma^\mu A_\mu\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_\gamma^2 A_\mu A^\mu .$$

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and the new mass term becomes

$$\tfrac{1}{2} \ldots ,$$

Mass fun … for weak

What about massive gauge bosons?

Introducing a Scalar Field (Simplified; REAL)

- Consider a scalar field ϕ with the potential

$$V(\phi) = \tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda\phi^4.$$

- The corresponding Lagrangian is given by

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi)$$
$$= \tfrac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda\phi^4.$$

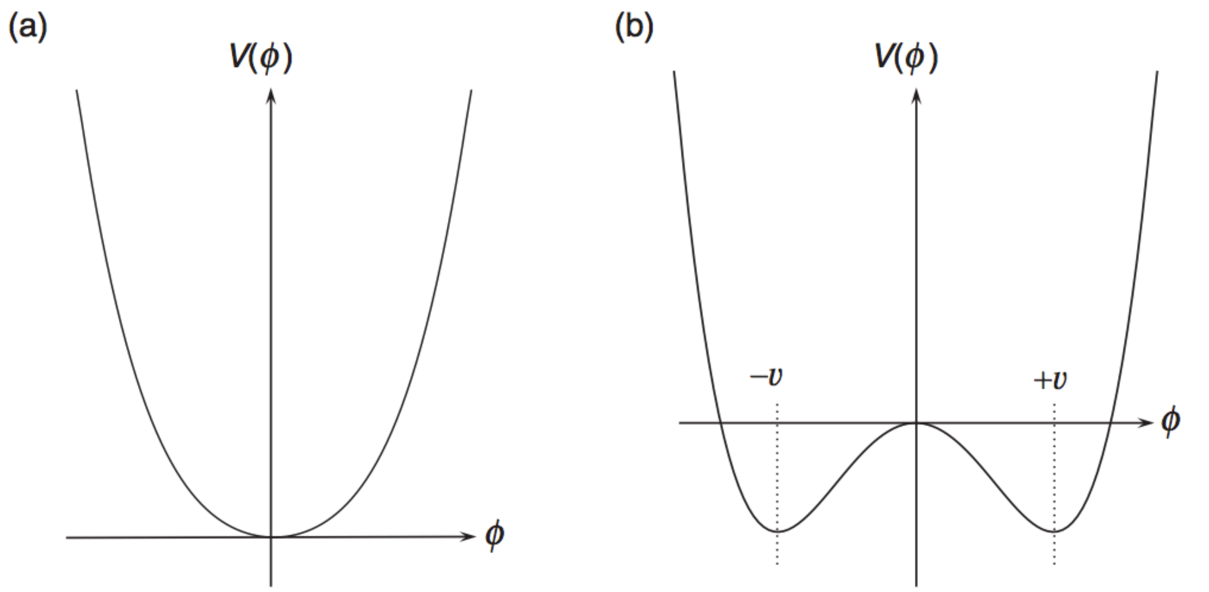

Fig. 17.5 The one-dimensional potential $V(\phi) = \mu^2\phi^2/2 + \lambda\phi^4/4$ for $\lambda > 0$ and the cases where (a) $\mu^2 > 0$ and (b) $\mu^2 < 0$.