PARTICLE PHYSICS

COLUMBIA SCIENCE HONORS PROGRAM

WEEK 7

OVERVIEW OF THE STANDARD MODEL (CONT'ED)

LIMITATIONS OF THE STANDARD MODEL

COURSE POLICIES

- Attendance
  - Up to four excused absences
    - Two with notes from parent/guardian
    - shpattendance@columbia.edu
  - Valid excuses:
    - Illness, family emergency, tests or athletic/academic competitions, mass transit breakdowns
  - Invalid excuses:
    - Sleeping in, missing the train…
  - I will take attendance during class
- No cell phones
- Ask questions!

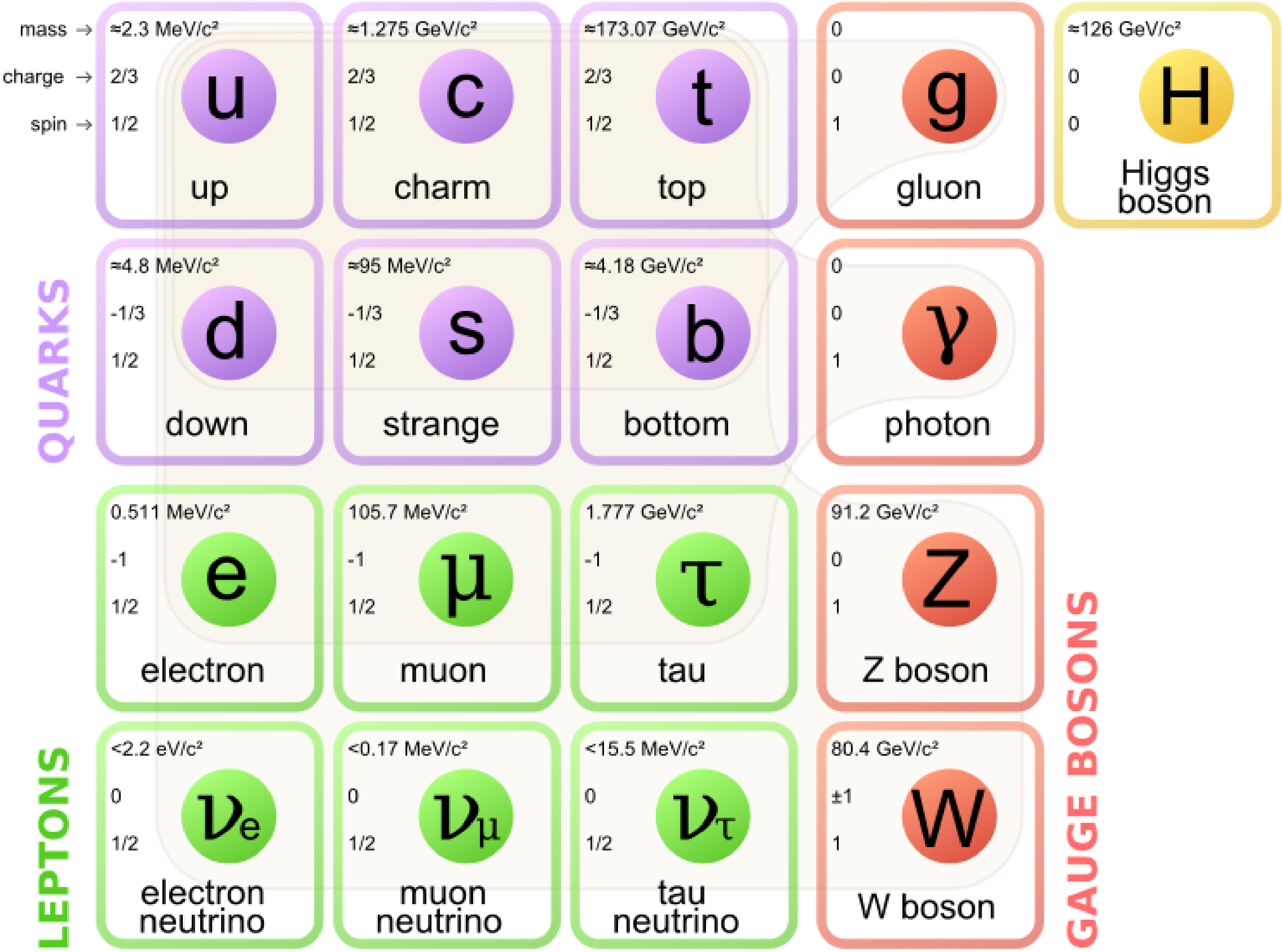
LECTURE MATERIALS

- https://twiki.nevis.columbia.edu/twiki/bin/view/Main/ScienceHonorsProgram
- Questions: cristovao.vilela@stonybrook.edu

LAST WEEK…

AN OVERVIEW OF THE STANDARD MODEL

THE WORLD, ACCORDING TO A PARTICLE PHYSICIST

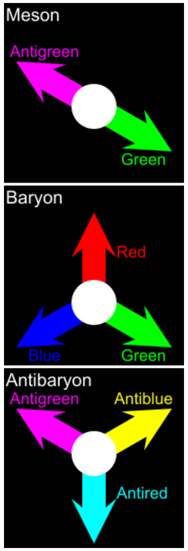


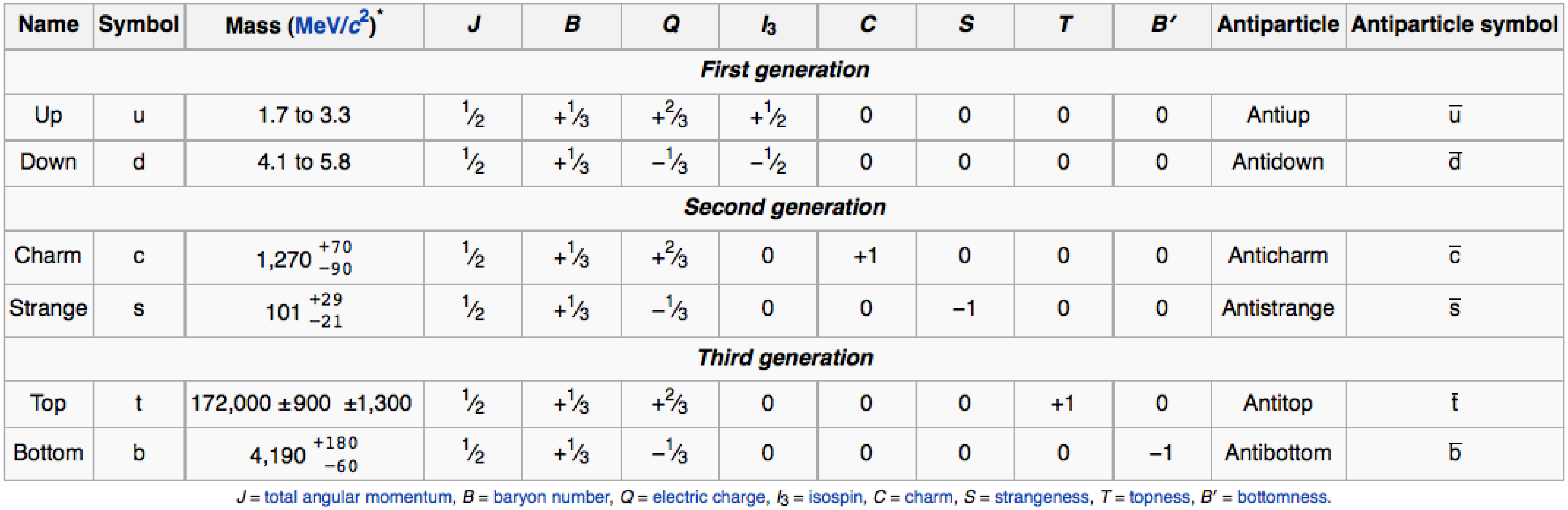
mass → charge → spin →	≈2.3 MeV/c^2 2/3 1/2 u up	≈1.275 GeV/c^2 2/3 1/2 c charm	≈173.07 GeV/c^2 2/3 1/2 t top	0 0 1 g gluon	≈126 GeV/c^2 0 0 H Higgs boson
QUARKS	≈4.8 MeV/c^2 -1/3 1/2 d down	≈95 MeV/c^2 -1/3 1/2 s strange	≈4.18 GeV/c^2 -1/3 1/2 b bottom	0 0 1 γ photon	
	0.511 MeV/c^2 -1 1/2 e electron	105.7 MeV/c^2 -1 1/2 μ muon	1.777 GeV/c^2 -1 1/2 τ tau	91.2 GeV/c^2 0 1 Z Z boson	
LEPTONS	<2.2 eV/c^2 0 1/2 ν_e electron neutrino	<0.17 MeV/c^2 0 1/2 ν_μ muon neutrino	<15.5 MeV/c^2 0 1/2 ν_τ tau neutrino	80.4 GeV/c^2 ±1 1 W W boson	GAUGE BOSONS

PARTICLE CHARGES

	Electric	Weak	Color
Quarks	✓	✓	✓

- Quarks:
  - There are **no** free quarks.
  - They form colorless composite objects, hadrons;
    - Baryons: qqq, $\overline{qqq}$
    - Mesons: $q\bar{q}$, $q\bar{q}$, $q\bar{q}$





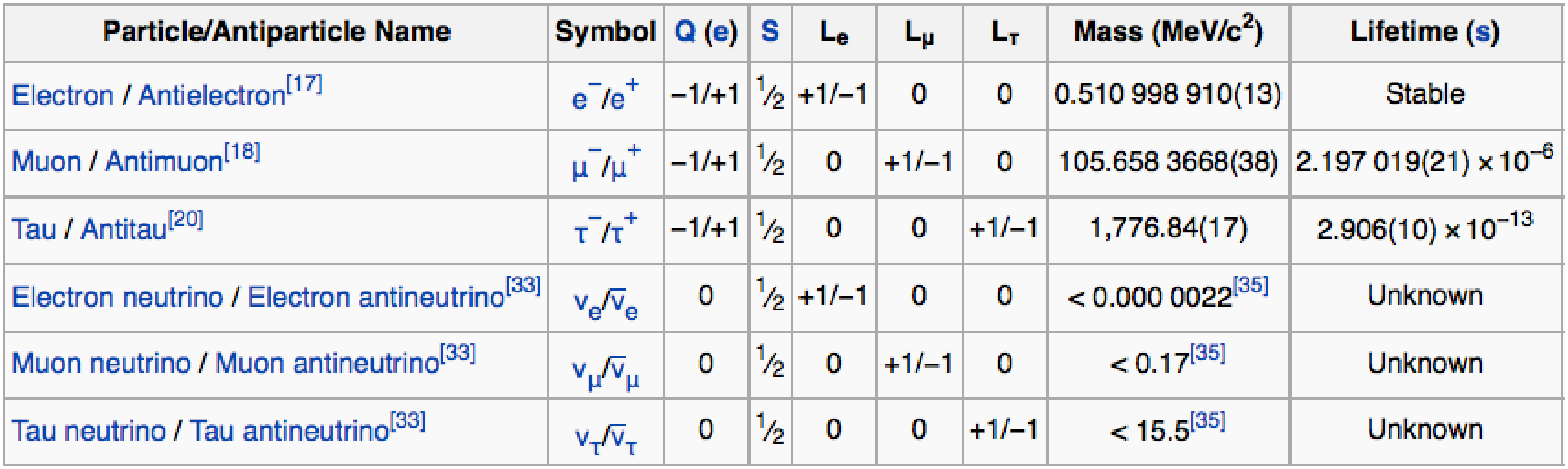
Name	Symbol	Mass (MeV/c^2)*	J	B	Q	I_3	C	S	T	B'	Antiparticle	Antiparticle symbol
First generation												
Up	u	1.7 to 3.3	½	+⅓	+⅔	+½	0	0	0	0	Antiup	$\bar{u}$
Down	d	4.1 to 5.8	½	+⅓	−⅓	−½	0	0	0	0	Antidown	$\bar{d}$
Second generation												
Charm	c	$1{,}270^{+70}_{-90}$	½	+⅓	+⅔	0	+1	0	0	0	Anticharm	$\bar{c}$
Strange	s	101^{+29}_{-21}	½	+⅓	−⅓	0	0	−1	0	0	Antistrange	$\bar{s}$
Third generation												
Top	t	172,000 ±900 ±1,300	½	+⅓	+⅔	0	0	0	+1	0	Antitop	$\bar{t}$
Bottom	b	$4{,}190^{+180}_{-60}$	½	+⅓	−⅓	0	0	0	0	−1	Antibottom	$\bar{b}$

J = total angular momentum, B = baryon number, Q = electric charge, I_3 = isospin, C = charm, S = strangeness, T = topness, B' = bottomness.

PARTICLE CHARGES

		Electric	Weak	Color
	Quarks	✓	✓	✓
Leptons	Charged	✓	✓	✗
	Neutral	✗	✓	✗

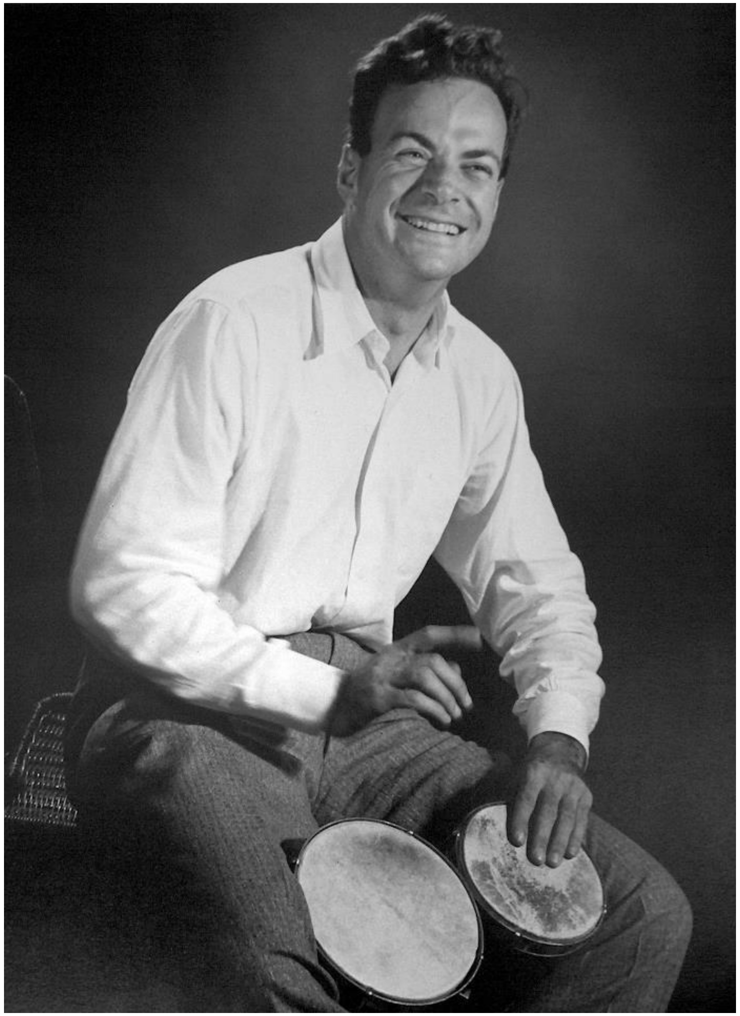
- Leptons
  - Exist as free particles



Particle/Antiparticle Name	Symbol	Q (e)	S	L_e	L_μ	L_τ	Mass (MeV/c^2)	Lifetime (s)
Electron / Antielectron[17]	e^-/e^+	−1/+1	½	+1/−1	0	0	0.510 998 910(13)	Stable
Muon / Antimuon[18]	μ^-/μ^+	−1/+1	½	0	+1/−1	0	105.658 3668(38)	$2.197\,019(21) \times 10^{-6}$
Tau / Antitau[20]	τ^-/τ^+	−1/+1	½	0	0	+1/−1	1,776.84(17)	$2.906(10) \times 10^{-13}$
Electron neutrino / Electron antineutrino[33]	$\nu_e/\bar{\nu}_e$	0	½	+1/−1	0	0	< 0.000 0022[35]	Unknown
Muon neutrino / Muon antineutrino[33]	$\nu_\mu/\bar{\nu}_\mu$	0	½	0	+1/−1	0	< 0.17[35]	Unknown
Tau neutrino / Tau antineutrino[33]	$\nu_\tau/\bar{\nu}_\tau$	0	½	0	0	+1/−1	< 15.5[35]	Unknown

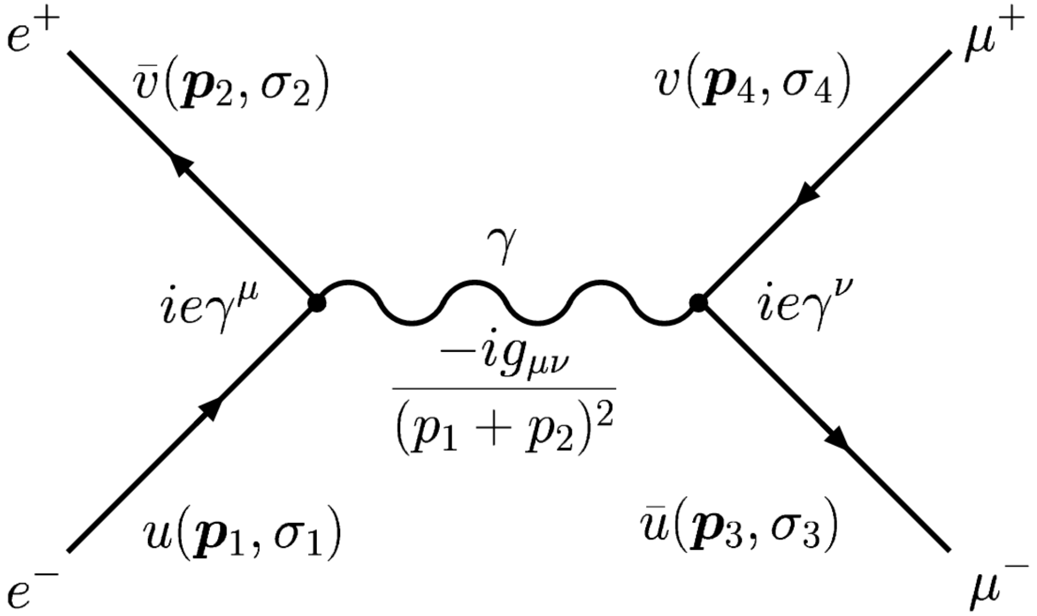
PARTICLE DYNAMICS: FEYNMAN DIAGRAMS

- Feynman Rules!
- 1948: introduced pictorial representation scheme for the mathematical expressions governing the behavior of subatomic particles.
  - Can be used to easily caclulate probability amplitudes
  - Other options: cumbersome mathematical derivations



FEYNMAN DIAGRAMS

AN EXAMPLE CALCULATION

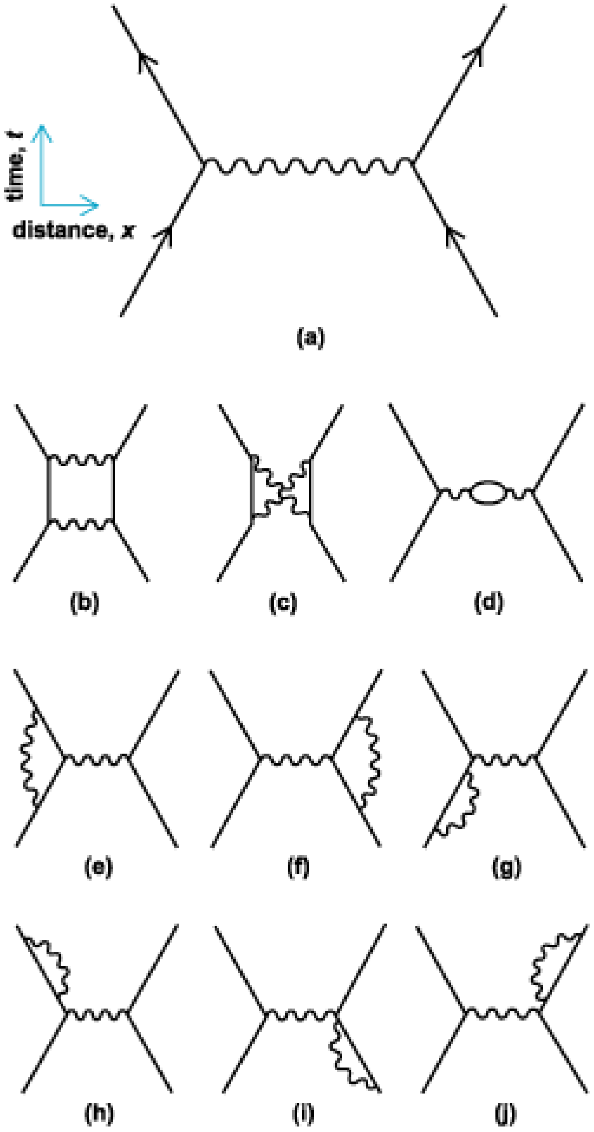


$$-i\mathcal{M} = [\bar{u}(\boldsymbol{p}_3, \sigma_3)(ie\gamma^\nu)v(\boldsymbol{p}_4, \sigma_4)] \frac{-ig_{\mu\nu}}{(p_1 + p_2)^2} [\bar{v}(\boldsymbol{p}_2, \sigma_2)(ie\gamma^\mu)u(\boldsymbol{p}_1, \sigma_1)]$$

QUANTUM ELECTRODYNAMICS

QED

- The vertices are interactions with the electromagnetic field.
- The straight lines are electrons and the wiggly ones are photons.
- Between interactions (vertices), particles propagate as free particles.
- The higher the number of vertices, the less likely for the interaction to happen.
  - Higher “order”.



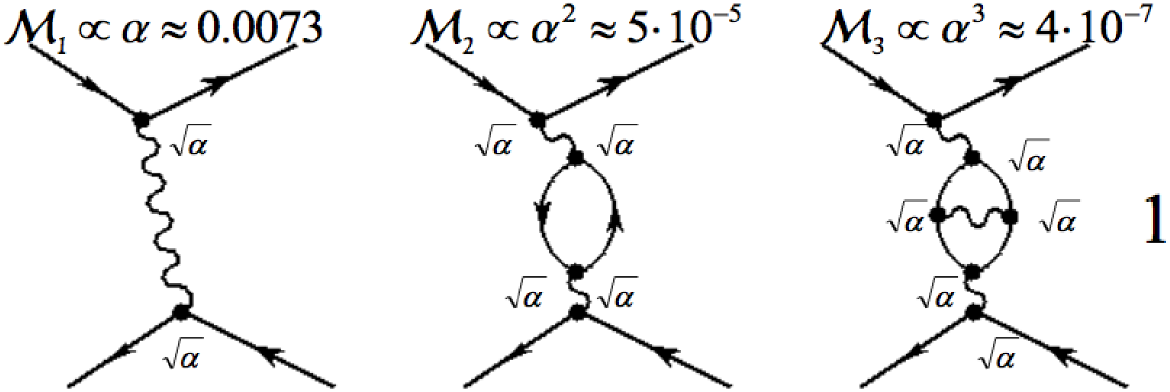
QUANTUM ELECTRODYNAMICS

QED

- Each vertex contributes a coupling constant √α, where α is a small dimensionless number:

$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

- Hence, higher-order diagrams get suppressed relative to diagrams with fewer vertices.



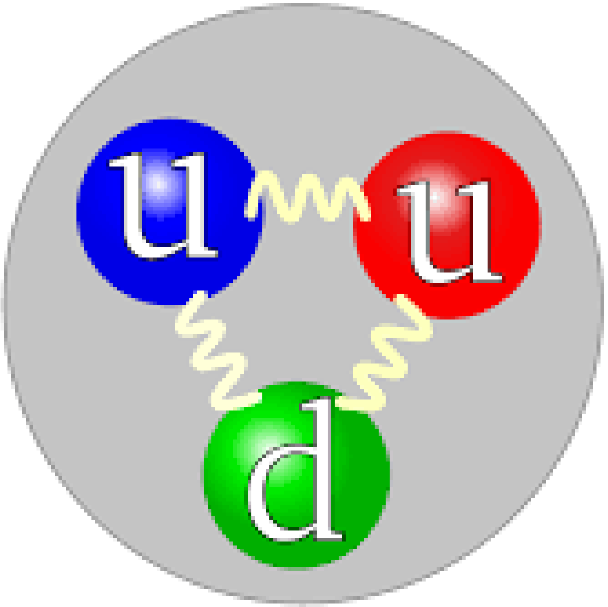
$$\begin{array}{ccccc} \mathcal{M}_1 & : & \mathcal{M}_2 & : & \mathcal{M}_3 \\ 18769 & : & 137 & : & 1 \end{array}$$

QUANTUM CHROMODYNAMICS

QCD

- Quarks and bound states:
  - Since quarks are spin-1/2 particles (fermions), they must obey the Pauli Exclusion Principle.
- **Pauli Exclusion Principle**: fermions in a bound state (e.g., the quarks inside a hadron) cannot have the same quantum numbers.

- Then, how can we squeeze three quarks into a baryon?
- Give them an additional charge, called color.
- This removes the quantum numbers degeneracy.



QUANTUM CHROMODYNAMICS
QCD

- Gluons carry a color and an anti-color.
- There are 9 possible combinations, but 1 is white, which is not allowed.
  - No evidence for colorless particles exchanging gluons.
- This leaves 8 types of gluon.

A quark changes color by emitting or absorbing gluons.

PERTURBATION THEORY

ASIDE

- Use a power series in a parameter ε (such that $\varepsilon << 1$) - known as perturbation series - as an approximation to the full solution.
- For example:

$$A = A_0 + \epsilon A_1 + \epsilon^2 A_2 + \ldots$$

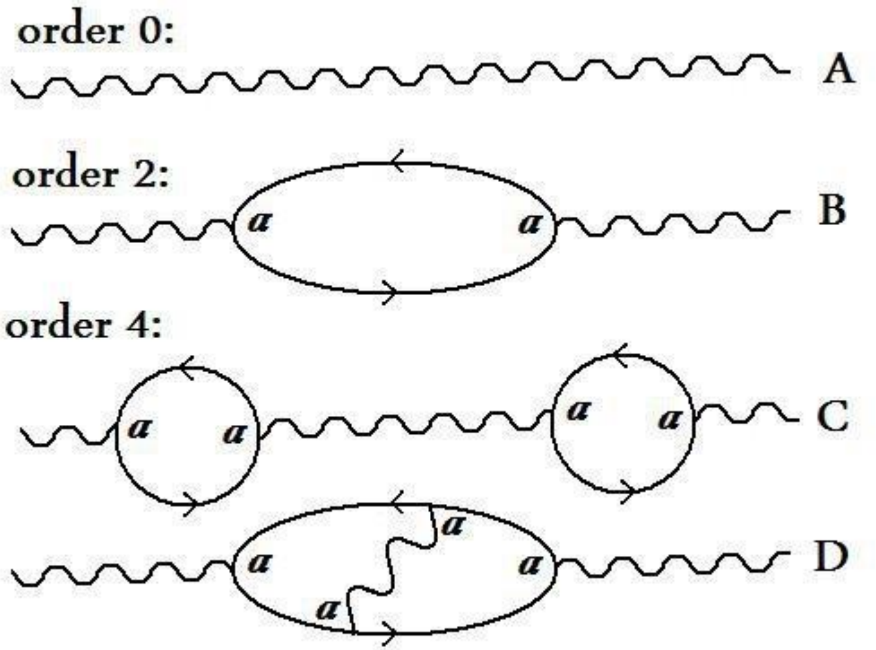
- In this example, A_0 is the "leading order" solution, while A_1, A_2, … represent higher order terms.
- **Note:** if ε is small, the higher-order terms in the series become successively smaller.
- Approximation:

$$A \approx A_0 + \epsilon A_1$$

PERTURBATION THEORY IN QFT

ASIDE

- Perturbation theory allows for well-defined predictions in quantum field theories (as long as they obey certain requirements).
- Quantum electrodynamics is one of those theories.
- Feynman diagrams correspond to the terms in the perturbation series!

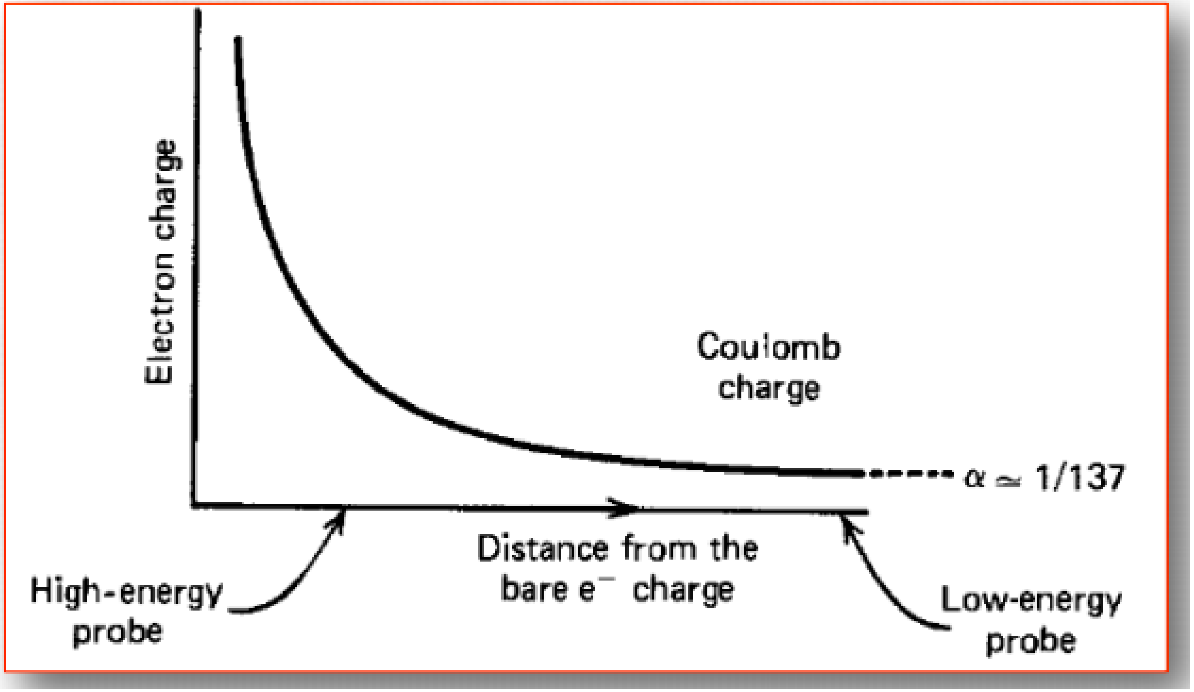


$$P = A + Ba^2 + (C + D)a^4 + \ldots$$

Diagrams define a series in a

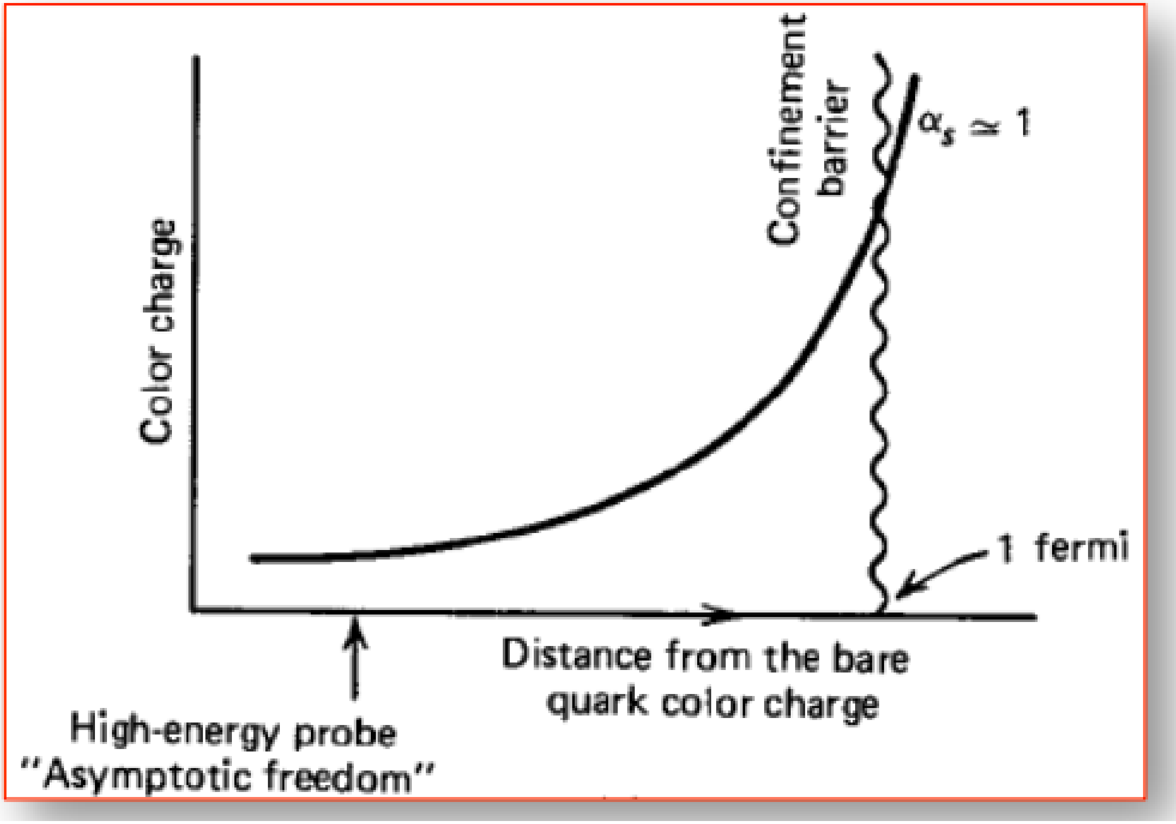
QCD VS QED

- Recall: In QED, each vertex contributes a coupling constant √α.
- α is not exactly a constant though… it “runs" with the scale of the interaction.



QCD VS QED

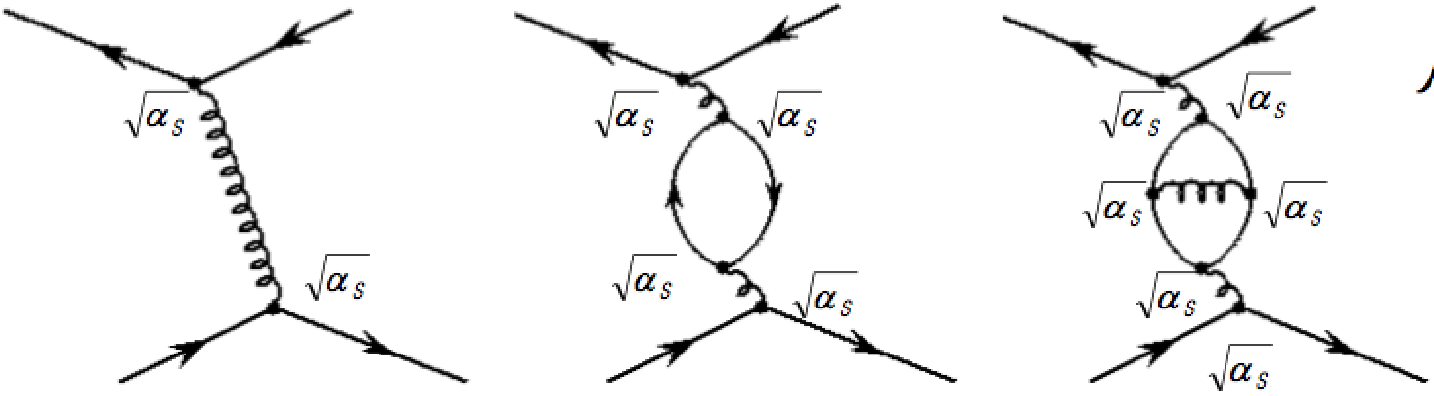
- The coupling constant for QCD, α_s, "runs" in a different way with energy.



QCD VS QED

- In QCD, the coupling between quarks and gluons, given by the number α_s, is much larger than 1/137 at low energies.
- In fact, at low energies, $\alpha_s >> 1$, making higher-order diagrams just as important as those with fewer vertices!
- This means we can't truncate the sum over diagrams.
  - Perturbation theory is not a good approximation!
- Calculations quickly become complicated!

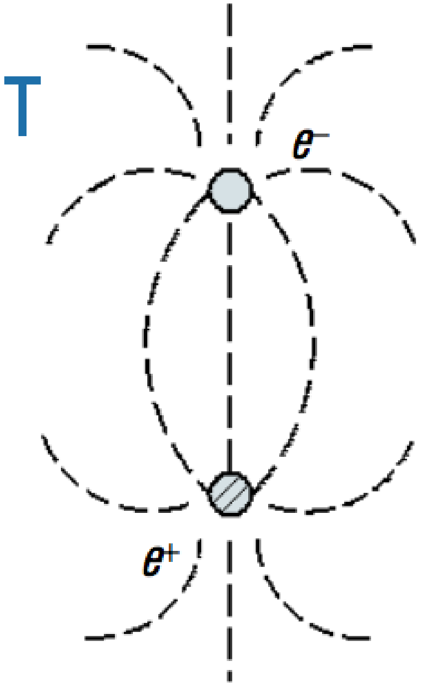
$$\mathcal{M}_1 \propto \alpha_S \approx 1 \qquad \mathcal{M}_2 \propto \alpha_S{}^2 \approx 1 \qquad \mathcal{M}_3 \propto \alpha_S{}^3 \approx 1$$



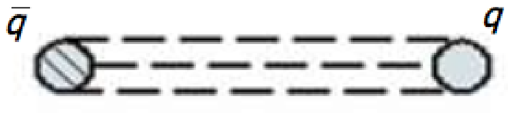
$$\begin{array}{ccccc} \mathcal{M}_1 & : & \mathcal{M}_2 & : & \mathcal{M}_3 \\ 1 & : & 1 & : & 1 \end{array}$$

UNDERSTANDING CONFINEMENT

- The mathematics of confinement are complicated, but we can understand them in terms of a very simple picture.
- Recall, the Coulomb field between a e+e- pair looks like $V(r) \sim 1/r$.
  - As we pull the pair apart, the attraction weakens.
- Imagine the color field between a quark-antiquark pair like Hooke's Law: $V(r) \sim r$.
  - As we pull the pair apart, the attraction between them increases.
- So, separating two quarks by a large r puts a large amount of energy into the color field: $V(r) \longrightarrow \infty$



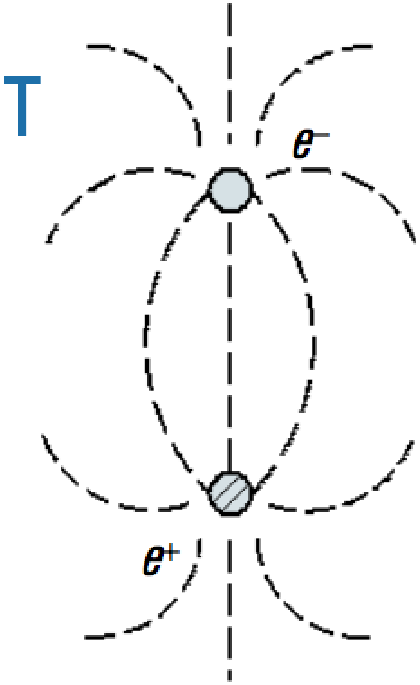
Dipole field for the Coulomb force between opposite electrical charges.



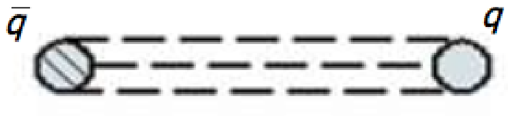
Dipole field between opposite-color quarks.

UNDERSTANDING CONFINEMENT

- How do we understand this picture?
- When a quark and anti-quark separate, their color interaction strengthens (more gluons appear in the color field).
- Through the interaction of the gluons with each other, the color lines of force are squeezed into a tube-like region.
- Contrast this with the Coulomb field: nothing prevents the lines of force from spreading out.
    - There is no self-coupling of photons to contain them.
- If the color tube has constant energy density per unit length k, the potential energy between quark and antiquark will increase with separation, V(r)~kr.



Dipole field for the Coulomb force between opposite electrical charges.

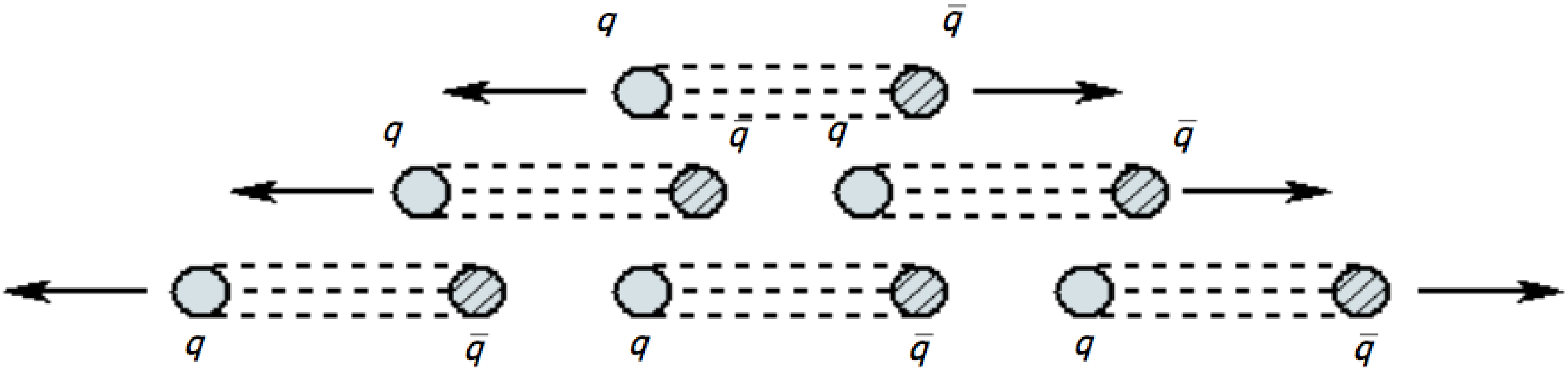


Dipole field between opposite-color quarks.

COLOR LINES AND HADRON PRODUCTION

WHY YOU CAN'T GET FREE QUARKS

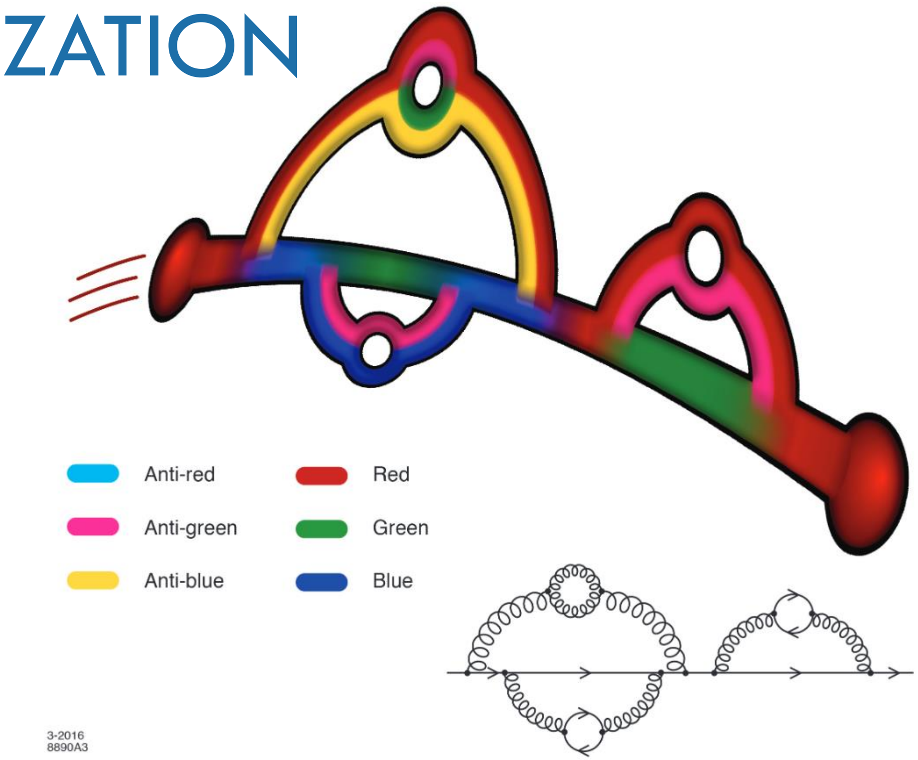
- Suppose we have a meson and we try to pull it apart. The potential energy in the quark-antiquark color field starts to increase.
- Eventually, the energy in the gluon field gets big enough that the gluons can pair-produce another quark-antiquark pair.
  - The new quarks pair up with the original quarks to form mesons, and thus our four quarks remain confined in colorless states.
- Experimentally, we see two particles!

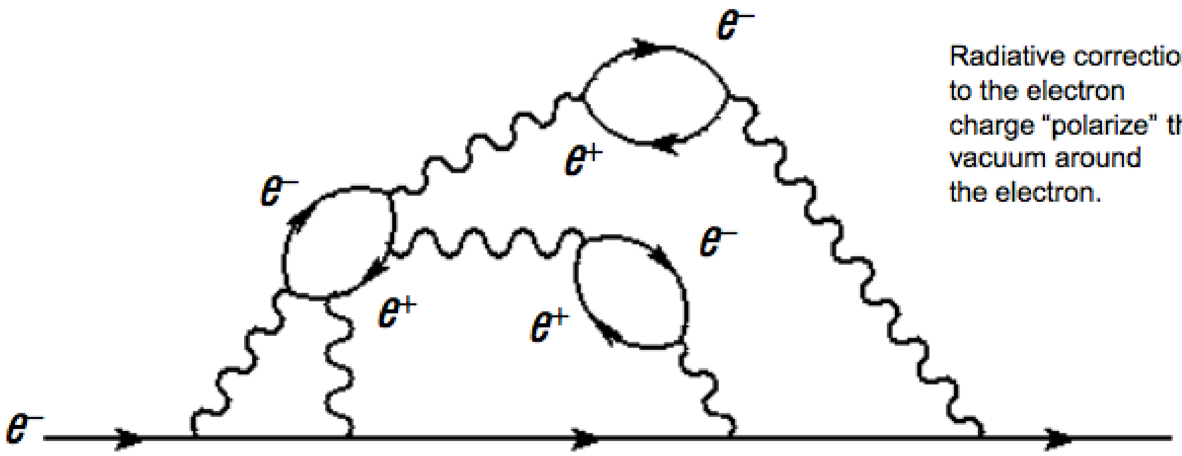


VACUUM POLARIZATION

QCD →

Color anti-screening





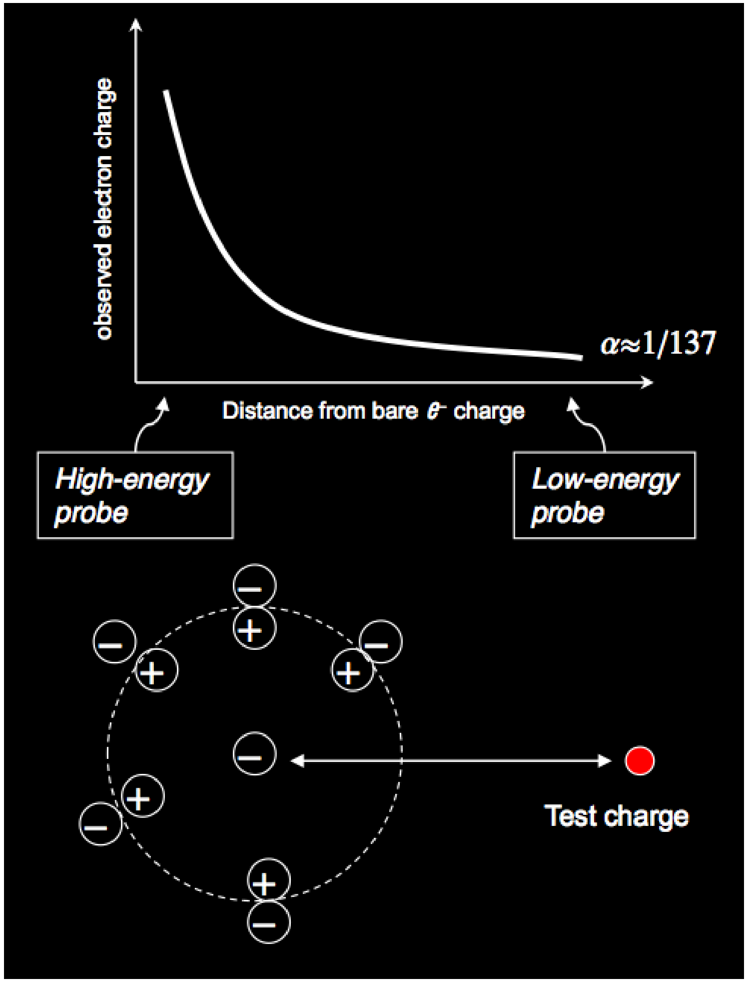
Radiative corrections to the electron charge "polarize" the vacuum around the electron.

←QED

Charge screening

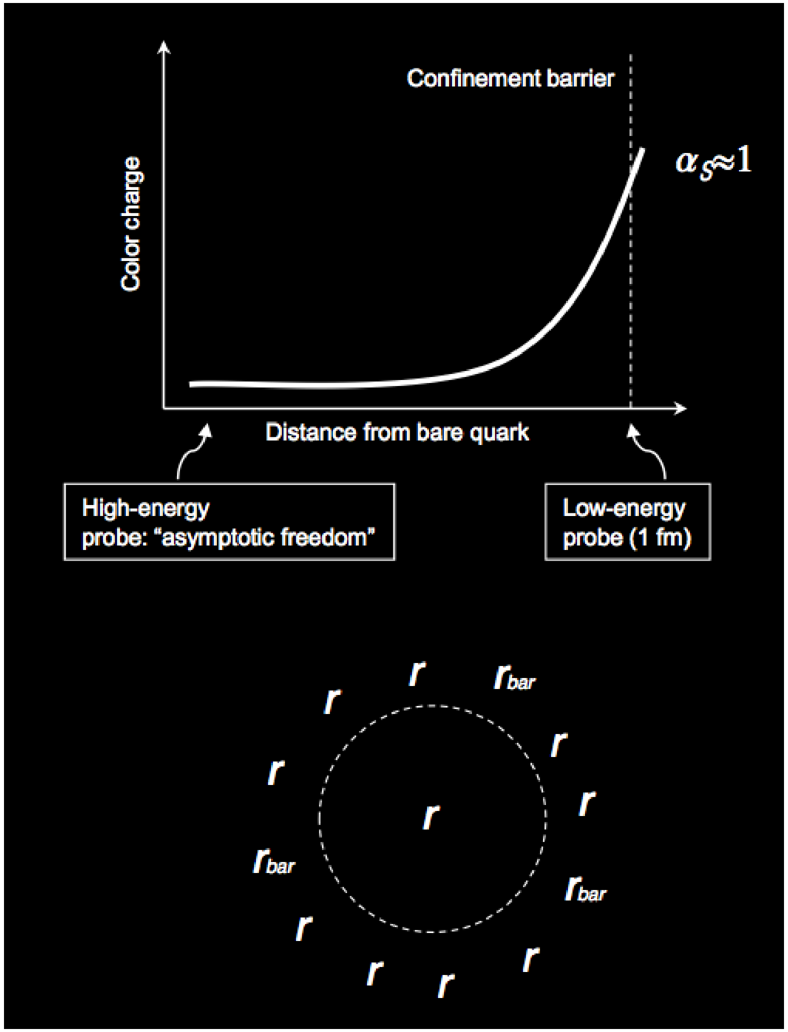
QED: CHARGE SCREENING

- Now, suppose we want to measure the charge of the electron by observing the Coulomb force experienced by a test charge.
- Far away from the electron, its charge is screened by a cloud of virtual positrons and electrons, so the effective charge is smaller than its **bare** charge.
- As we move closer in, fewer positrons are blocking our line of sight to the electron.
- Hence, with decreasing distance, the effective charge of the electron increases.
- **We can think of this as α increasing with energy.**



QCD: CHARGE ANTI-SCREENING

- In QCD, the gluon self-interaction reverses the result of QED:
  - A red charge is preferentially surrounded by other red charges.
- By moving the test probe closer to the original quark, the probe penetrates a sphere of mostly red charge, and the measured red charge decreases.
- This is "antiscreening".
- **We can think of this as α_s decreasing with energy.**



RUNNING CONSTANTS

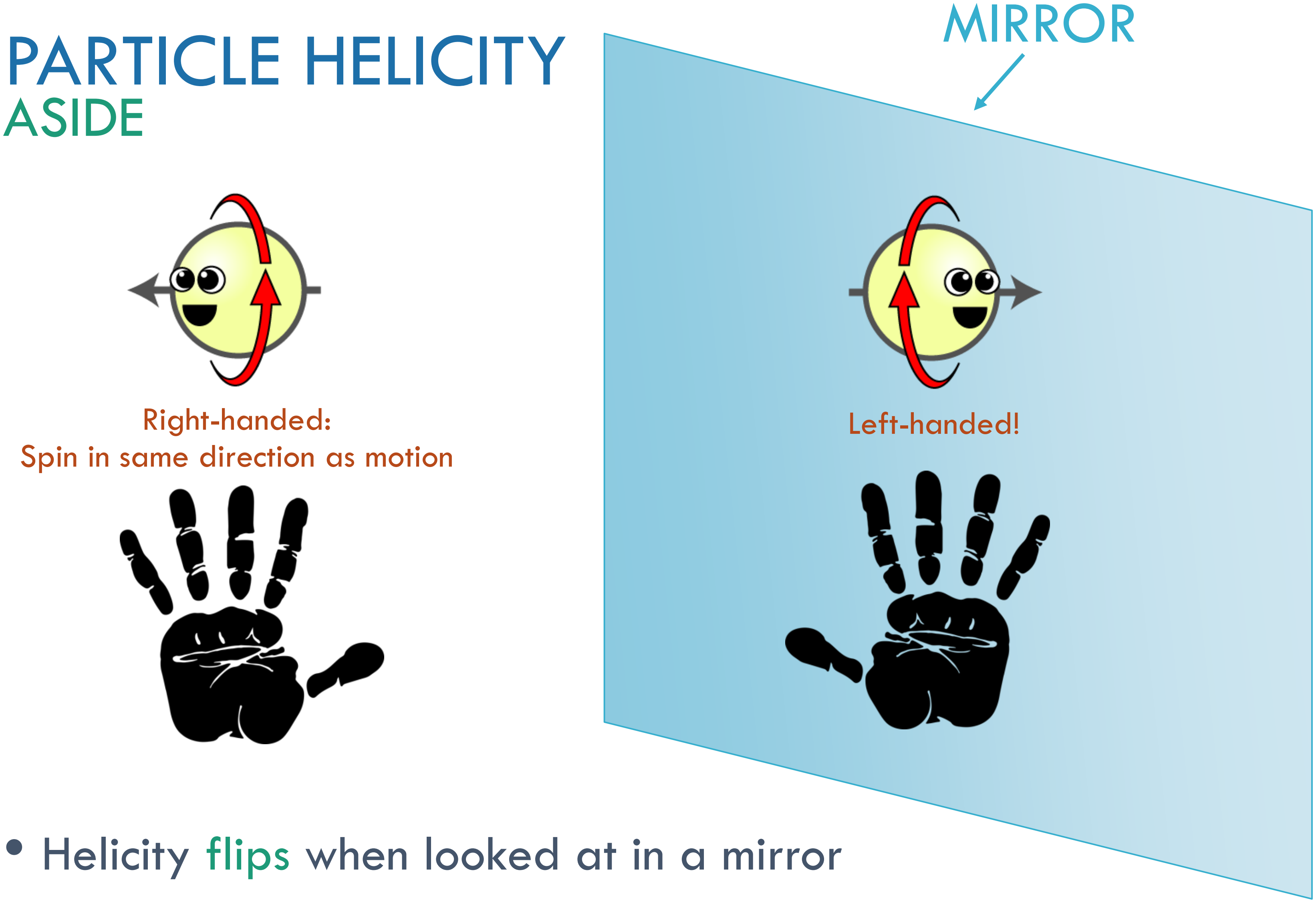
- As we probe an electron at increasingly higher energies, its effective charge increases.
- This can be rephrased in the following way: as interactions increase in energy, the QED coupling strength α between charges and photons also increases.
  - This should not really be a surprise; after all, the coupling strength of EM depends directly on the electron charge.
- Since α is not a constant, but a (slowly-varying) function of energy, it is called a **running coupling constant**.
- In QCD, the net effect is that the quark color charge and α_s decrease as the interaction energy goes up.

UNDERLYING CAUSE

SELF-INTERACTIONS OF THE MEDIATORS

- Gluon self-interaction!
- W and Z (weak force mediators) also self-interact.
  - Similar behavior.
  - The weak coupling constant also decreases as the energy scale goes up.

PARTICLE HELICITY
ASIDE



- Helicity flips when looked at in a mirror
  - Equivalent to inverting one of the space coordinates (z → -z)

http://www.quantumdiaries.org/

WEAK INTERACTIONS

- Unlike electromagnetism and the strong interaction, the weak interaction is mediated by massive bosons:
  - $m_Z = 91.19$ GeV
  - $m_W = 80.39$ GeV
- This makes it extremely short-range

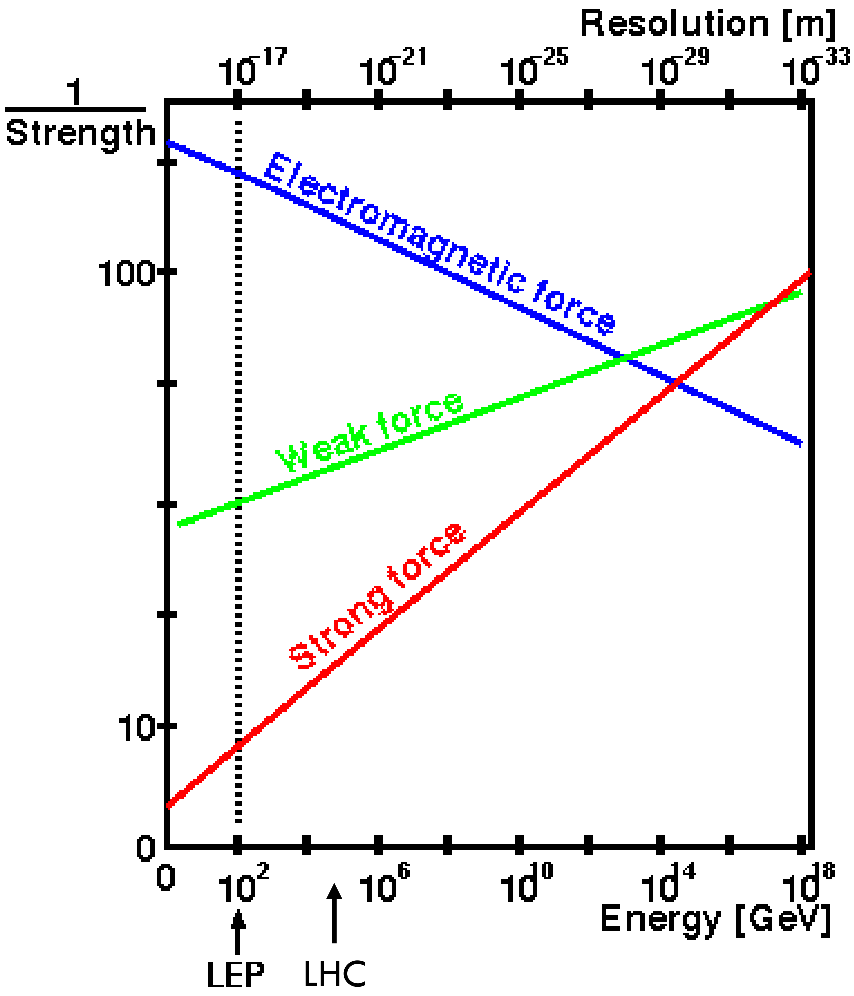
$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

- And very weak at low energies
- Only left-handed particles* participate in weak interactions
  - Nature is different if looked at in a mirror!
  - * and right-handed anti-particles

FORCE UNIFICATION

ASIDE

- At laboratory energies near M_W O(100) GeV, the measured values of the coupling constants are quite different.
- However, their “running” trends suggest that they approach a common value near 10^{16} GeV.
- This is an insanely high energy!
- The Standard Model provides no explanation for what may happen beyond this unification scale, nor why the forces have such different strengths at low energies.



PARTICLE/FIELD FORMULATION

PARTICLE/FIELD FORMULATION

- In particle physics, we define fields like φ(x,t) at every point in spacetime.
- These fields don't just sit there; they fluctuate about some minimum energy state.
- The oscillations combine to form wavepackets.
- The wavepackets move around in the field and interact with each other. We interpret them as elementary particles.
- Terminology: the wavepackets are called the **quanta** of the field φ(x,t).

PARTICLE/FIELD FORMULATION

- How do we describe interactions and fields mathematically?
- Classically,

  Lagrangian L = kinetic energy - potential energy

- Particle physics:
  - Same concept, using Dirac equation to describe free spin-1/2 particles:

$$L = \bar{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi$$

Field

Ψ = wavefunction

m = mass

γ^{μ} = μ^{th} gamma matrix

∂_{μ} = partial derivative

LAGRANGIAN MECHANICS

- Developed by Euler, Lagrange, and others during the mid-1700's.
- This is an energy-based theory that is equivalent to Newtonian mechanics (a force-based theory, if you like).

$$\frac{\partial \mathcal{L}}{\partial x_i} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_i}\right) = 0$$

- **Lagrangian:** quantity that allows us to infer the dynamics of a system.

STANDARD MODEL LAGRANGIAN

$$\mathcal{L} = \overset{\text{Free Fields}}{\mathcal{L}_0} + \overset{\text{Interaction}}{\mathcal{L}'}$$

STANDARD MODEL LAGRANGIAN

Free Fields

Interaction

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

Gauge Bosons

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi$$

Fermions

$$F_{\mu\nu}F^{\mu\nu} = G_{\mu\nu}G^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}$$

STANDARD MODEL LAGRANGIAN

Free Fields

Interaction

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

Gauge Bosons

$$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\gamma^\mu \partial_\mu \psi$$

Fermions

$$\mathcal{L}' = e\bar{\psi}\gamma^\mu A_\mu \psi$$

Fermion-Boson
Coupling

$$eA_\mu = \frac{g_s}{2}\lambda_\nu G^\nu_\mu + \frac{g}{2}\vec{\tau}\,\vec{W}_\mu + \frac{g'}{2} Y B_\mu$$

$$F_{\mu\nu}F^{\mu\nu} = G_{\mu\nu}G^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}$$

STANDARD MODEL LAGRANGIAN

Free Fields

Interaction

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

Gauge Bosons

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi$$

Fermions

$$\mathcal{L}' = e\bar{\psi}\gamma^\mu A_\mu\psi$$

Fermion-Boson Coupling

Gluons and strong interaction

Electroweak bosons and interaction

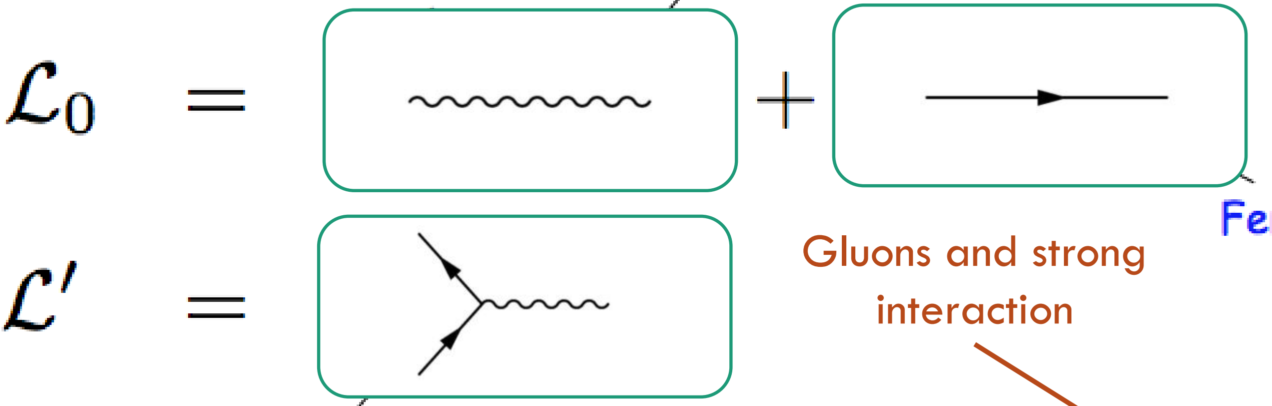
$$\begin{aligned} eA_\mu &= \boxed{\frac{g_s}{2}\lambda_\nu G^\nu_\mu} + \boxed{\frac{g}{2}\vec{\tau}\,\vec{W}_\mu + \frac{g'}{2}Y B_\mu} \\ F_{\mu\nu}F^{\mu\nu} &= \boxed{G_{\mu\nu}G^{\mu\nu}} + \boxed{W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}} \end{aligned}$$

STANDARD MODEL LAGRANGIAN

Free Fields

Interaction

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$



Gauge Bosons

Fermions

Fermion-Boson Coupling

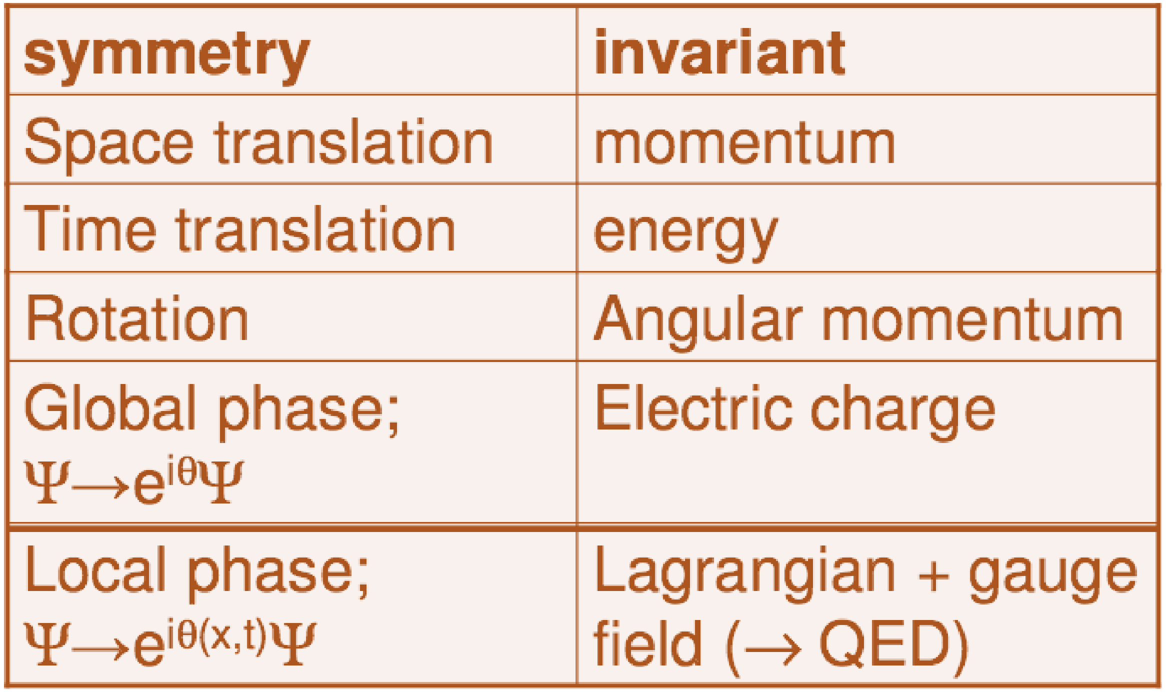
Electroweak bosons and interaction

Gluons and strong interaction

$$\begin{aligned} eA_\mu &= \frac{g_s}{2}\lambda_\nu G^\nu_\mu + \frac{g}{2}\vec{\tau}\,\vec{W}_\mu + \frac{g'}{2} Y B_\mu \\ F_{\mu\nu}F^{\mu\nu} &= G_{\mu\nu}G^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu} \end{aligned}$$

SYMMETRIES AND INVARIANCE

- Noether’s theorem (1915):
  - Every symmetry under some operation corresponds to a conservation law



symmetry	invariant
Space translation	momentum
Time translation	energy
Rotation	Angular momentum
Global phase; $\Psi \rightarrow e^{i\theta}\Psi$	Electric charge
Local phase; $\Psi \rightarrow e^{i\theta(x,t)}\Psi$	Lagrangian + gauge field (→ QED)

QED FROM LOCAL GAUGE INVARIANCE

- Apply local gauge symmetry to Dirac equation:

$$\bar{\Psi} \rightarrow e^{i\theta(x,t)}\bar{\Psi},\ \Psi \rightarrow e^{-i\theta(x,t)}\Psi$$

This type of transformation leaves quantum mechanical amplitudes invariant.

- Consider very small changes in a field:

$$\Psi \rightarrow \Psi + \delta\Psi = \Psi - i\theta(x,t)\Psi \quad \text{ie. } \delta\Psi = -i\theta(x,t)\Psi$$

- The effect on the Lagrangian is:

$$L = \bar{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi \Rightarrow \delta L = \bar{\Psi}\gamma^{\mu}\partial_{\mu}\theta(x,t)\Psi$$

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For the Lagrangian to remain invariant: δL = 0

QED FROM LOCAL GAUGE INVARIANCE

- To satisfy $\delta L=0$, we "engineer" a mathematical "trick":
  1. Introduce a gauge field A_μ to interact with fermions, and A_μ transform as: $A_\mu \rightarrow A_\mu' = A_\mu + 1/e\, \partial_\mu \theta(x,t)$
  2. In resulting Lagrangian, replace $\partial_\mu \rightarrow D_\mu = \partial_\mu + ieA_\mu$

- In that case, L is redefined:

$$L = \overline{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

The new Lagrangian is invariant under local gauge transformations.

QED FROM LOCAL GAUGE INVARIANCE

ONE MORE THING…

- Need to add kinetic term for field (field strength):

Define $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Add term $-1/4 F_{\mu\nu}F^{\mu\nu}$ (Lorentz invariant, matches Maxwell's equations)

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Final lagrangian (for QED!):

$$L = -1/4F_{\mu\nu}F^{\mu\nu} + \Psi(i\gamma^\mu D_\mu - m)\Psi$$

- No mass term is allowed for A_μ, otherwise the Lagrangian is not gauge invariant
  - The gauge field is massless!

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is not ga

  - The g

We have mathematically engineered a quantum field that couples to fermions, obeys Maxwell's equations and is massless!

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- No mass grangian

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We have mathematically engineered a quantum field that couples to fermions, obeys Maxwell's equations and is massless!
The photon!

THE HIGGS MECHANISM

QCD AND WEAK LAGRANGIANS

- Follow similar reasoning as **QED**, but allow for self-interaction of gauge bosons.
  - Jargon: QCD and weak interactions based on non-abelian groups.
- In non-abelian groups gauge invariance is achieved by adding n^2-1 massless gauge bosons for SU(n).
  - SU(n): gauge group.
  - SU(3): 8 massless gluons for QCD ✓
  - SU(2): 3 massless gauge bosons (W_1,W_2,W_3) for weak force
  - If mixing with U(1), we get (W_1,W_2,W_3) and B: electroweak force.

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By the same mechanism as the **massless photon** arises from QED, a set of **massless bosons** arise when the theory is extended to include the weak nuclear force.

But Nature tells us they have mass!

HIGGS MECHANISM

- A theoretically proposed mechanism which gives rise to elementary particle masses: W^+, W^- and Z bosons (and solves other problems…)

- It actually predicts the mass of W^+, W^- and Z^0 bosons:
  - The W^+, W^- bosons should have a mass of 80.390 ± 0.018 GeV
  - The Z^0 boson should have a mass of 91.1874 ± 0.0021 GeV
  - Measurements:

✓ 80.387 ± 0.019 GeV

✓ 91.1876 ± 0.0021 GeV

- Beware: it ends up also providing a mechanism for fermion masses, but it doesn't make any prediction for those…

HIGGS MECHANISM

Higgs Field

$$\boxed{\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'} + \mathcal{L}_{\phi'}$$

Massless case, from previous slide

HIGGS MECHANISM

Yukawa Couplings

Higgs Field

$$\boxed{\mathcal{L} \quad = \quad \mathcal{L}_0 + \mathcal{L}'} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\phi'}$$

Massless case, from previous slide

HIGGS MECHANISM

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Massless case, from previous slide

$$\mathcal{L}_\phi \quad = \quad (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - V(\phi)$$

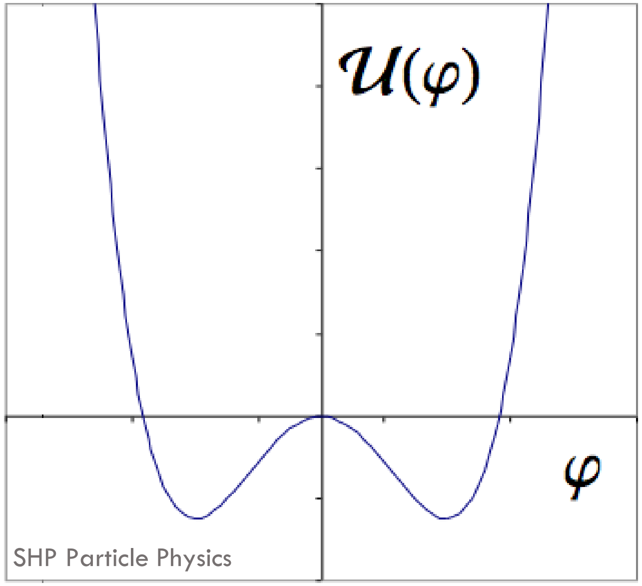
Higgs Potential

$$\mathcal{L}_{\text{Yuk}} \quad = \quad c_f(\bar{\psi}_L \psi_R \phi + \bar{\psi}_R \psi_L \phi)$$

Higgs Fermion Interaction

V(Φ) AND SPONTANEOUS SYMMETRY BREAKING

- When we speak of symmetry in the Standard Model, we usually mean gauge symmetry.
- This has to do with the form of the Lagrangian in terms of a given field φ.
- For simplicity, let's consider a simple (and suggestive!) example in which the Lagrangian has reflection symmetry about the φ=0 axis:



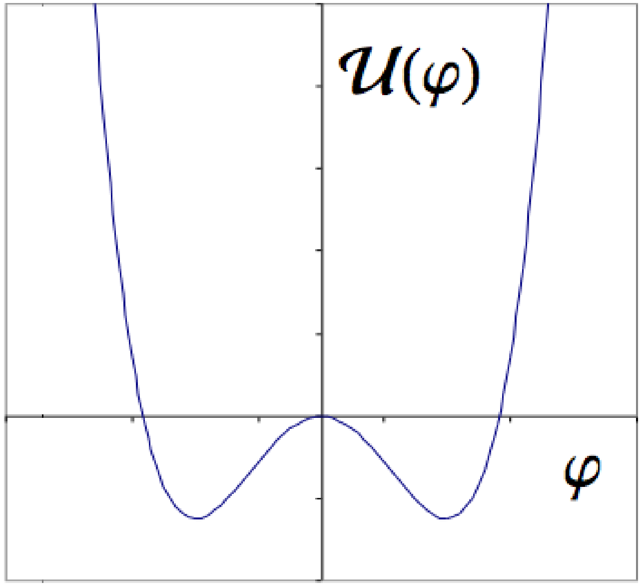
Lagrangian term for Higgs field, over all spacetime:

$$\mathcal{L}(\varphi)=\frac{1}{2}(\partial\varphi)^2+\frac{1}{2}m^2\varphi^2-\frac{\lambda^2}{4}\varphi^4$$

"potential energy": (recall: *L = T-U*)

$U(\varphi) = -1/2\, m^2\varphi^2 + \lambda^2/4\, \varphi^4$

V(Φ) AND SPONTANEOUS SYMMETRY BREAKING

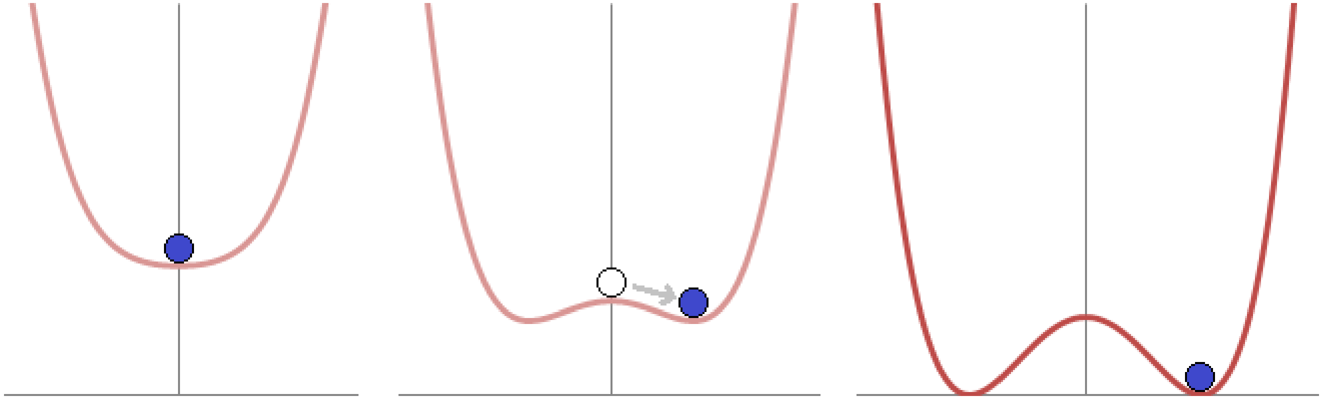


Calculation must be formulated in terms of deviations from one or the other of these ground states.

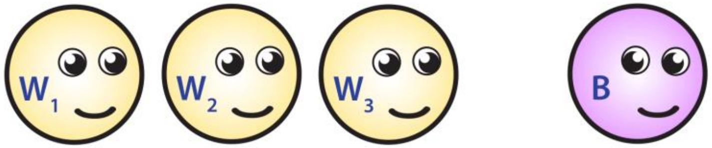
- To get a correct interpretation of the Lagrangian, and to be able to use perturbative calculations, we need to find one of its minima (a ground state) and look at small fluctuations of φ about that minimum.
- For this Lagrangian, the calculations must be formulated as small fluctuations of the field about one of two minima.
- Switching to either one of the two minima in U, effectively breaks the symmetry and introduces a new massive particle!

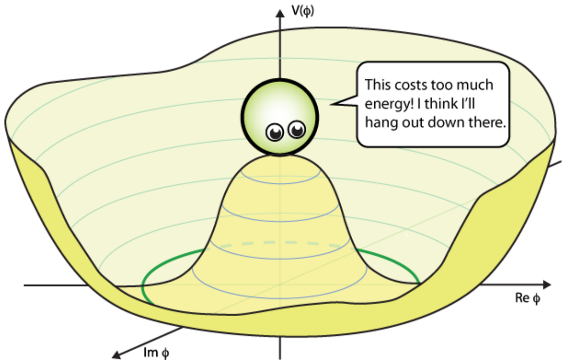
V(Φ) AND SPONTANEOUS SYMMETRY BREAKING

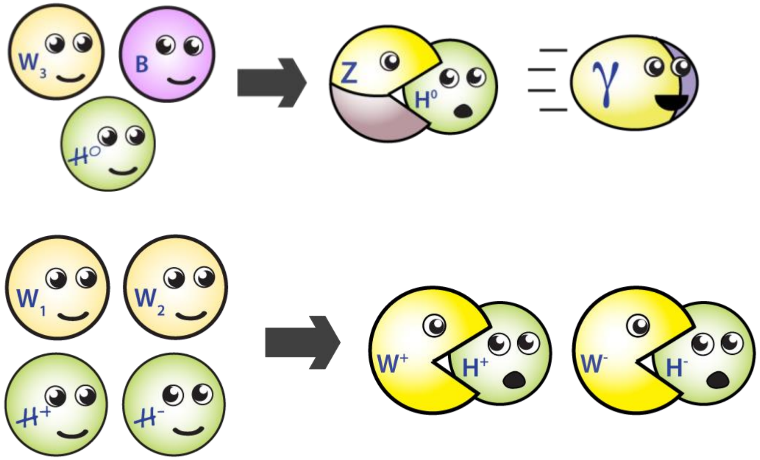
- The phenomenon we have just considered is called *spontaneous symmetry breaking*.
- Why symmetry breaking? Our choice of a ground state “breaks" the obvious reflection symmetry of the original Lagrangian.
- What about the spontaneous part?
  - The choice of a ground state is arbitrary in this system. There is no external agency that favors one over the other, or even forces the choice to begin with.

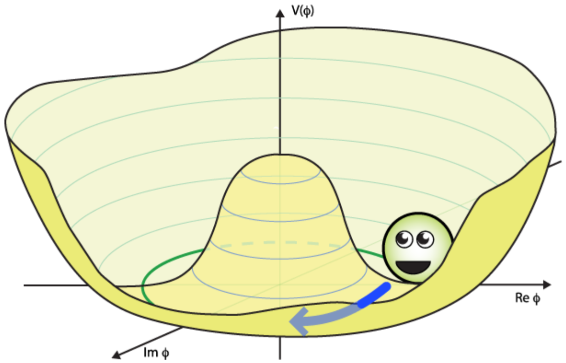


V(Φ) AND SPONTANEOUS SYMMETRY BREAKING









http://www.quantumdiaries.org/

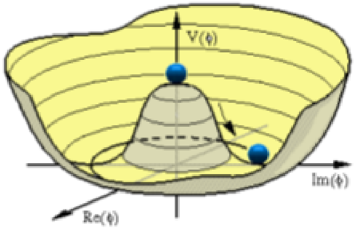
HIGGS MECHANISM

Yukawa Couplings

Higgs Field

$$\boxed{\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'} + \mathcal{L}_{\mathrm{Yuk}} + \mathcal{L}_{\phi'}$$

Massless case, from previous slide



$$\mathcal{L}_\phi = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - V(\phi)$$

Higgs Potential

$$\mathcal{L}_{\mathrm{Yuk}} = c_f(\bar{\psi}_L \psi_R \phi + \bar{\psi}_R \psi_L \phi)$$

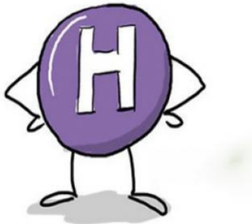
Higgs Fermion Interaction

- When φ "picks a ground state" (i.e. when the gauge symmetry of a Lagrangian is spontaneously broken), all fermion fields and weak bosons become massive!
- Physically, the "massless" bosons acquire mass by interacting with a new "massive" scalar field called the Higgs field. Hence, this process is called the Higgs mechanism.

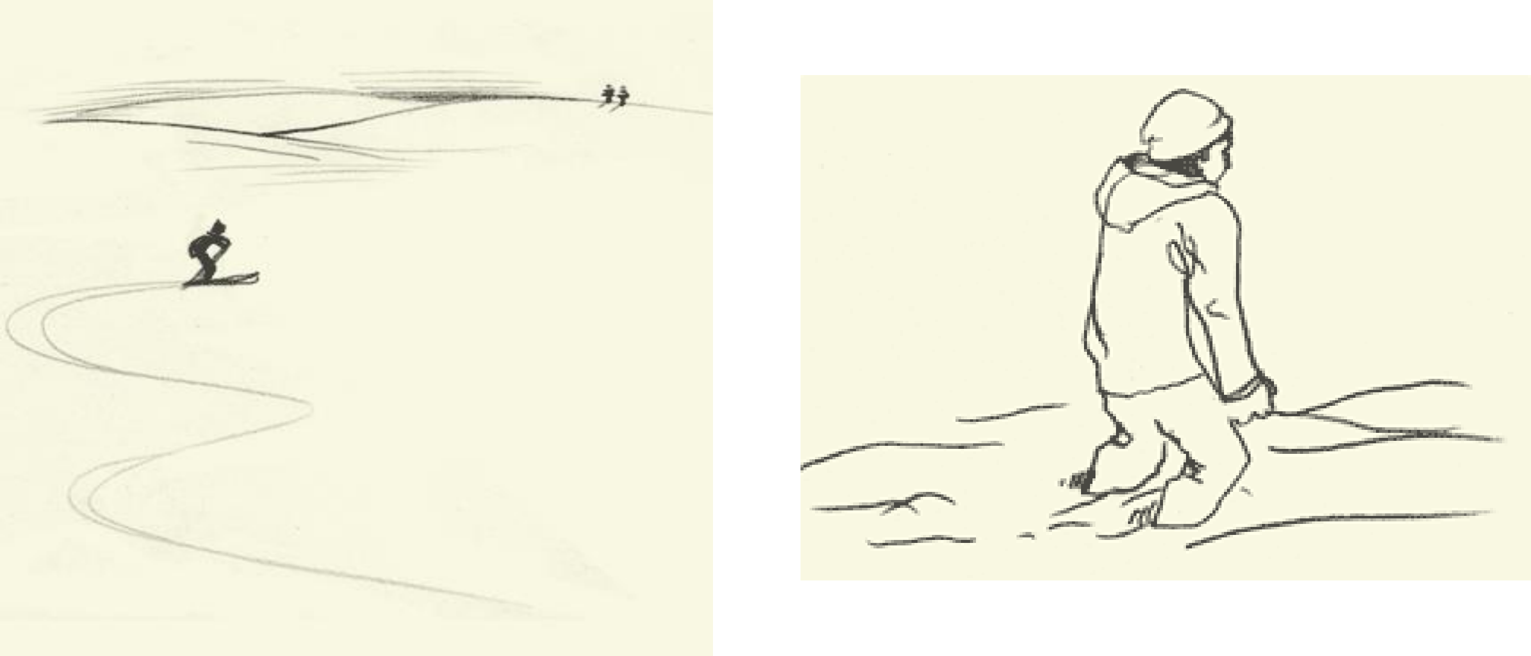
ELECTROWEAK UNIFICATION

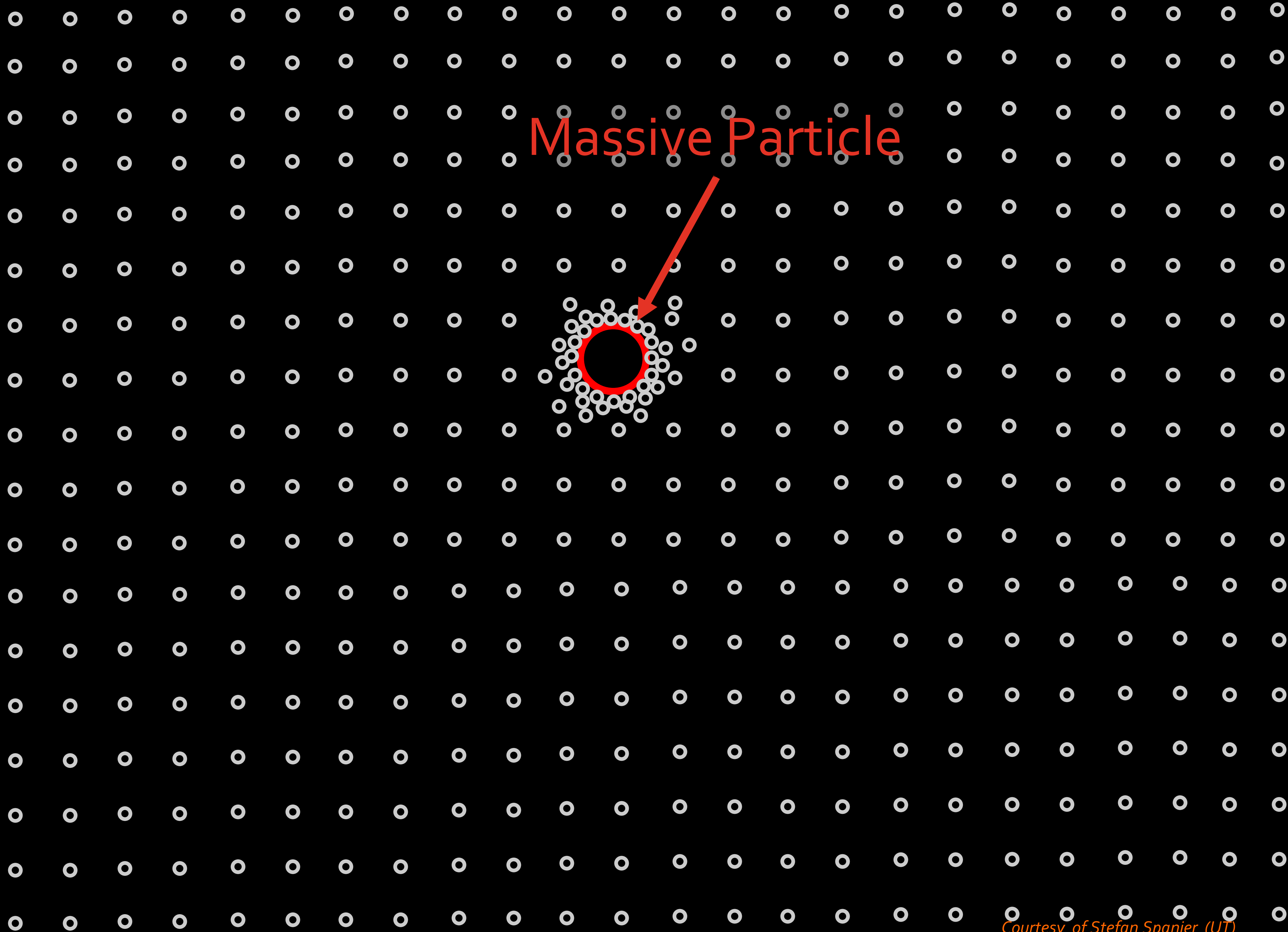
- In the end, the SU(2)xU(1) part of the Standard Model is called the *electroweak theory*, because electromagnetism and the weak force start out mixed together in this overall gauge symmetry.
- SU(2)xU(1) predicts four massless bosons, which are not apparent at everyday energies.
- Analogous to our simple example, the ground state of the SU(2)xU(1) theory (where we live) is one in which this gauge symmetry is **hidden**.
- Result: the four massless gauge bosons appear to us as the massive W^+, W^-, Z and the massless photon. The explicit U(1) symmetry of QED is preserved.

HIGGS MECHANISM



- A force field that permeates the Universe and slows particles down to below the speed of light. This is the equivalent to having mass.





Courtesy of Stefan Spanier (UT)

THE HIGGS PARTICLE

- Spin-0 (scalar) particle which gives massive force mediators their mass.

To understand the Higgs mechanism, imagine that a room full of physicists chattering quietly is like space filled with the Higgs field ...

THE HIGGS PARTICLE

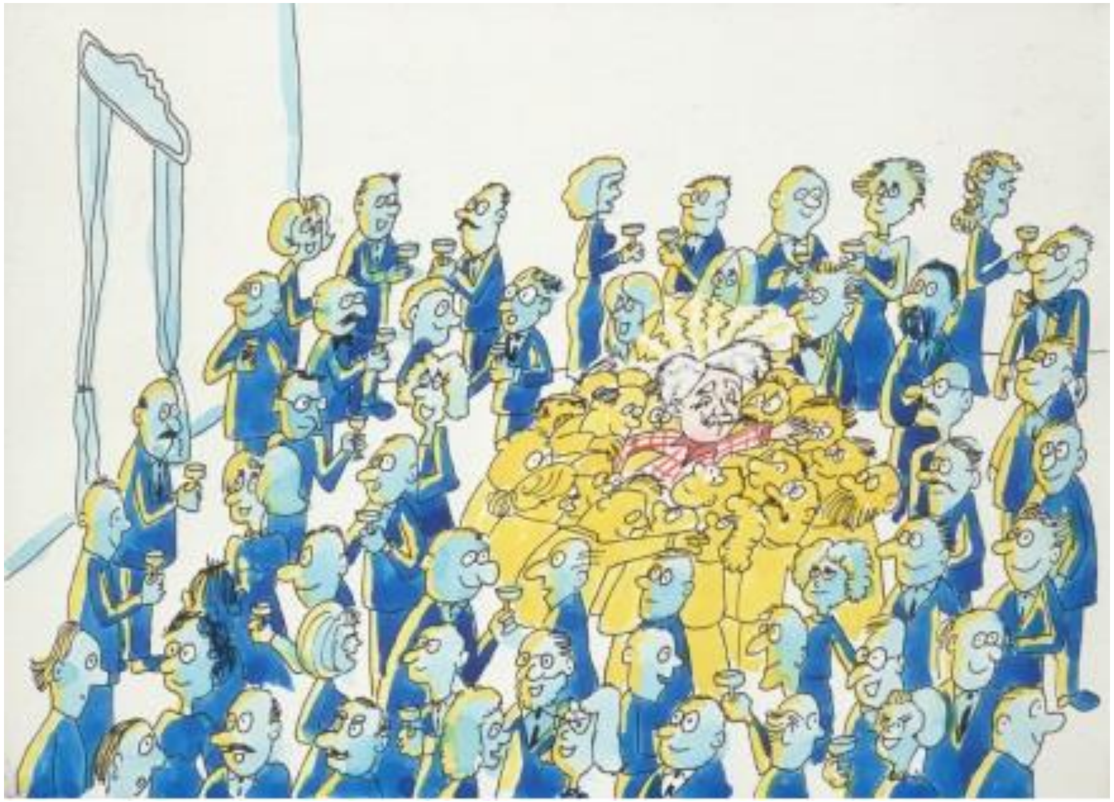
- Spin-0 (scalar) particle which gives massive force mediators their mass.

… a well-known scientist walks in, creating a disturbance as he moves across the room and attracting a cluster of admirers with each step …

THE HIGGS PARTICLE

- Spin-0 (scalar) particle which gives massive force mediators their mass.

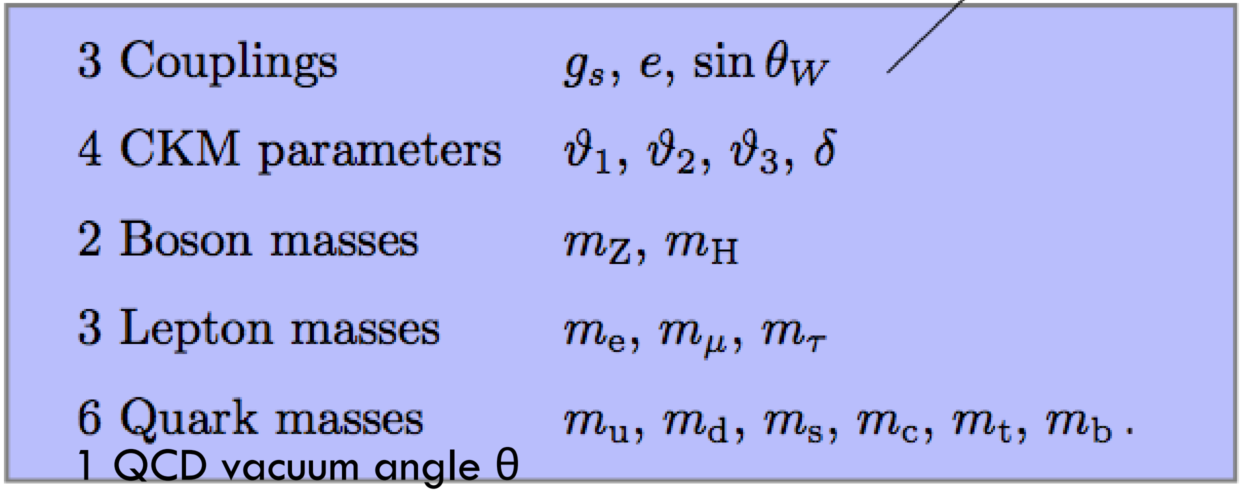
... this increases his resistance to movement, in other words, he acquires mass, just like a particle moving through the Higgs field...



THE STANDARD MODEL
HOW SIMPLE, REALLY?

- The Standard Model does not predict:

Determine experimentally



3 Couplings	g_s, e, $\sin\theta_W$
4 CKM parameters	ϑ_1, ϑ_2, ϑ_3, δ
2 Boson masses	m_Z, m_H
3 Lepton masses	m_e, m_μ, m_τ
6 Quark masses	m_u, m_d, m_s, m_c, m_t, m_b .

1 QCD vacuum angle θ

19 free SM parameters
no neutrino masses

$$\begin{aligned} m_W^2 &= \tfrac{1}{2}\, g^2\, \rho_0^2 \\ m_Z^2 &= \tfrac{1}{2}\, (g^2 + g'^2)\, \rho_0^2 \\ m_H^2 &= 4\, \lambda \rho_0^2 \end{aligned}$$

$$\begin{aligned} g &= e/\sin\theta_W \\ g' &= e/\cos\theta_W \end{aligned}$$

$$m_f = c_f \rho_0$$

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19 free SM parameters

Plus 7 from neutrino mass

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$$\begin{aligned} g &= e/\sin\theta_W \\ g' &= e/\cos\theta_W \end{aligned}$$

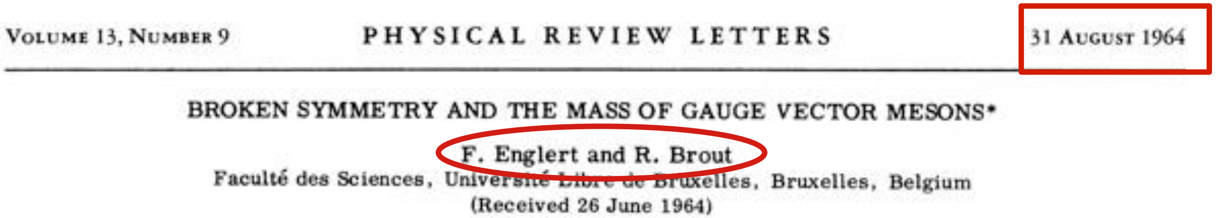
$$m_f = c_f \rho_0$$

THE STANDARD MODEL
PREDICTS RELATIONSHIPS

- All observables can be predicted in terms of **26** free parameters (including neutrino masses, mixing parameters).
- If we have > 26 measurements of those observables, we overconstrain the SM.
  - Overconstrain: we don't have any more ad hoc inputs AND we can test the consistency of the model.
- In practice:
  - Pick well measured set of observables.
  - Calculate other observables in terms of these well known quantities.
  - Test predictions, measure observables, and compare to theory.

THE HIGGS BOSON DISCOVERY

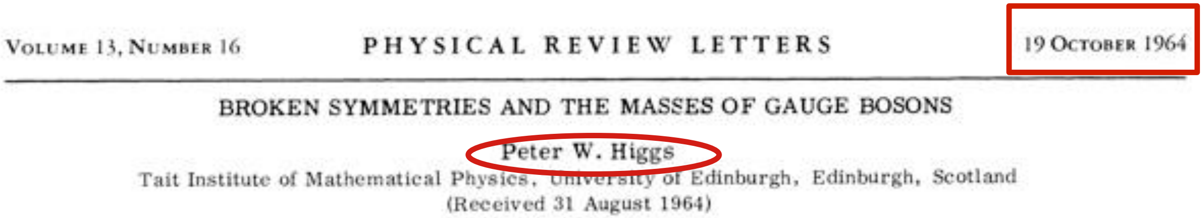
Higgs Boson
Predicted in 1964 and discovered at the LHC in 2012!



Volume 13, Number 9 PHYSICAL REVIEW LETTERS 31 August 1964

BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

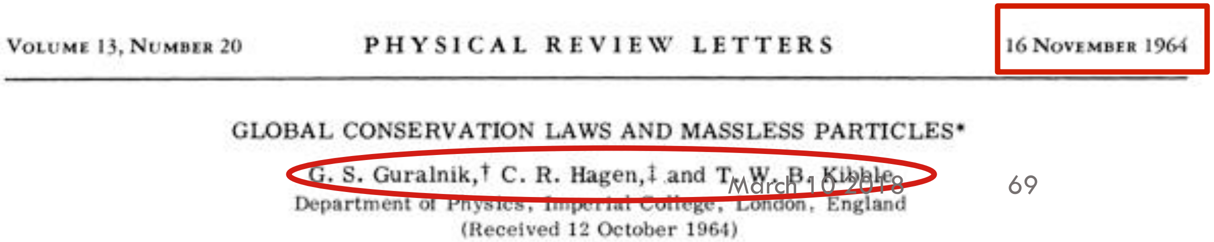
F. Englert and R. Brout
Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium
(Received 26 June 1964)



Volume 13, Number 16 PHYSICAL REVIEW LETTERS 19 October 1964

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

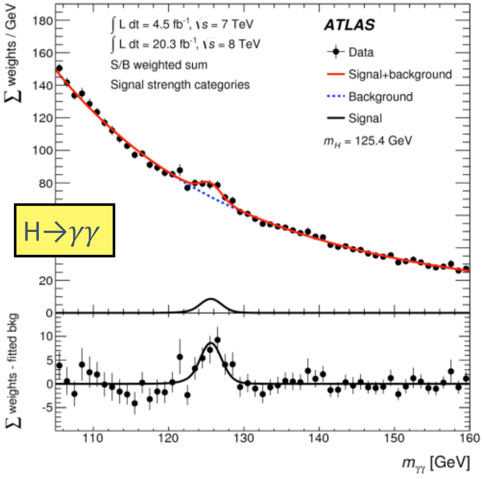


Volume 13, Number 20 PHYSICAL REVIEW LETTERS 16 November 1964

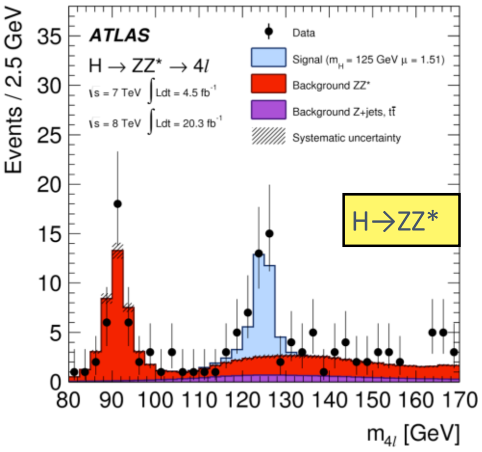
GLOBAL CONSERVATION LAWS AND MASSLESS PARTICLES*

G. S. Guralnik,† C. R. Hagen,‡ and T. W. B. Kibble
Department of Physics, Imperial College, London, England
(Received 12 October 1964)

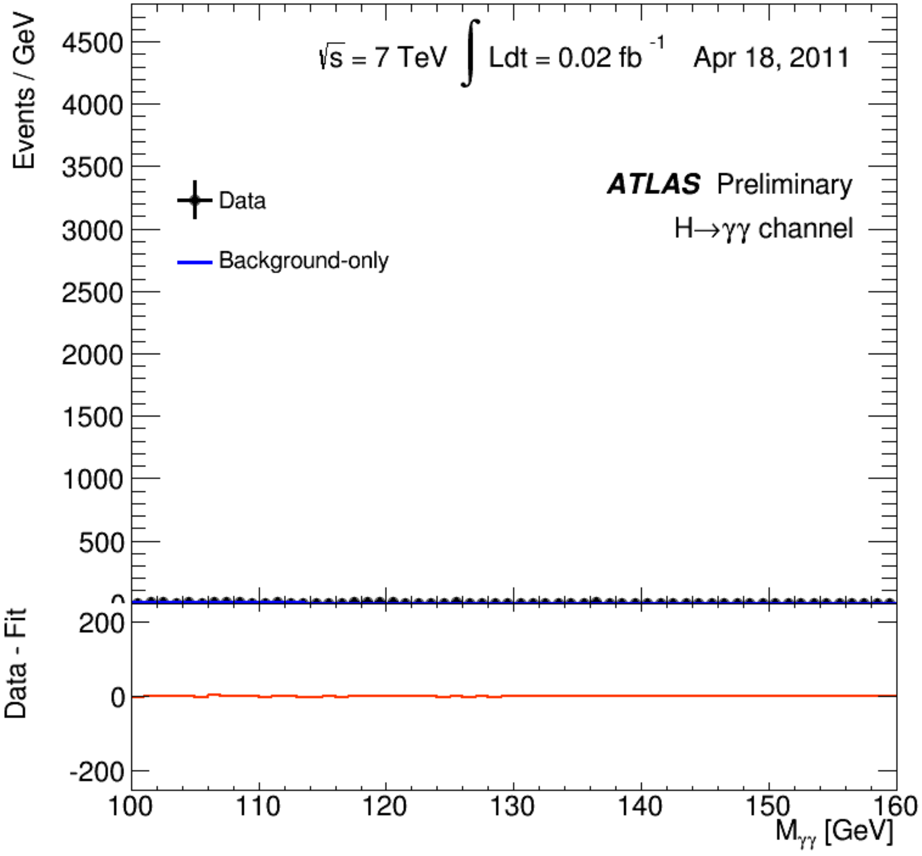
THE HIGGS DISCOVERY (BY ATLAS + CMS)

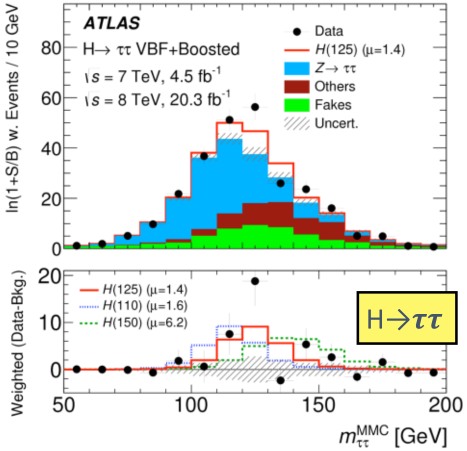


H→γγ

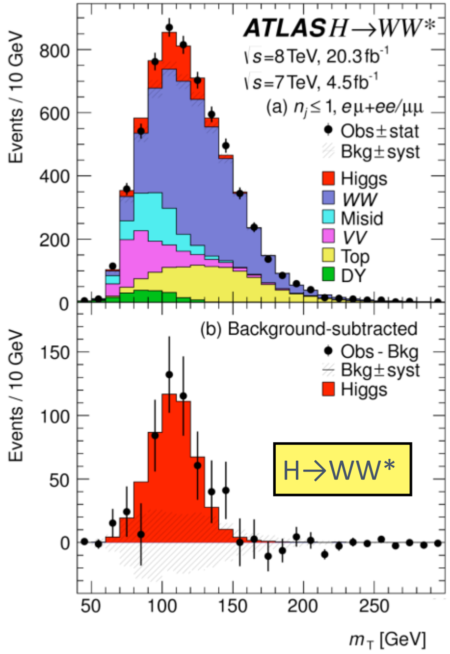


H→ZZ*





H→ττ



H→WW*

THE MOST PRECISELY TESTED THEORY

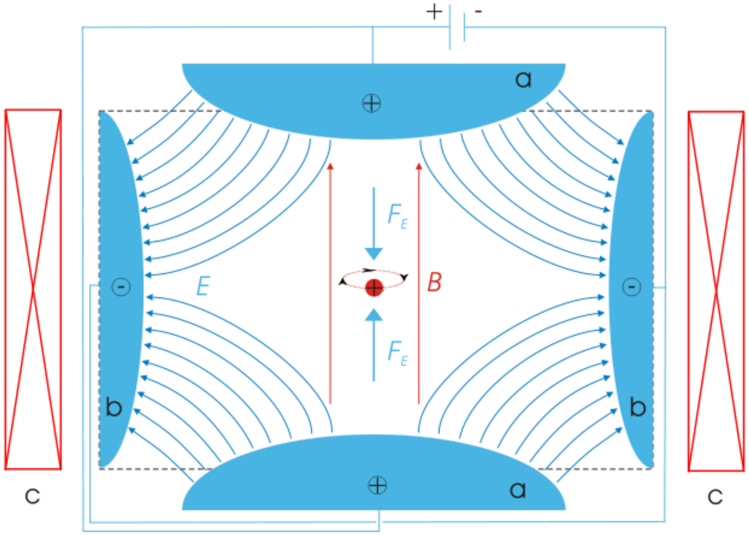
QED

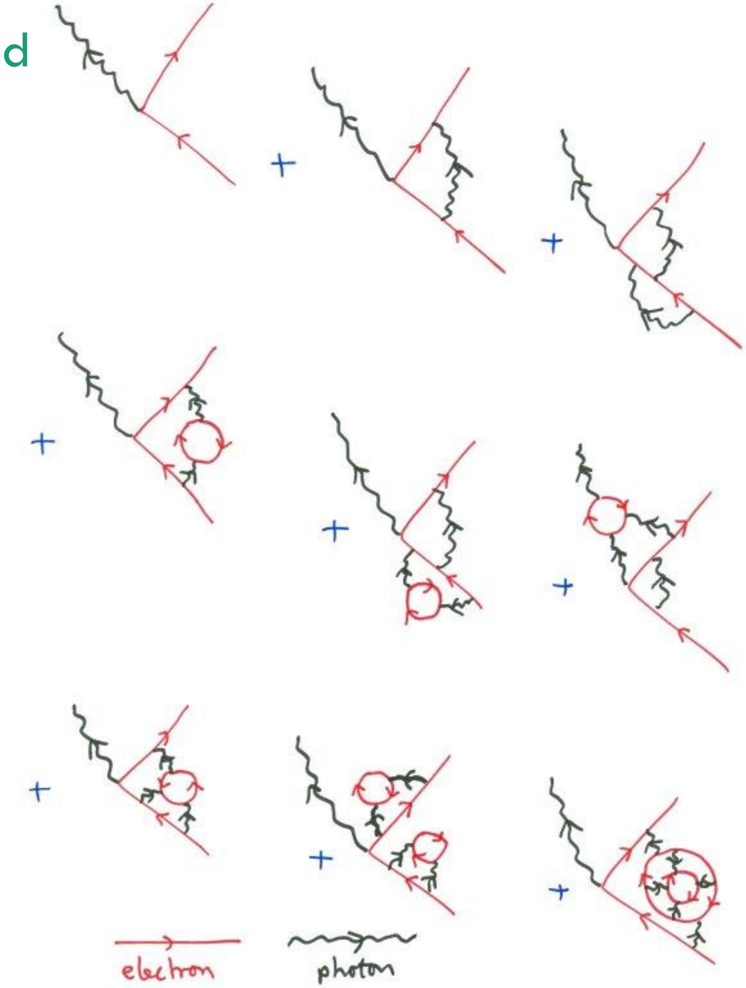
- Magnetic moment of the electron calculated up to order α^5 in QED:

$$a_e = 0.001\,159\,652\,181\,643(764)$$

- Measured in single electron Penning trap experiments:

$$a_e = 0.001\,159\,652\,180\,73(28)$$





THE MOST PRECISELY TESTED THEORY
QED

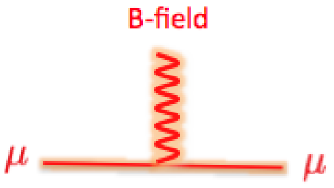
- Equivalent measurements in muons are an active area of research
- Very small differences between prediction:

$$\alpha_\mu^{\text{SM}} = \alpha_\mu^{\text{QED}} + \alpha_\mu^{\text{EW}} + \alpha_\mu^{\text{Hadron}} = 0.001\,165\,9\boxed{18}\,04(51)$$

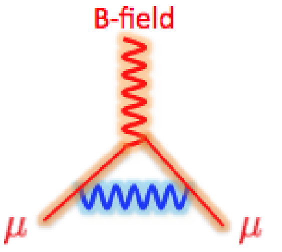
- And measurement:

$$a_\mu = 0.001\,165\,9\boxed{20}\,91(54)(33),$$

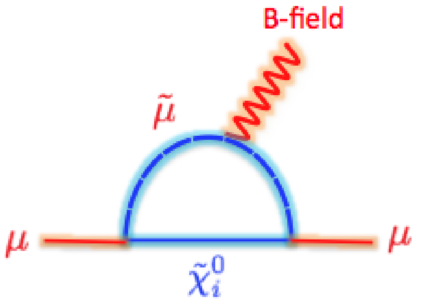
- Might indicate **new** physics beyond the Standard Model



"Dirac Interaction"
g=2

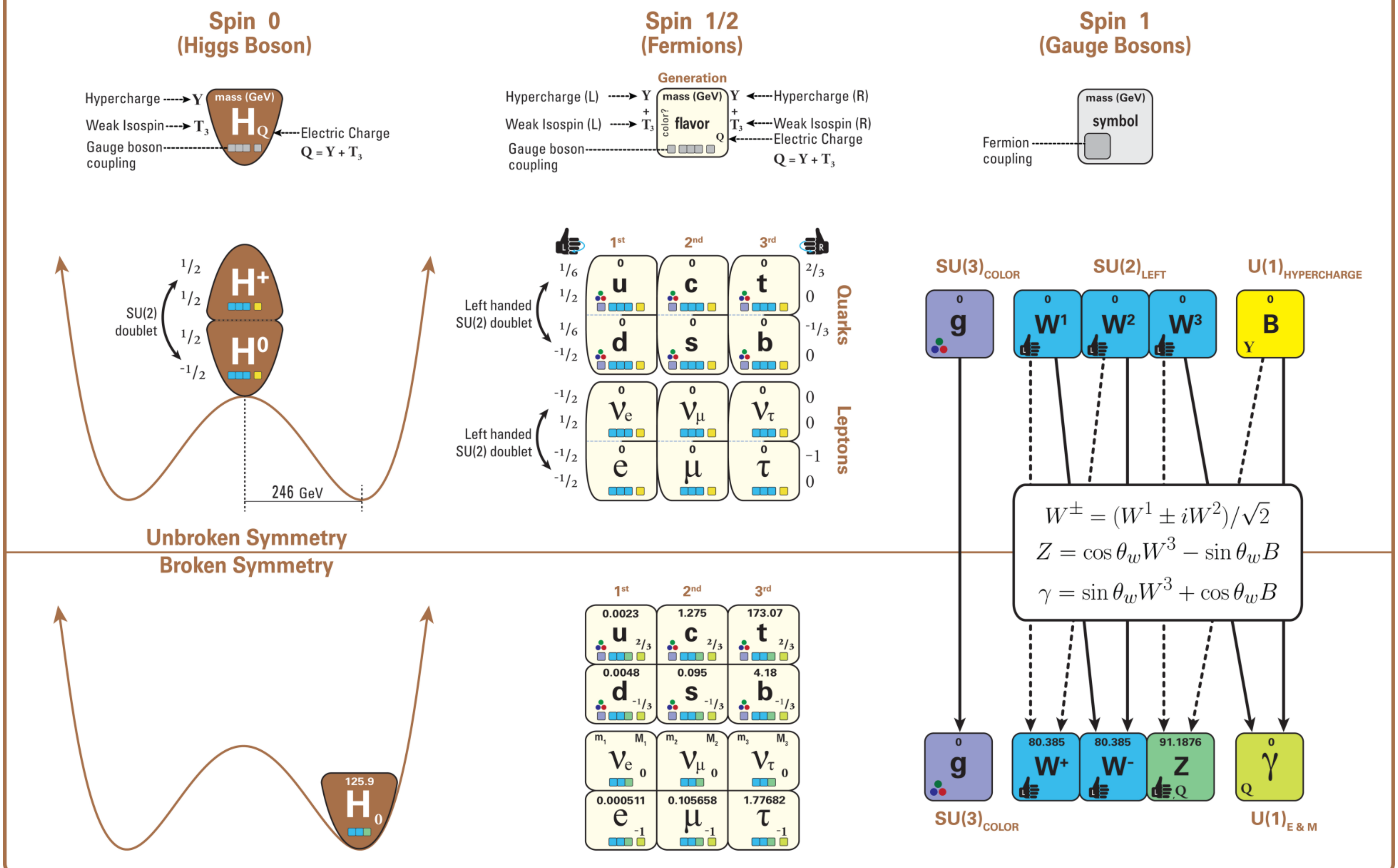


QED "Schwinger Interaction"
(g-2)=α/π



BSM SUSY interaction

The Standard Model of Particle Physics



Unbroken symmetry (fermions, mass 0):

Y (L)	T_3 (L)	1st	2nd	3rd	Y (R)	T_3 (R)
1/6	1/2	u	c	t	2/3	0
1/6	-1/2	d	s	b	-1/3	0
-1/2	1/2	ν_e	ν_μ	ν_τ	0	0
-1/2	-1/2	e	μ	τ	-1	0

Unbroken symmetry (gauge bosons, mass 0): g (SU(3)$_{COLOR}$); W^1, W^2, W^3 (SU(2)$_{LEFT}$); B, Y (U(1)$_{HYPERCHARGE}$)

$$W^{\pm} = (W^1 \pm iW^2)/\sqrt{2}$$
$$Z = \cos\theta_w W^3 - \sin\theta_w B$$
$$\gamma = \sin\theta_w W^3 + \cos\theta_w B$$

Broken symmetry (fermions, mass in GeV and charge):

1st	2nd	3rd
0.0023 u 2/3	1.275 c 2/3	173.07 t 2/3
0.0048 d -1/3	0.095 s -1/3	4.18 b -1/3
m_1 M_1 ν_e 0	m_2 M_2 ν_μ 0	m_3 M_3 ν_τ 0
0.000511 e -1	0.105658 μ -1	1.77682 τ -1

Broken symmetry (gauge bosons, mass in GeV):

g	W^+	W^-	Z	γ
0	80.385	80.385	91.1876	0

THE STANDARD MODEL LAGRANGIAN

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm Dirac} + \mathcal{L}_{\rm mass} + \mathcal{L}_{\rm gauge} + \mathcal{L}_{{\rm gauge}/\psi} \ . \qquad (1)$$

Here,

$$\mathcal{L}_{\rm Dirac} = i\bar{e}^i_{\rm L}\partial\!\!\!/ e^i_{\rm L} + i\bar{\nu}^i_{\rm L}\partial\!\!\!/ \nu^i_{\rm L} + i\bar{e}^i_{\rm R}\partial\!\!\!/ e^i_{\rm R} + i\bar{u}^i_{\rm L}\partial\!\!\!/ u^i_{\rm L} + i\bar{d}^i_{\rm L}\partial\!\!\!/ d^i_{\rm L} + i\bar{u}^i_{\rm R}\partial\!\!\!/ u^i_{\rm R} + i\bar{d}^i_{\rm R}\partial\!\!\!/ d^i_{\rm R} \ ; \qquad (2)$$

$$\mathcal{L}_{\rm mass} = -v\left(\lambda^i_e\bar{e}^i_{\rm L}e^i_{\rm R} + \lambda^i_u\bar{u}^i_{\rm L}u^i_{\rm R} + \lambda^i_d\bar{d}^i_{\rm L}d^i_{\rm R} + {\rm h.c.}\right) - M^2_{\rm W}W^+_\mu W^{-\mu} - \frac{M^2_W}{2\cos^2\theta_W}Z_\mu Z^\mu \ ; \qquad (3)$$

$$\mathcal{L}_{\rm gauge} = -\frac{1}{4}(G^a_{\mu\nu})^2 - \frac{1}{2}W^+_{\mu\nu}W^{-\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{WZA} \ , \qquad (4)$$

where

$$\begin{aligned} G^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_3 f^{abc}A^b_\mu A^c_\nu \\ W^\pm_{\mu\nu} &= \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu \\ Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \ , \qquad (5) \end{aligned}$$

and

$$\begin{aligned} \mathcal{L}_{WZA} = \ & ig_2\cos\theta_{\rm W}\left[\left(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu\right)\partial^\mu Z^\nu + W^+_{\mu\nu}W^{-\mu}Z^\nu - W^-_{\mu\nu}W^{+\mu}Z^\nu\right] \\ & +ie\left[\left(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu\right)\partial^\mu A^\nu + W^+_{\mu\nu}W^{-\mu}A^\nu - W^-_{\mu\nu}W^{+\mu}A^\nu\right] \\ & +g^2_2\cos^2\theta_{\rm W}\left(W^+_\mu W^-_\nu Z^\mu Z^\nu - W^+_\mu W^{-\mu}Z_\nu Z^\nu\right) \\ & +g^2_2\left(W^+_\mu W^-_\nu A^\mu A^\nu - W^+_\mu W^{-\mu}A_\nu A^\nu\right) \\ & +g_2 e\cos\theta_{\rm W}\left[W^+_\mu W^-_\nu\left(Z^\mu A^\nu + Z^\nu A^\mu\right) - 2W^+_\mu W^{-\mu}Z_\nu A^\nu\right] \\ & +\frac{1}{2}g^2_2\left(W^+_\mu W^-_\nu\right)\left(W^{+\mu}W^{-\nu} - W^{+\nu}W^{-\mu}\right) \ ; \qquad (6) \end{aligned}$$

and

$$\mathcal{L}_{{\rm gauge}/\psi} = -g_3 A^a_\mu J^{\mu a}_{(3)} - g_2\left(W^+_\mu J^\mu_{W^+} + W^-_\mu J^\mu_{W^-} + Z_\mu J^\mu_Z\right) - eA_\mu J^\mu_A \ , \qquad (7)$$

where

$$\begin{aligned} J^{\mu a}_{(3)} &= \bar{u}^i\gamma^\mu T^a_{(3)}u^i + \bar{d}^i\gamma^\mu T^a_{(3)}d^i \\ J^\mu_{W^+} &= \frac{1}{\sqrt{2}}\left(\bar{\nu}^i_{\rm L}\gamma^\mu e^i_{\rm L} + V^{ij}\bar{u}^i_{\rm L}\gamma^\mu d^j_{\rm L}\right) \\ J^\mu_{W^-} &= (J^\mu_{W^+})^* \\ J^\mu_Z &= \frac{1}{\cos\theta_{\rm W}}\left[\frac{1}{2}\bar{\nu}^i_{\rm L}\gamma^\mu\nu^i_{\rm L} + \left(-\frac{1}{2} + \sin^2\theta_{\rm W}\right)\bar{e}^i_{\rm L}\gamma^\mu e^i_{\rm L} + (\sin^2\theta_{\rm W})\bar{e}^i_{\rm R}\gamma^\mu e^i_{\rm R}\right. \\ & \quad + \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_{\rm W}\right)\bar{u}^i_{\rm L}\gamma^\mu u^i_{\rm L} + \left(-\frac{2}{3}\sin^2\theta_{\rm W}\right)\bar{u}^i_{\rm R}\gamma^\mu u^i_{\rm R} \\ & \quad \left. + \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_{\rm W}\right)\bar{d}^i_{\rm L}\gamma^\mu d^i_{\rm L} + \left(\frac{1}{3}\sin^2\theta_{\rm W}\right)\bar{d}^i_{\rm R}\gamma^\mu d^i_{\rm R}\right] \\ J^\mu_A &= (-1)\bar{e}^i\gamma^\mu e^i + \left(\frac{2}{3}\right)\bar{u}^i\gamma^\mu u^i + \left(-\frac{1}{3}\right)\bar{d}^i\gamma^\mu d^i \ . \qquad (8) \end{aligned}$$

THE STANDARD MODEL LAGRANGIAN

SUMMARY

- The Standard Model is a framework that describes the elementary particles and their electromagnetic, weak and strong interactions.
  - Furthermore, it provides an explanation for the electromagnetic and weak interactions in terms of a spontaneously broken electroweak symmetry.
- The Standard Model predicts cross-sections, couplings.
- Latest success: the discovery of the Higgs boson!

- However, the Standard Model is not completely satisfactory:
  - 26 free parameters…
  - We know it is incomplete: it does not fully describe nature.

STANDARD MODEL LIMITATIONS

STANDARD MODEL LIMITATIONS
OBSERVED DISCREPANCIES

- How can we describe gravity and the three Standard Model forces consistently?
  - We know gravity exists, but incompatible with Gauge theories…
- What is the mass of neutrinos, and what is its origin?
  - We know neutrino mass is finite.
  - Simple extensions to the Standard Model may be able to accommodate neutrino mass.
- What are dark matter and dark energy?
  - We know they exist but we don't know what they are. Theories beyond the Standard Model might offer solutions…
- How did we end up in a matter dominated universe?
  - The Standard Model might explain this…

STANDARD MODEL LIMITATIONS

PHILOSOPHICAL NUISANCES (“WHY” RATHER THAN “WHAT/HOW”)

- Why are there three generations of matter?
  - “Because” this is required for CP violation? And CP violation is required for a matter-dominated universe: us?
    - Anthropic principle – perhaps not great Science…
- Why are some of the Standard Model parameters “unnaturally” large / small?
  - Parameters that look like “1” give theorists a warm and fuzzy feeling.
- Why are the masses what they are?
  - And why are they so different?! $m_t/m_\nu \sim 10^{14}$!

FACING THE STANDARD MODEL LIMITATIONS

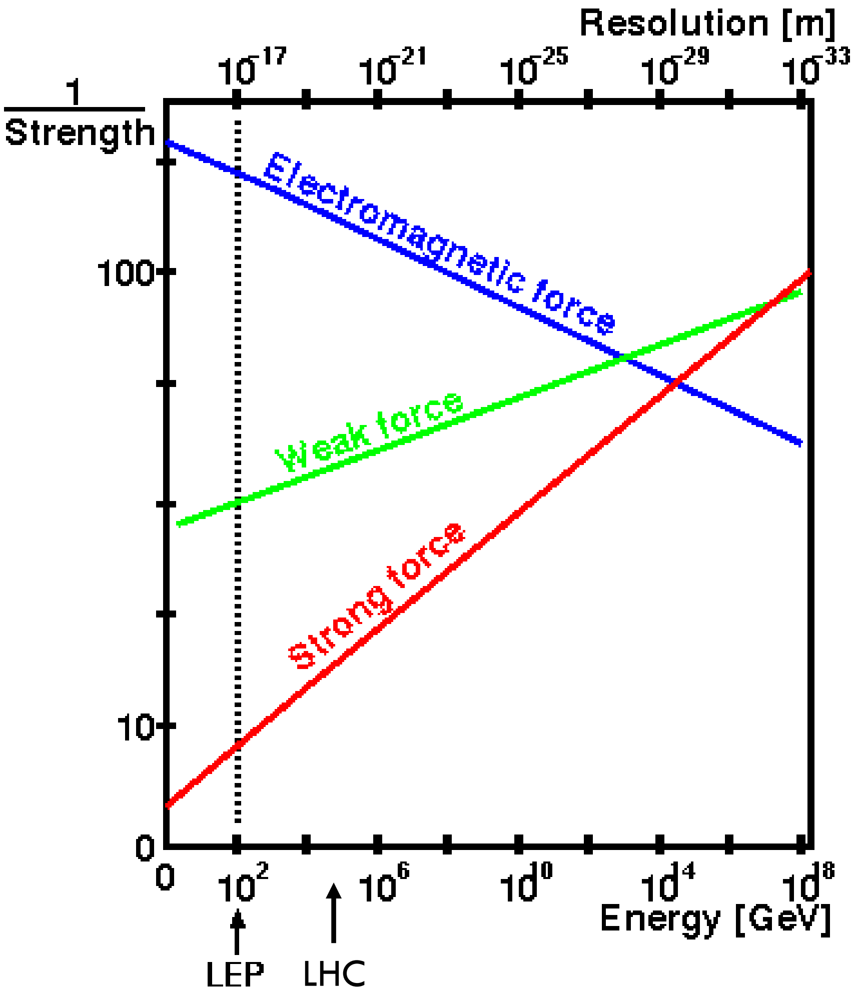
- Over the history of particle physics, a great deal of time has been spent by both theorists and experimentalists to try to resolve the limitations of the Standard Model.
- Generally, the strategy has been to extend the Standard Model in a way or another:
  - Grand Unified Theories.
    - Predicts proton decay.
  - Supersymmetry.
    - Might explain dark matter.
- A longer term, and more ambitious project has been to formulate a Theory of Everything, that includes gravity.
  - So far these theories have lacked predictive power…

GUT SCALE

- Grand unification scale:

  10^{16} GeV

- The Standard Model provides no explanation for what may happen beyond this unification scale, nor why the forces have such different strengths at low energies.



GRAND UNIFIED THEORIES

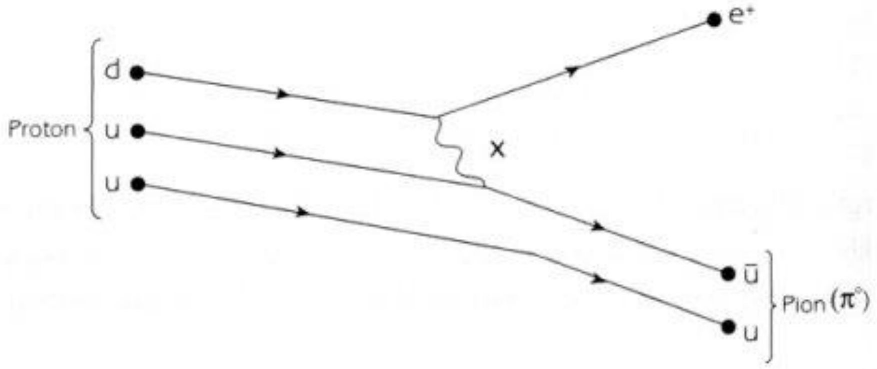
- In the 1970's, people started to think a lot about how to combine the SU(3), SU(2), and U(1) gauge symmetries of the Standard Model into a more fundamental, global symmetry.
- The first such Grand Unified Theory is a 1974 model based on SU(5) symmetry.
- This model groups all of the known fermions – i.e., the leptons and quarks – into multiplets.
- Inside the multiplets, quarks and leptons can couple to each other and transform into one another.
- In essence, this theory imposes a grand symmetry: all fermions, whether quarks or leptons, are fundamentally the same.

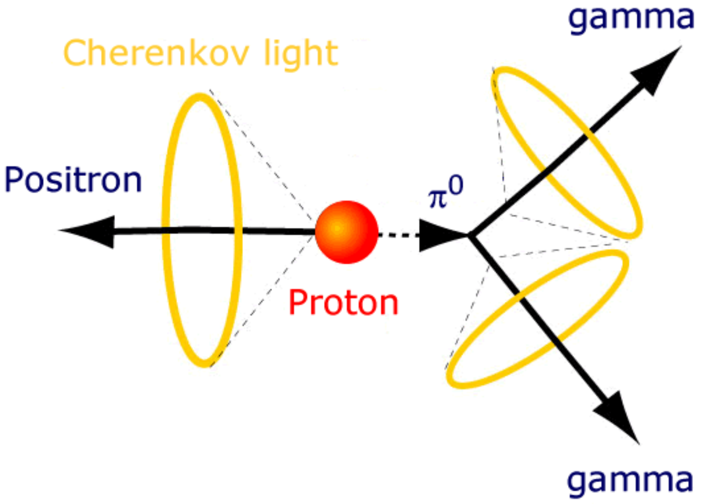
THE SU(5) GUT

- In this early model, interactions between the quarks and leptons are mediated by two new massive bosons, called the **X** and **Y**.
- To conserve electric and color charge, the X and Y have electric charges of -4e/3 and -e/3, and one of three possible colors. They are also incredibly massive, close to the grand unification scale of 10^{16} GeV.
- Hence, including both particles and antiparticles, the model predicts 12 types of X and Y.
- In addition to these 12, there are also 8 gluons, 3 weak bosons, and 1 photon, for a total of 24. This makes sense, for recall that a theory exhibiting SU(n) gauge symmetry requires the existence of n^2-1gauge bosons.

SU(5) GUT OBSERVATIONAL CONSEQUENCES

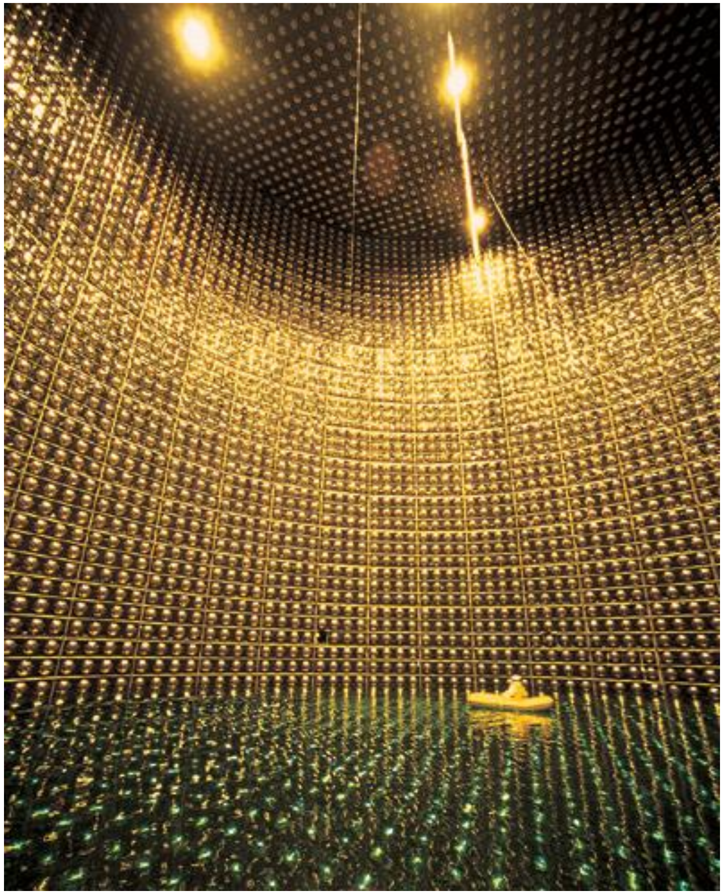
- The SU(5) GUT implies that it would take huge energies to even hope to see an X or Y particle “in the wild”.
- However, even at “normal” (i.e. low) energies, virtual X and Y exchanges can take place.
- This is major: if quarks can decay into leptons via virtual X and Y exchange, then “stable” particles might actually be unstable!
- Example: the proton could possibly decay via exchange of a virtual X.





PROTON DECAY

- The instability of the proton is one of the few tests of GUT physics that would be manifest at everyday energies.
- Computations show that relative to most elementary particles, the proton is very stable; its lifetime according to the SU(5) GUT is 10^{30} years!
- How can we detect such an effect?
- Put many protons together – e.g., in a huge tank of water – and wait for some to decay…
- The Super-Kamiokande water Cherenkov detector, was built to search for proton decay, although it has made major contributions to understanding neutrino oscillations!



PROTON DECAY

Neil did not see a proton decay, but he was very excited about neutrino oscillations!

PROTON DECAY

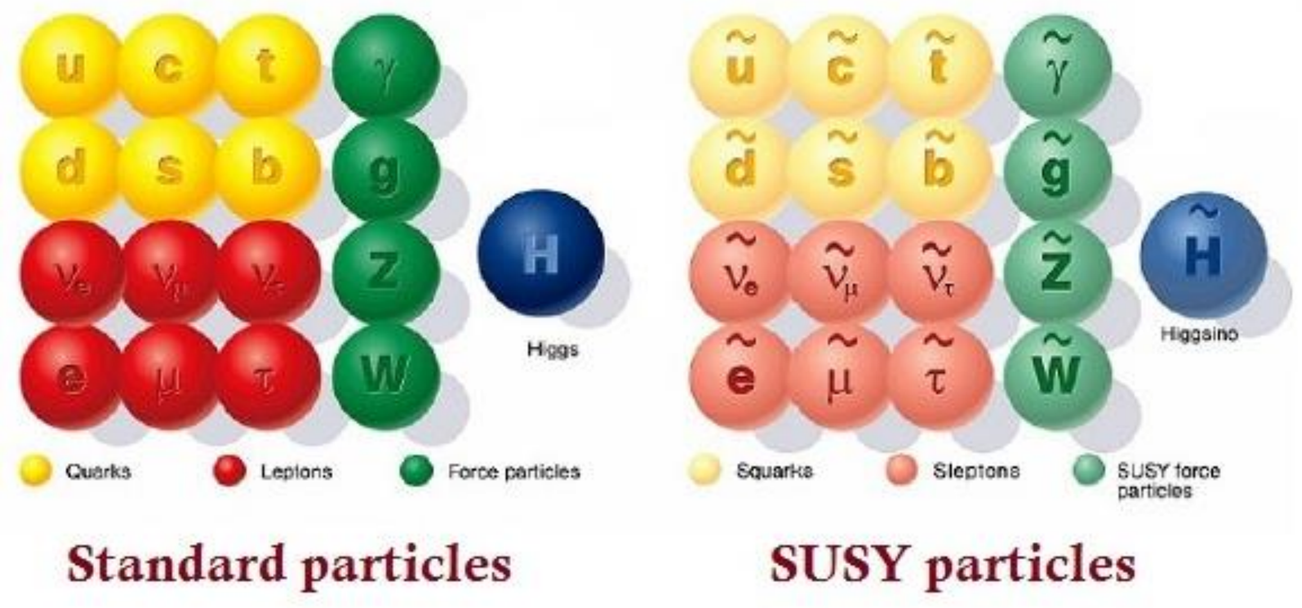
- Even though the proton lifetime is very long, a kiloton of material contains about 10^{32} protons, so about one decay per day should occur in such a sample.
- The Super-Kamiokande detector holds 50 kilotons of water viewed by 11000 photomultiplier tubes. It is located underground, shielded from background noise due to cosmic radiation at the Earth's surface.
- **No** proton decays have been observed despite more than two decades of searching, leading to lower limits on the proton lifetime of about 10^{34} years.
- Is this a disaster for the theory? Not necessarily. GUT physics can be saved if we introduce supersymmetry…

SUPERSYMMETRY

- The idea behind GUTs like SU(5) symmetry is to present different particles as transformed versions of each other.
- The SU(5) GUT treats quarks and leptons as the same underlying "something", unifying the fermions in the Standard Model.
- However, these early approaches to grand unification failed to incorporate the gauge bosons and Higgs scalars of the Standard Model into a unified scheme.
- Hence, some eventually suggested that the bosons and fermions should be united somehow by invoking a new kind of symmetry: supersymmetry (or SUSY for short).
- Aside: theorists did not develop supersymmetry explicitly for the Standard Model; the field started obscurely in the early 1970's during investigations of the spacetime symmetries in quantum field theory.

SUPERSYMMETRY

- So what, exactly, does supersymmetry predict?
- It says that for each known boson, there should be a fermion of identical mass and charge, and similarly for each known fermion, there exists a boson of identical mass and charge.

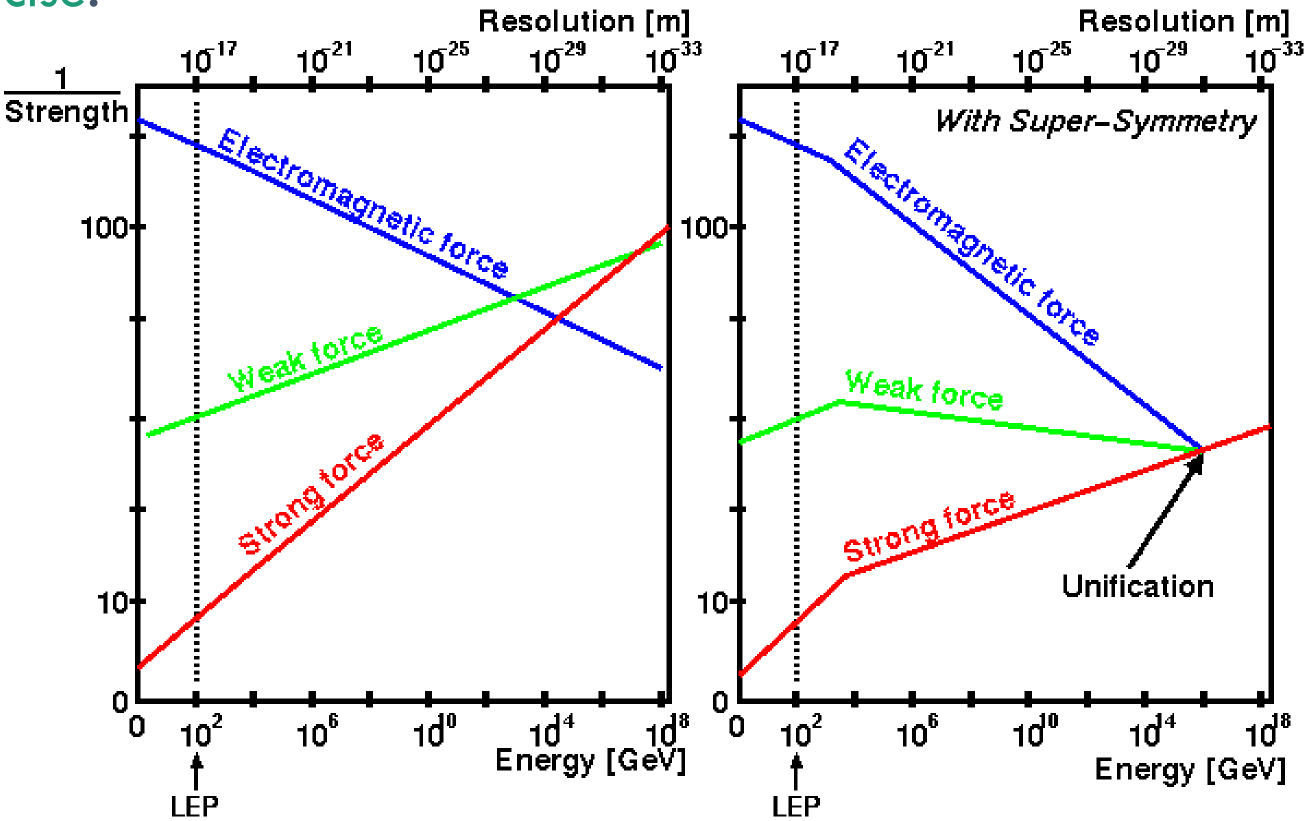


SUPERSYMMETRY

- So what, exactly, does supersymmetry predict?
- It says that for each known boson, there should be a fermion of identical mass and charge, and similarly for each known fermion, there exists a boson of identical mass and charge.
- If this is the case, why haven't we observed a selectron of mass 0.511 MeV?
  - Supersymmetry is a broken symmetry at our energy scale.
  - All sparticles are expected to be more massive than their Standard Model partners.

SUPERSYMMETRY AND THE GUT SCALE

- If instead of the Standard Model, we start from the Minimal Supersymmetric Standard Model, the alignment of the three coupling constants at the GUT scale becomes much more precise.



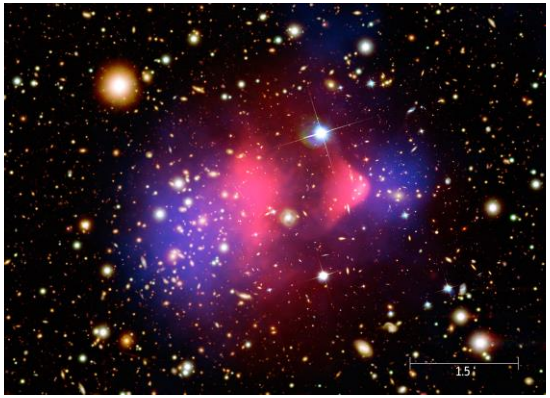
SUPERSYMMETRY AT THE LHC

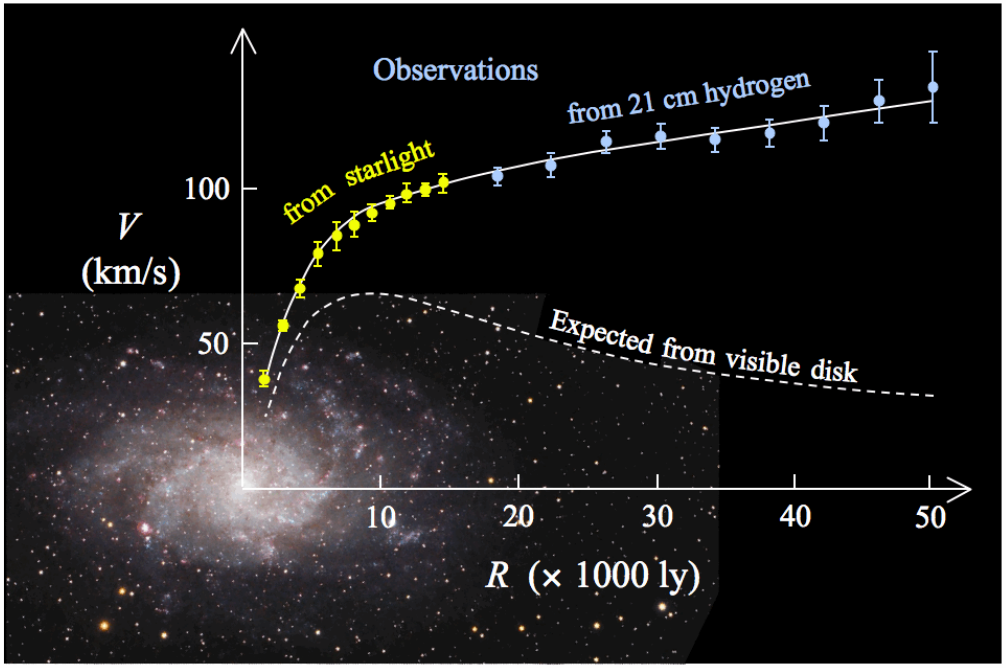
- If SUSY is a symmetry of nature, some of the sparticles might have a mass around 1000 GeV.
- We hope to see evidence for SUSY at the LHC.
  - But so far no candidates have been found…

Excited LHC physicists still haven't found SUSY.

DARK MATTER

- Galaxies rotate faster than expected, implying more mass than what is seen.
- A host of other observations point in the same direction…
- Alternatively, general relativity might need modification.
  - Probably not the solution…
- Scientific consensus leaning towards the weakly interacting massive particle hypothesis.
  - We need to observe it in the lab!





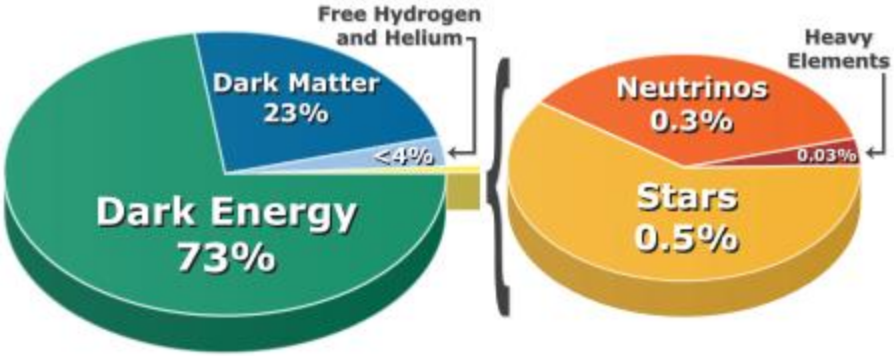
Bullet cluster

SUPERSYMMETRY AND DARK MATTER

- The neutrinos that we know are not good candidates to explain dark matter.
  - They are "dark" but also too light – moving too fast to cluster around galaxies.
- To account for dark matter, we need a particle that looks a lot like a neutrino, but more massive.
  - No electric charge or color.
- Lightest Supersymmetric Particle
  - We haven't seen proton decay so, if SUSY exists, decays of sparticles into particles must be highly supressed.
    - We call this R-parity.
  - Hence the LSP is stable, much heavier than neutrinos, and in several SUSY models, charge neutral.
    - A dark matter candidate!

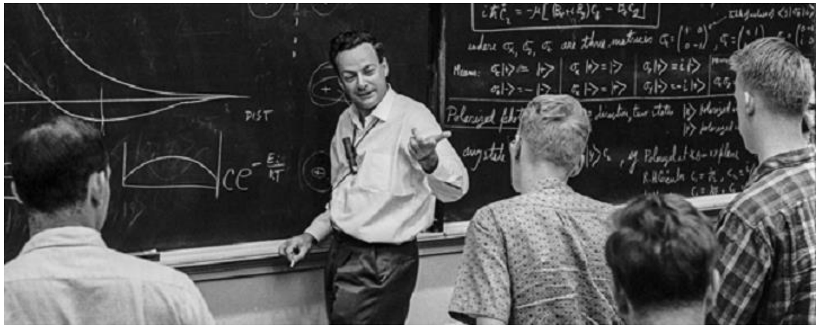
…AND THERE'S ALSO DARK ENERGY

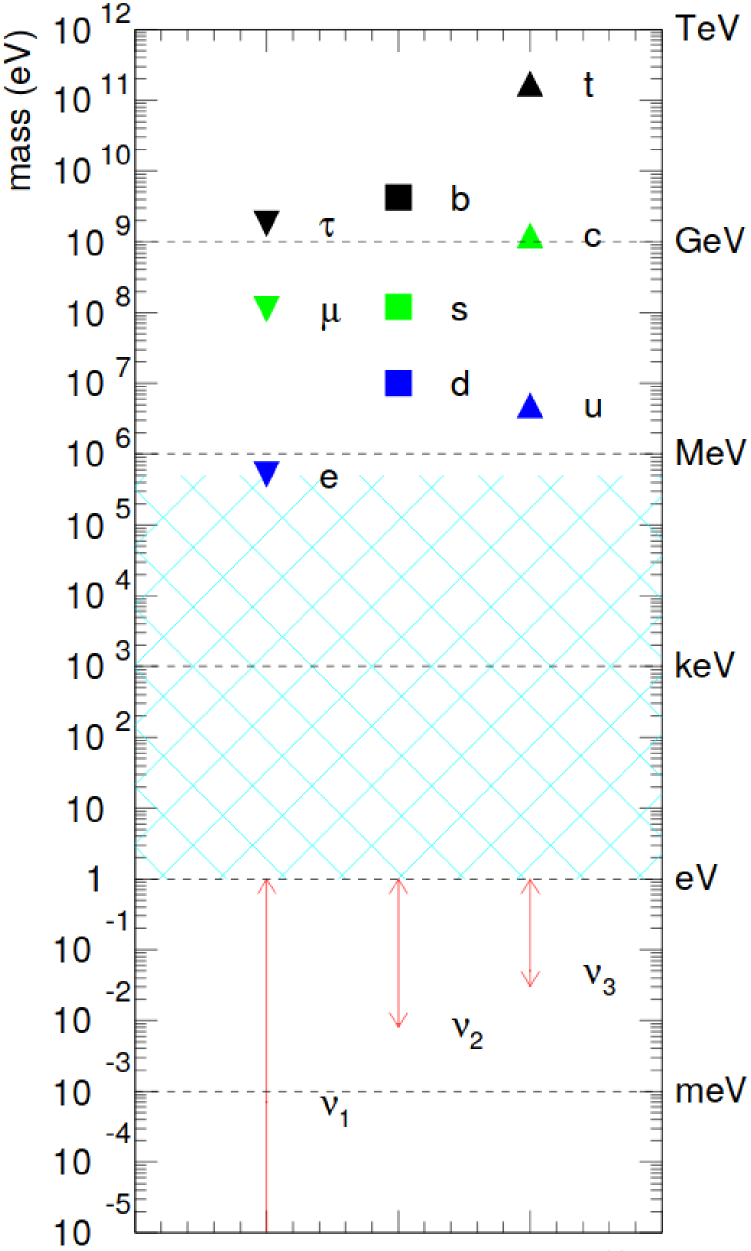
- The universe is expanding at an accelerating rate!
  - This can be accommodated in general relativity by having a non-zero cosmological constant (Λ), also called vacuum energy.
- What is the origin of this vacuum energy? Quantum field theories tend to predict vacuum energies that are far too large…



PARTICLE MASSES

- "There remains one especially unsatisfactory feature [of the Standard Model of particle physics]: the observed masses of the particles, m. There is no theory that adequately explains these numbers. We use the numbers in all our theories, but we do not understand them – what they are, or where they come from. I believe that from a fundamental point of view, this is a very interesting and serious problem."
  - R. P. Feynman



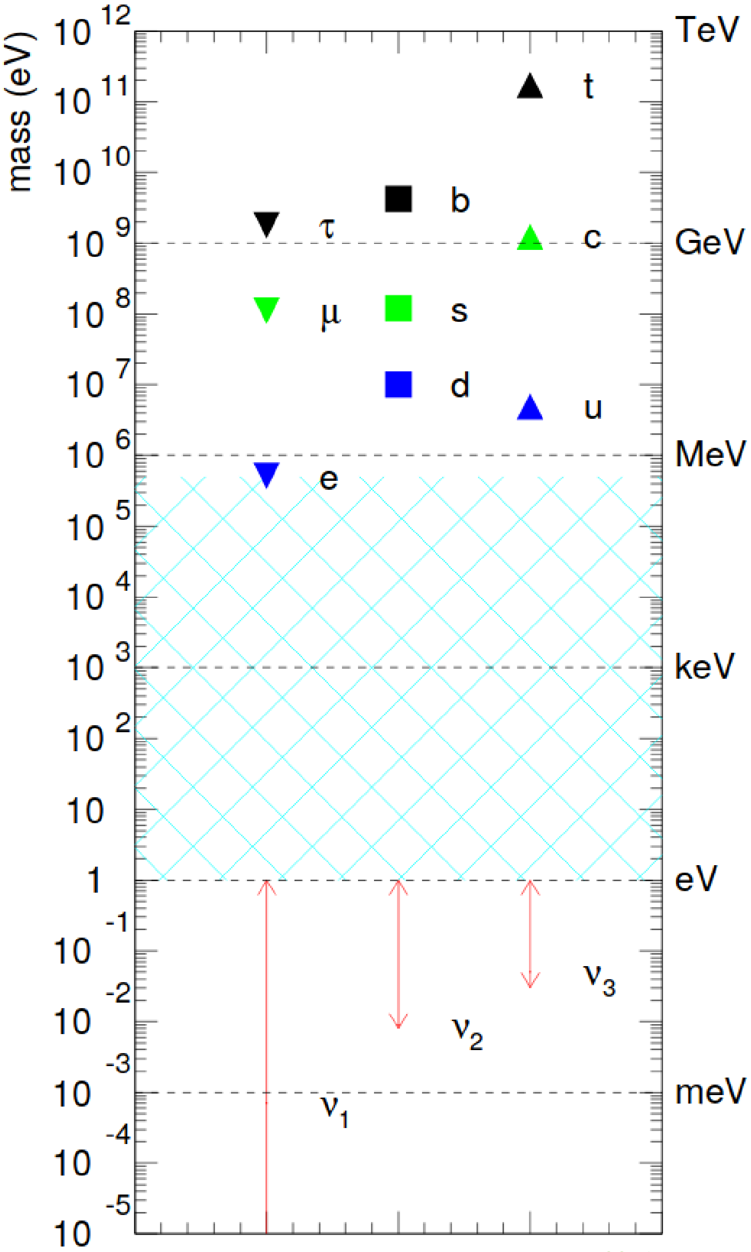


de Gouvêa, Neutrino 2016

PARTICLE MASSES

- Is there a theory that can accurately predict the masses of the fermions?
  - None so far…
  - Would it elucidate the nature of the three generations?

- Why are neutrino masses so much smaller than the other fermion masses?
  - Is the mechanism that generates neutrino masses qualitatively different from the mechanism that generates other fermion masses?



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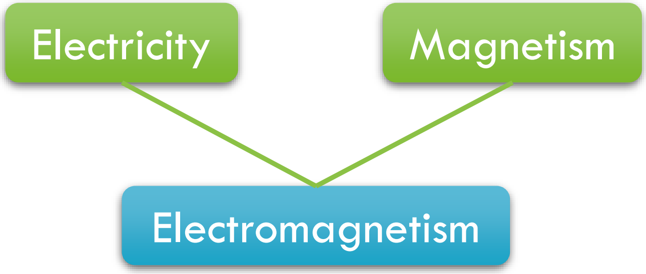
A THEORY OF EVERYTHING?

Electricity

Magnetism

A THEORY OF EVERYTHING?



Electricity

Magnetism

Electromagnetism

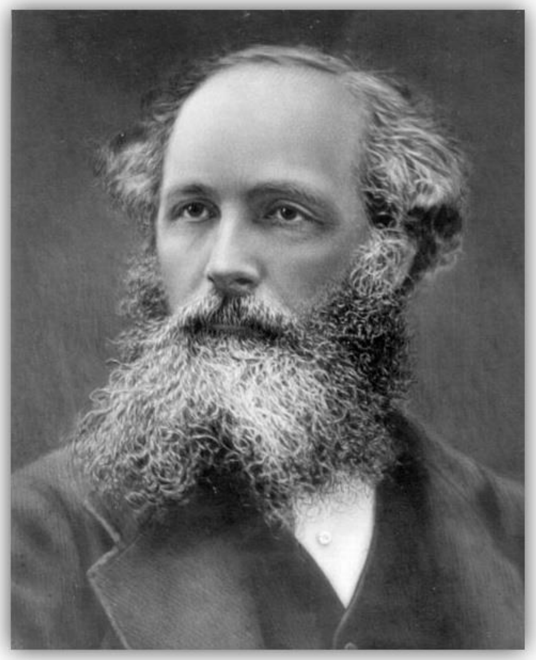
$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

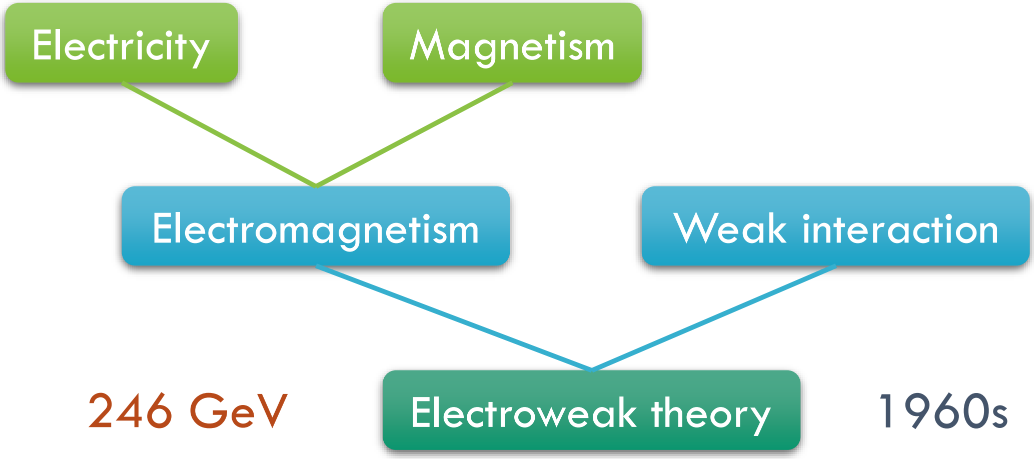
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

James Clerk Maxwell
1800s



A THEORY OF EVERYTHING?



Electricity

Magnetism

Electromagnetism

Weak interaction

246 GeV

Electroweak theory

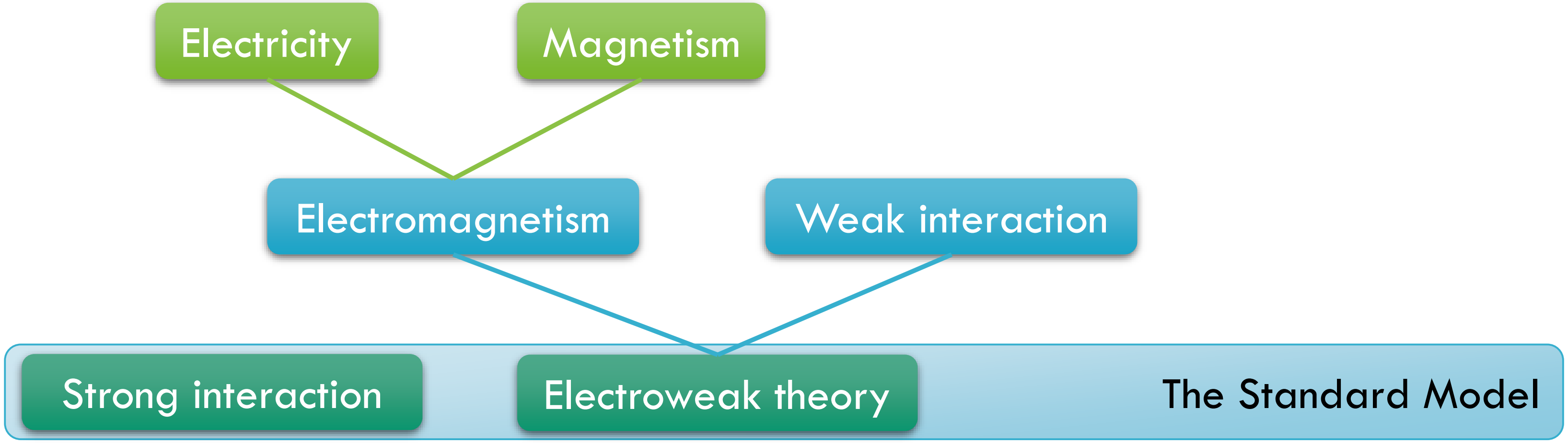
1960s

Sheldon Lee Glashow

Abdus Salam

Steven Weinberg

A THEORY OF EVERYTHING?



Electricity

Magnetism

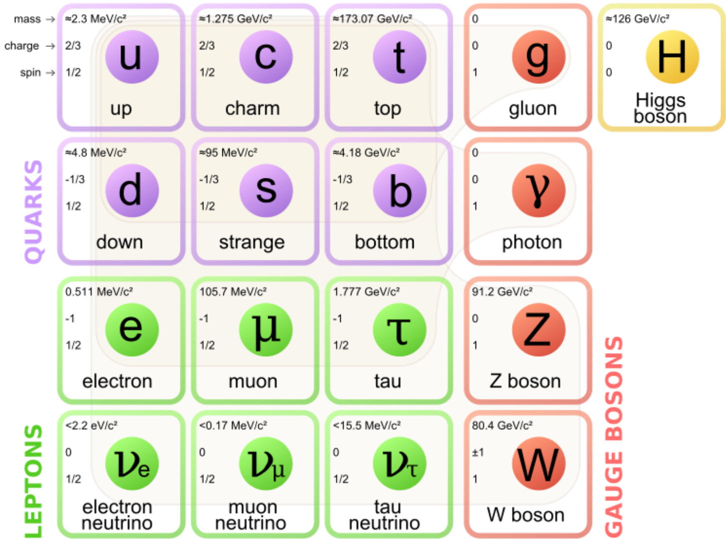
Electromagnetism

Weak interaction

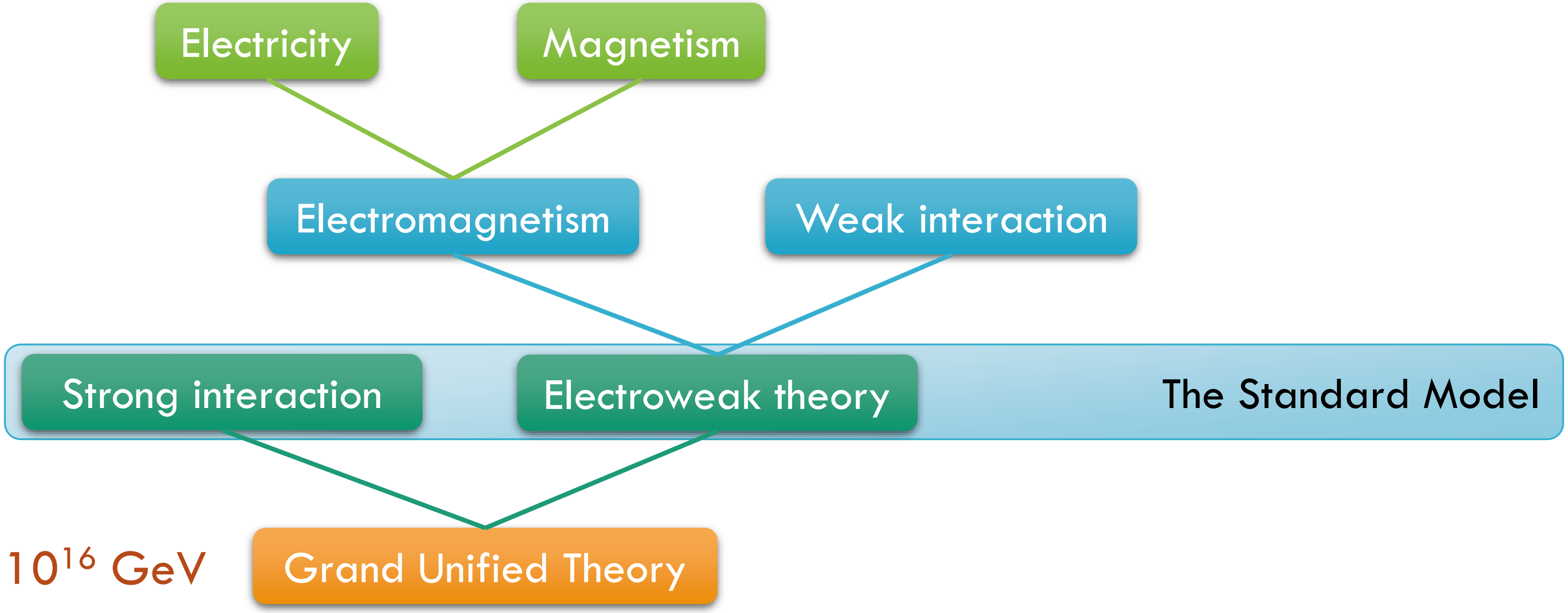
Strong interaction

Electroweak theory

The Standard Model



A THEORY OF EVERYTHING?



Electricity

Magnetism

Electromagnetism

Weak interaction

Strong interaction

Electroweak theory

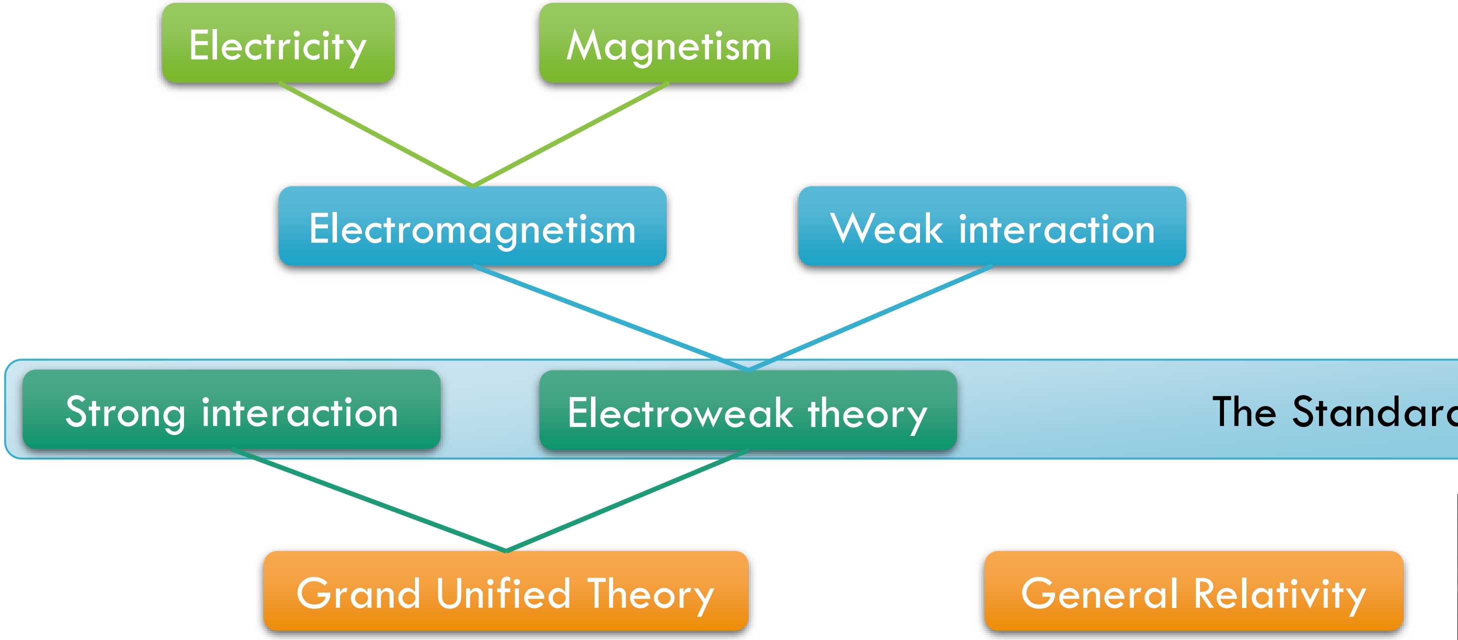
The Standard Model

10^{16} GeV

Grand Unified Theory

SU(5)? SO(10)? SU(8)? O(16)?
1970s – today…

A THEORY OF EVERYTHING?



Electricity

Magnetism

Electromagnetism

Weak interaction

Strong interaction

Electroweak theory

The Standard Model

Grand Unified Theory

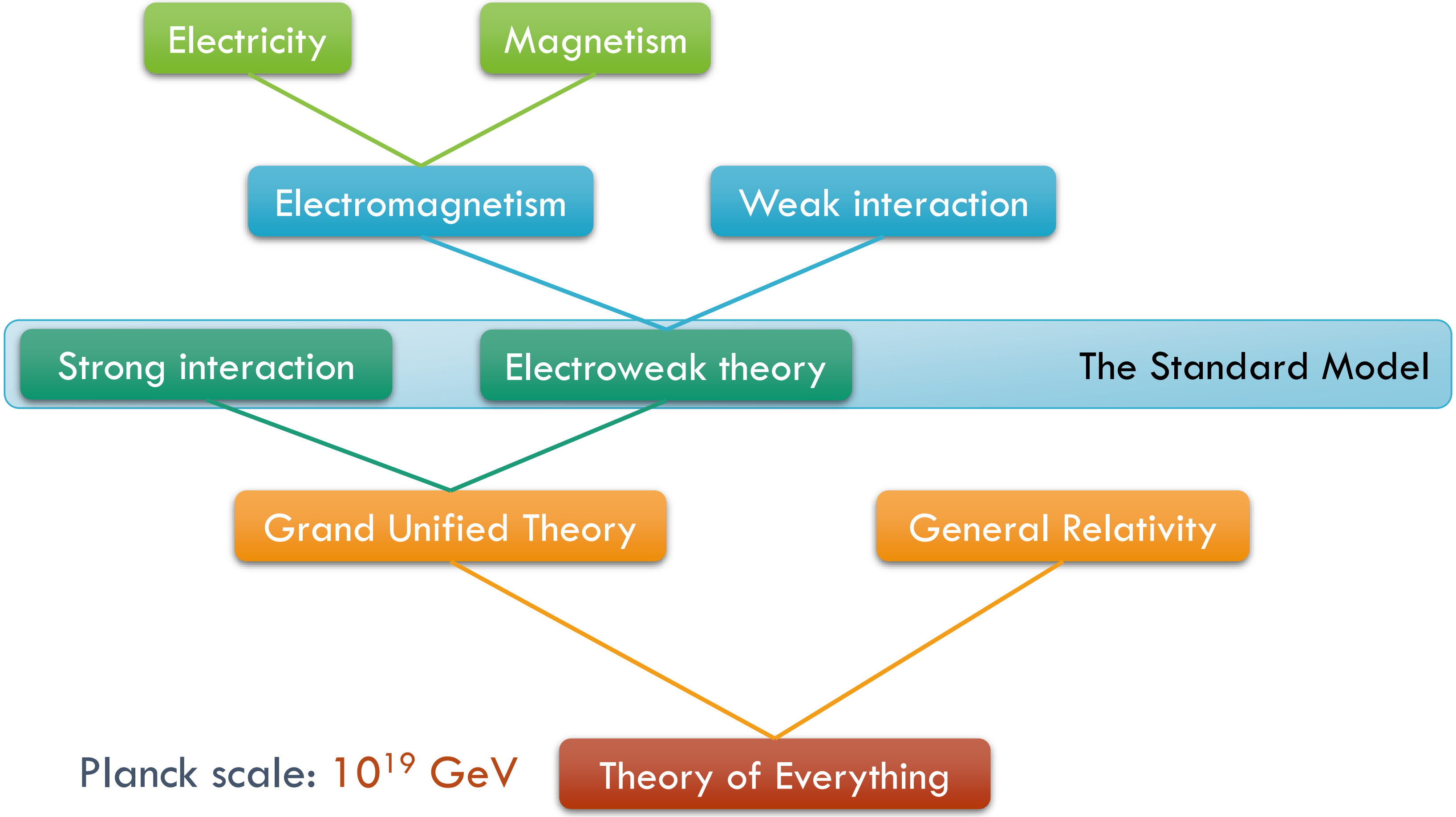
General Relativity

SU(5)? SO(10)? SU(8)? O(16)?
1970s – today…

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Albert Einstein
1925

A THEORY OF EVERYTHING?



Electricity

Magnetism

Electromagnetism

Weak interaction

Strong interaction

Electroweak theory

The Standard Model

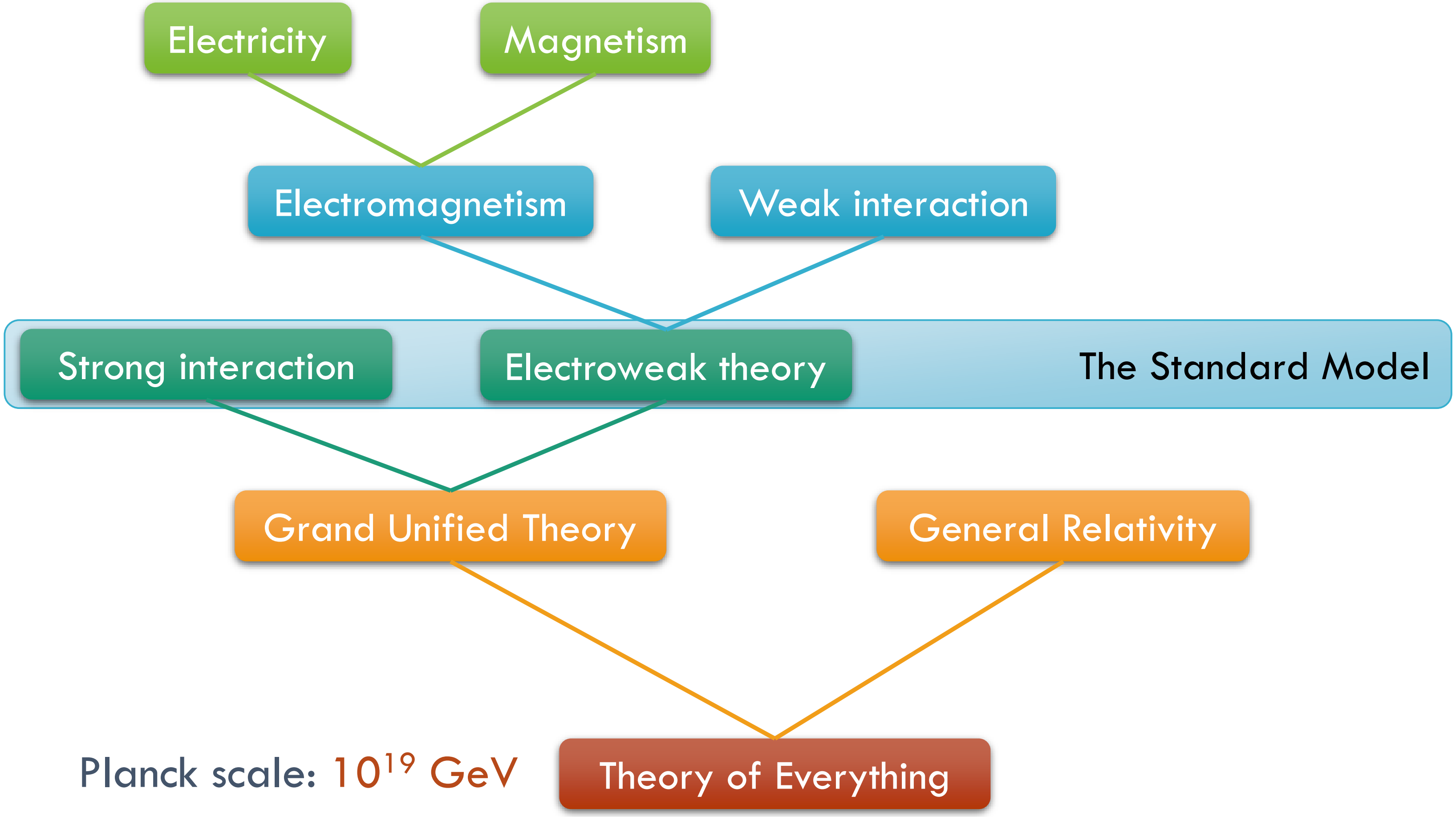
Grand Unified Theory

General Relativity

Theory of Everything

Planck scale: 10^{19} GeV

A THEORY OF EVERYTHING?



Planck scale: 10^{19} GeV

???????

A THEORY OF EVERYTHING?

- No one knows how to build a theory of everything.
    - General relativity and quantum mechanics are fundamentally different:
        - GR predicts smooth spacetime; QM predicts vacuum fluctuations.
    - A Grand Unified Theory is probably an intermediate step.
- String theory / M-theory
    - Particles are replaced by one-dimensional “strings”.
        - Different vibration modes give rise to different particle properties.
    - Needs extra dimensions to work! (9+1; 10+1)
    - Looks like supersymmetry at low energies.
- Loop quantum gravity
    - Granular description of space – quantization!
    - Needs only three space dimensions…

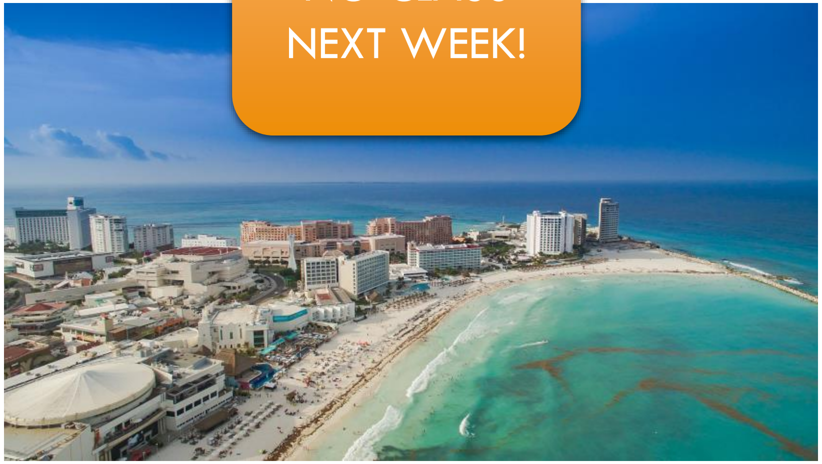
STANDARD MODEL LIMITATIONS SUMMARY

- We know that the Standard Model is incomplete: it does not fully account for experimental observations.
  - It doesn't describe gravity – and it seems very difficult to extend it in order to do so.
  - There isn't a unique way of describing neutrino mass in the SM context.
  - It doesn't provide a candidate for dark matter.

- Offers unsatisfactory answers to some fundamental questions:
  - Doesn't predict particle masses.
  - Or why there are three generations of matter.
  - Very large differences in the scale of some parameters remain unexplained.
    - Including some "lucky" cancelations.

SCHEDULE

1. ~~Introduction~~
2. ~~History of Particle Physics~~
3. ~~Special Relativity~~
4. ~~Quantum Mechanics~~
5. ~~Experimental Methods~~
6. ~~The Standard Model - Overview~~
7. The Standard Model - Limitations
8. Neutrino Theory } Cris
9. Neutrino Experiment } José
10. LHC and Experiments
11. The Higgs Boson and Beyond } Inês
12. Particle Cosmology } Cris



NO CLASS NEXT WEEK!

BONUS

STANDARD MODEL LAGRANGIAN

- Encompasses all theory.
  - From the Lagrangian to cross-section predictions:

$$\boxed{\sigma \sim \langle\, f\,|\mathbf{S}|\,i\,\rangle^{\,2}}$$

Inelastic

Cross Section

[for |i⟩ ≠ |f⟩]

$$[\mathrm{Def.}:\ |\,t=+\infty\,\rangle \equiv \mathbf{S}|\,t=-\infty\,\rangle\,]$$

Time Evolution

From Schrödinger-Equation

[Dirac picture]

$$|\,t\,\rangle = |\,t_0\,\rangle - i\int_{t_0}^{t}\mathrm{d}t'\,\mathbf{H}'(t')|\,t'\,\rangle$$

$$\mathbf{H}'(t) = -\int \mathcal{L}'(x,t)\,\mathrm{d}^3x$$

Lagrangian of Interaction

$$\langle\, f\,|\mathbf{S}|\,i\,\rangle \cong \delta_{fi} - i\int_{-\infty}^{\infty}\mathrm{d}t'\langle\, f\,|\mathbf{H}'(t')|\,i\,\rangle$$

➥ Feynman rules

INTEGRAL OVER "ALL POSSIBLE PATHS"

- Were does the integral notion come from?
- Recall, QM picture of free particle motion: there is some amplitude for a free electron to travel along any path from the source to some point p. Not just the straight, classical trajectory!
  - The word "path" here doesn't only refer to a x(y) path in space, but also the time at which it passes each point in space.
  - In 3D, a path (sometimes called wordline) is defined by three functions x(t), y(t) and z(t). An electron has an amplitude associated with a given path.
- The total amplitude for the electron to arrive at some final point is the sum of the amplitudes of all possible paths. Since there are an infinite number of paths, the sum turns into an integral.

THE QUANTUM MECHANICAL AMPLITUDE

- *Feynman: Each path has a corresponding probability amplitude. The amplitude ψ for a system to travel along a given path x(t) is:*

$$\psi[x(t)] = \text{const} \cdot e^{iS[x(t)]/\hbar}$$

where the object S[x(t)] is called the **action** corresponding to x(t).

- The total amplitude is the sum of contributions from each path:

$$\sum_{\text{over all paths}} \psi[x(t)]$$

THE QUANTUM MECHANICAL AMPLITUDE

$$\psi[x(t)] = \text{const}\cdot\, e^{iS[x(t)]/\hbar}$$

1. What is $e^{iS[x(t)]/\hbar}$?
2. What is the action $S[x(t)]$?

UNDERSTANDING THE PHASE

- You may not have seen numbers like $e^{i\theta}$, so let's review.
- Basically, $e^{i\theta}$ is just a fancy way of writing sinusoidal functions; from Euler's famous formula:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

- Note: those of you familiar with complex numbers (of the form $z=x+iy$) know that $e^{i\theta}$ is the phase of the so-called polar form of z, in which $z=re^{i\theta}$, with:

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

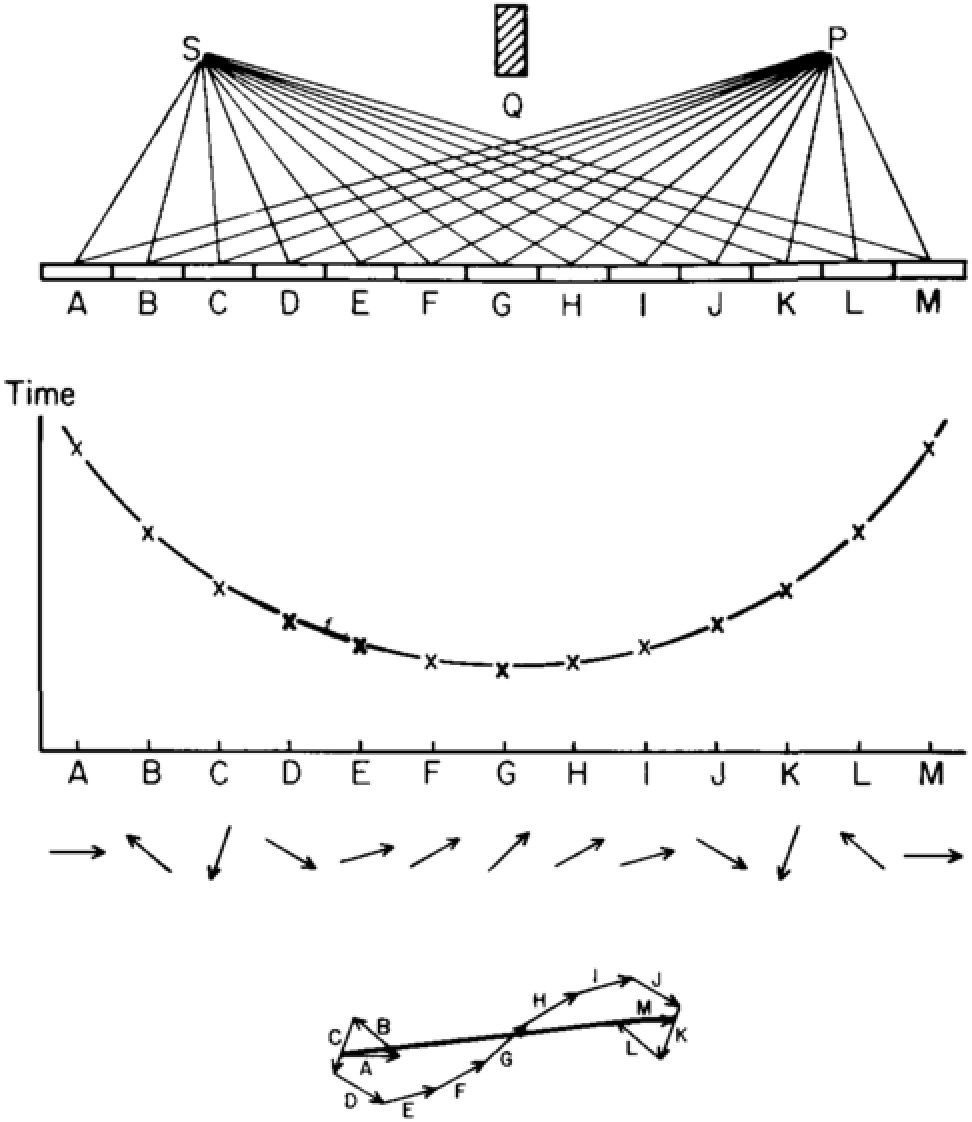
COMMENTS ON THE AMPLITUDE

- Now we can understand the probability amplitude $\psi[x(t)] \sim e^{iS[x(t)]/\hbar}$ a little better.
- The amplitude is a sinusoidal function – a wave – that oscillates along the worldline x(t). The frequency of oscillation is determined by how rapidly the action S changes along the path.
- The probability that a particle will take a given path (up to some overall multiplication constant) is:

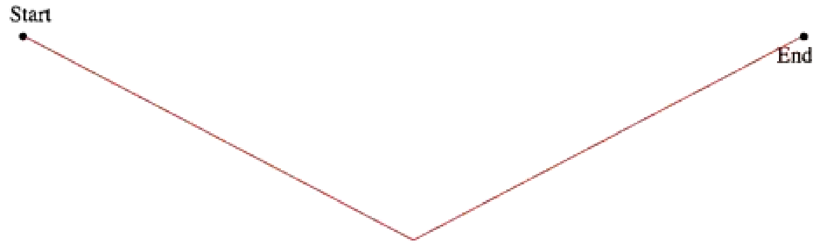
$$
\begin{aligned}
P &\propto |\psi|^2 = \psi^* \psi \\
&\propto e^{-iS[x(t)]/\hbar} e^{iS[x(t)]/\hbar} \\
&= e^{iS[x(t)]/\hbar - iS[x(t)]/\hbar} = e^0 \\
&= 1
\end{aligned}
$$

- This is the same for every worldline. According to Feynman, the particle is equally likely to take any path through space and time!
  - Contributions from "crazy" paths will likely be supressed by interference!

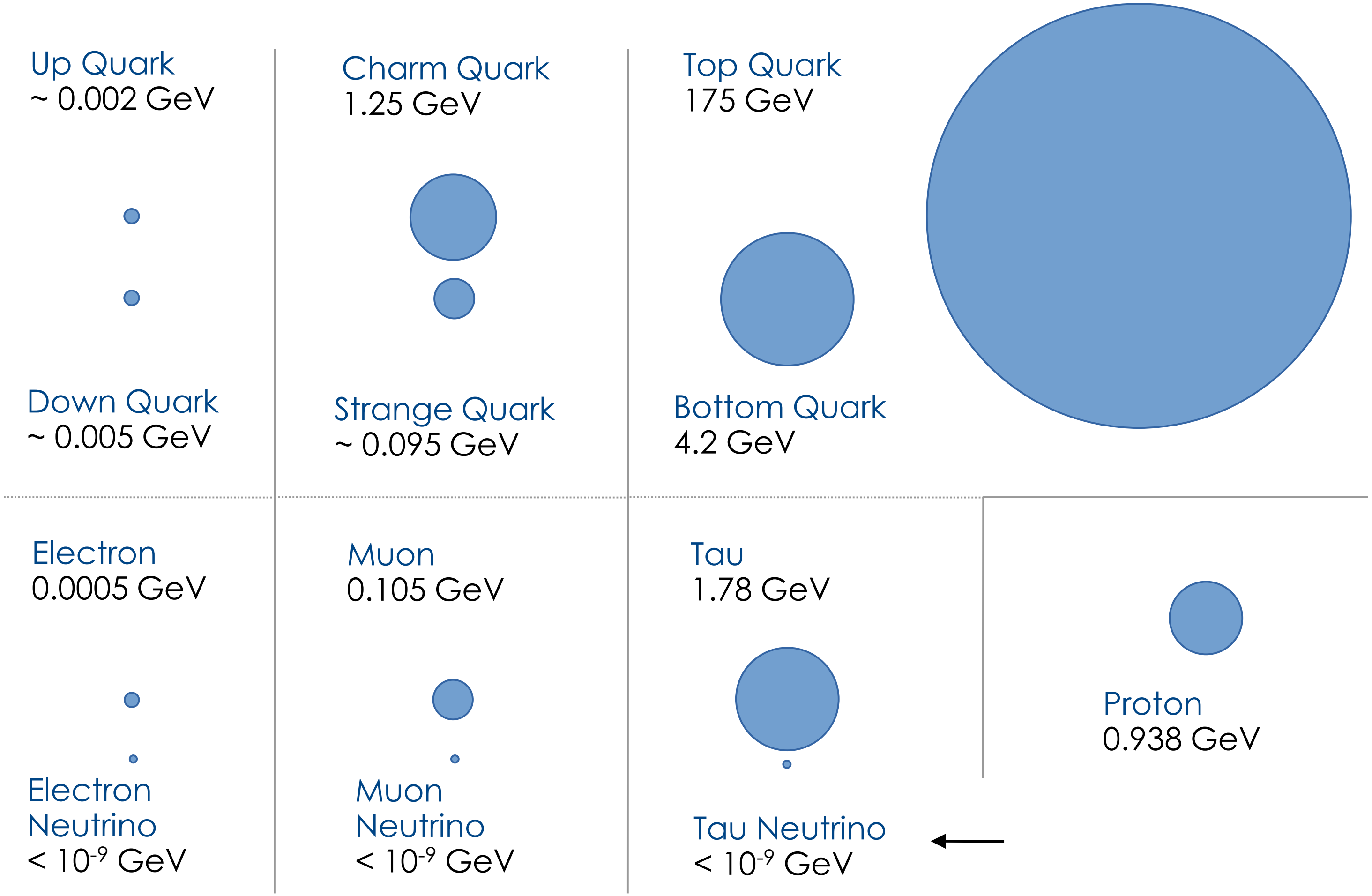
COMMENTS ON THE AMPLITUDE



COMMENTS ON THE AMPLITUDE



MASSES OF SM FERMIONS



Up Quark
~ 0.002 GeV

Down Quark
~ 0.005 GeV

Charm Quark
1.25 GeV

Strange Quark
~ 0.095 GeV

Top Quark
175 GeV

Bottom Quark
4.2 GeV

Electron
0.0005 GeV

Electron
Neutrino
< 10^{-9} GeV

Muon
0.105 GeV

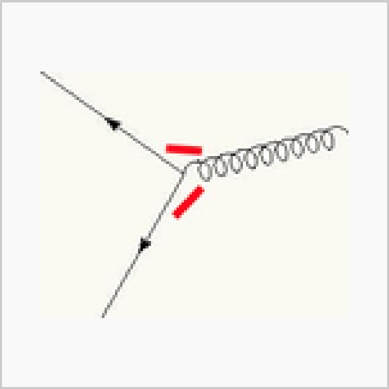
Muon
Neutrino
< 10^{-9} GeV

Tau
1.78 GeV

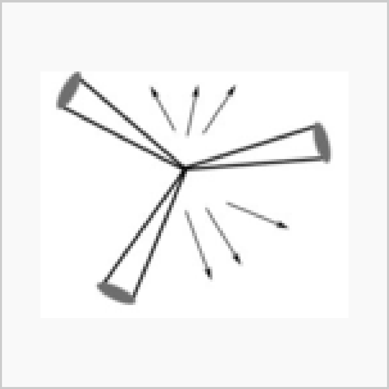
Tau Neutrino
< 10^{-9} GeV

Proton
0.938 GeV

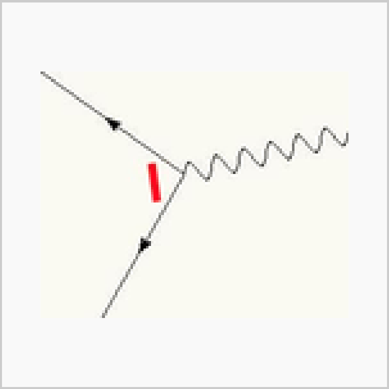
GLUON DISCOVERY AT TASSO, PETRA



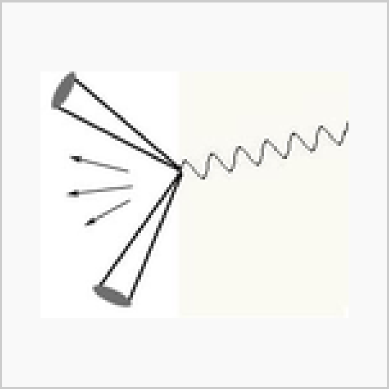
Two quarks (solid lines) and a gluon (curly line) fly apart, with the strings (red bars) primarily between the gluon and each quark.



As a result, three jets (cones) form, with extra hadrons (arrows) found where the strings formed.



For comparison, physicists looked at events with two quarks and a photon (wavy line). Here the string forms only between quarks.



Therefore extra hadrons are found only between the two jets, which is inconsistent with observations.