

# PARTICLE PHYSICS

COLUMBIA SCIENCE HONORS PROGRAM

WEEK 7

OVERVIEW OF THE STANDARD MODEL (CONT'ED)

LIMITATIONS OF THE STANDARD MODEL

# COURSE POLICIES

- Attendance
  - Up to four excused absences
    - Two with notes from parent/guardian
    - [shpattendance@columbia.edu](mailto:shpattendance@columbia.edu)
  - Valid excuses:
    - Illness, family emergency, tests or athletic/academic competitions, mass transit breakdowns
  - Invalid excuses:
    - Sleeping in, missing the train...
    - I will take attendance during class
- No cell phones
- Ask questions!

# LECTURE MATERIALS

- <https://twiki.nevis.columbia.edu/twiki/bin/view/Main/ScienceHonorsProgram>
- Questions: [cristovao.vilela@stonybrook.edu](mailto:cristovao.vilela@stonybrook.edu)

LAST WEEK...

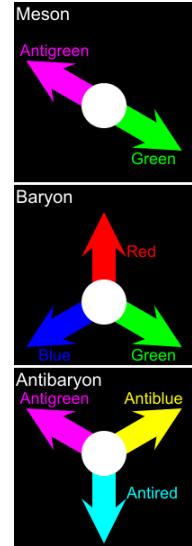
# AN OVERVIEW OF THE STANDARD MODEL

# THE WORLD, ACCORDING TO A PARTICLE PHYSICIST

mass → $\approx 2.3 \text{ MeV}/c^2$ charge → $2/3$ spin → $1/2$	<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b>g</b> gluon	<b>H</b> Higgs boson
$\approx 4.8 \text{ MeV}/c^2$ $-1/3$ $1/2$	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom	<b><math>\gamma</math></b> photon	
$0.511 \text{ MeV}/c^2$ $-1$ $1/2$	<b>e</b> electron	<b><math>\mu</math></b> muon	<b><math>\tau</math></b> tau	<b>Z</b> Z boson	
$<2.2 \text{ eV}/c^2$ $0$ $1/2$	<b><math>\nu_e</math></b> electron neutrino	<b><math>\nu_\mu</math></b> muon neutrino	<b><math>\nu_\tau</math></b> tau neutrino	<b>W</b> W boson	

# PARTICLE CHARGES

Quarks



- **Quarks:**

- There are **no** free quarks.
- They form **colorless** composite objects, hadrons;
  - **Baryons:**  $qqq$ ,  $\overline{qqq}$
  - **Mesons:**  $q\bar{q}$ ,  $q\bar{q}$ ,  $q\bar{q}$

Name	Symbol	Mass (MeV/c <sup>2</sup> ) <sup>*</sup>	J	B	Q	<i>I</i> <sub>3</sub>	C	S	T	B'	Antiparticle	Antiparticle symbol
<i>First generation</i>												
Up	u	1.7 to 3.3	½	+1/3	+2/3	+1/2	0	0	0	0	Antiup	ȳ
Down	d	4.1 to 5.8	½	+1/3	-1/3	-1/2	0	0	0	0	Antidown	ȳ
<i>Second generation</i>												
Charm	c	1,270 <sup>+70</sup> <sub>-90</sub>	½	+1/3	+2/3	0	+1	0	0	0	Anticharm	ȳ
Strange	s	101 <sup>+29</sup> <sub>-21</sub>	½	+1/3	-1/3	0	0	-1	0	0	Antistrange	ȳ
<i>Third generation</i>												
Top	t	172,000 ± 900 ± 1,300	½	+1/3	+2/3	0	0	0	+1	0	Antitop	ȳ
Bottom	b	4,190 <sup>+180</sup> <sub>-60</sub>	½	+1/3	-1/3	0	0	0	0	-1	Antibottom	ȳ

*J* = total angular momentum, *B* = baryon number, *Q* = electric charge, *I*<sub>3</sub> = isospin, *C* = charm, *S* = strangeness, *T* = topness, *B'* = bottomness.

# PARTICLE CHARGES

	Electric	Weak	Color
Leptons	Quarks	✓	✓
	Charged	✓	✓
	Neutral	✗	✓

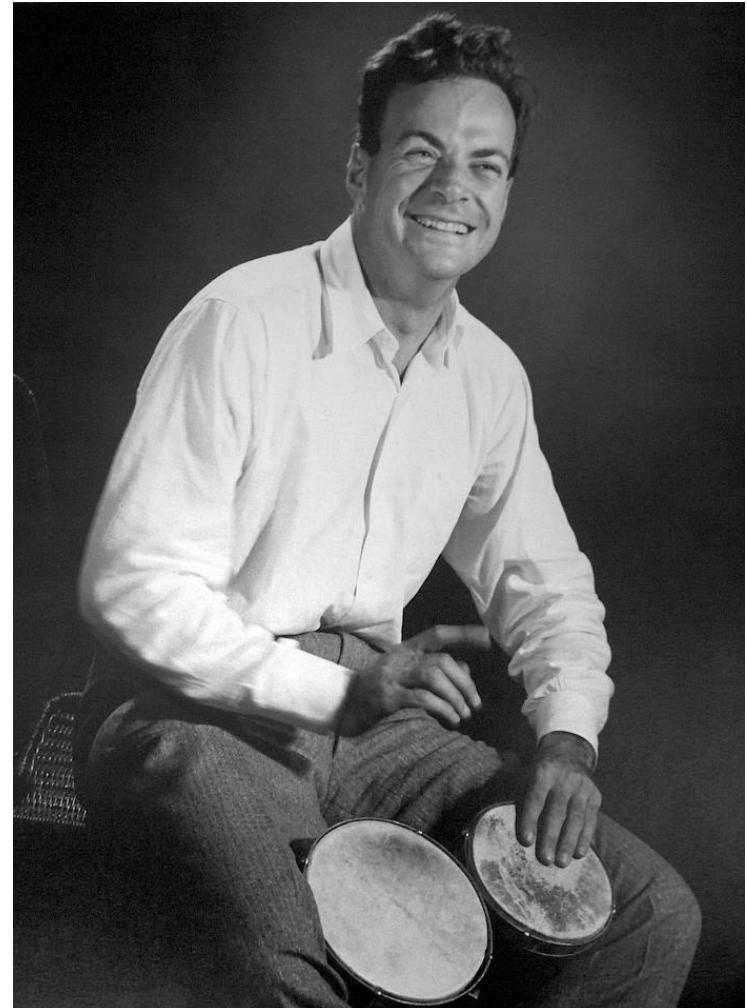
- Leptons

- Exist as **free** particles

Particle/Antiparticle Name	Symbol	Q (e)	S	L <sub>e</sub>	L <sub>μ</sub>	L <sub>τ</sub>	Mass (MeV/c <sup>2</sup> )	Lifetime (s)
Electron / Antielectron <sup>[17]</sup>	e <sup>-</sup> /e <sup>+</sup>	-1/+1	1/2	+1/-1	0	0	0.510 998 910(13)	Stable
Muon / Antimuon <sup>[18]</sup>	μ <sup>-</sup> /μ <sup>+</sup>	-1/+1	1/2	0	+1/-1	0	105.658 3668(38)	2.197 019(21) × 10 <sup>-6</sup>
Tau / Antitau <sup>[20]</sup>	τ <sup>-</sup> /τ <sup>+</sup>	-1/+1	1/2	0	0	+1/-1	1,776.84(17)	2.906(10) × 10 <sup>-13</sup>
Electron neutrino / Electron antineutrino <sup>[33]</sup>	ν <sub>e</sub> /̄ν <sub>e</sub>	0	1/2	+1/-1	0	0	< 0.000 0022 <sup>[35]</sup>	Unknown
Muon neutrino / Muon antineutrino <sup>[33]</sup>	ν <sub>μ</sub> /̄ν <sub>μ</sub>	0	1/2	0	+1/-1	0	< 0.17 <sup>[35]</sup>	Unknown
Tau neutrino / Tau antineutrino <sup>[33]</sup>	ν <sub>τ</sub> /̄ν <sub>τ</sub>	0	1/2	0	0	+1/-1	< 15.5 <sup>[35]</sup>	Unknown

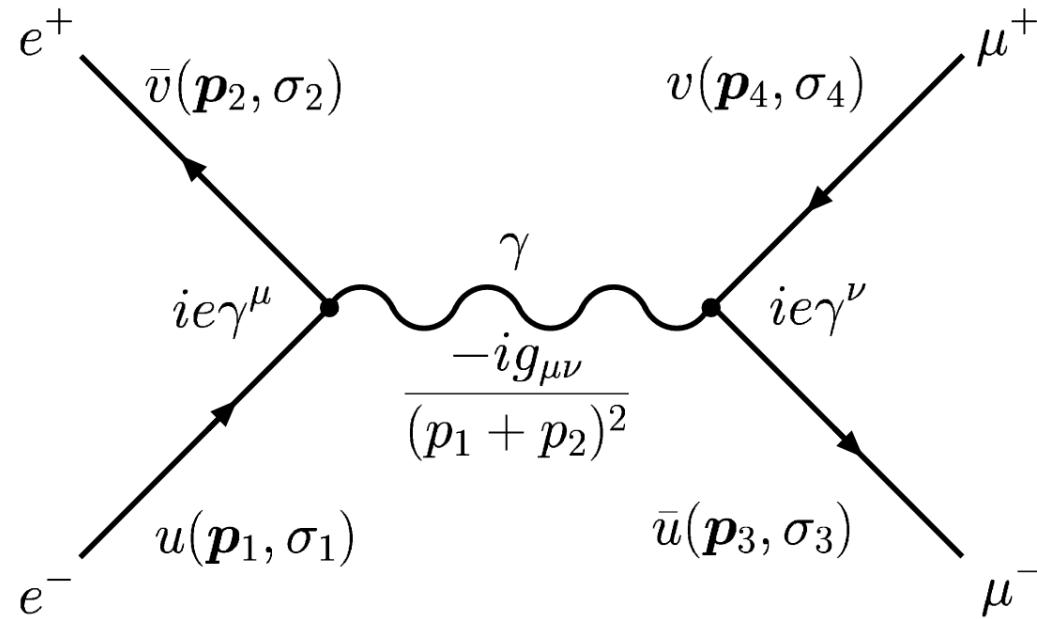
# PARTICLE DYNAMICS: FEYNMAN DIAGRAMS

- Feynman Rules!
- 1948: introduced **pictorial representation** scheme for the mathematical expressions governing the behavior of subatomic **particles**.
  - Can be used to easily **calculate** probability amplitudes
  - Other options: cumbersome mathematical derivations



# FEYNMAN DIAGRAMS

## AN EXAMPLE CALCULATION

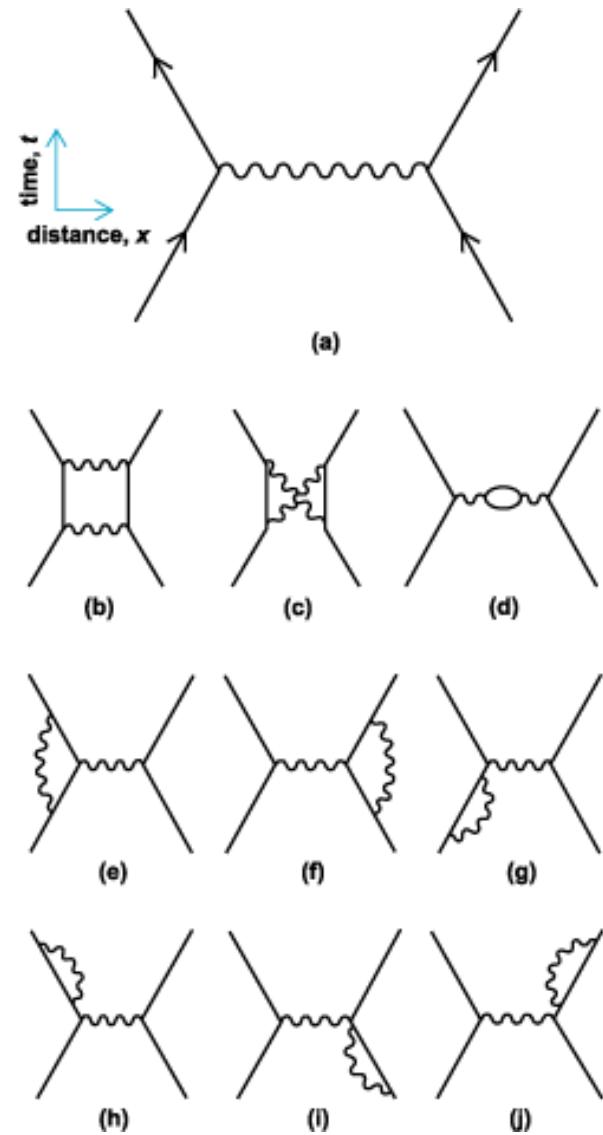


$$-i\mathcal{M} = [\bar{u}(\mathbf{p}_3, \sigma_3)(ie\gamma^\nu)v(\mathbf{p}_4, \sigma_4)] \frac{-ig_{\mu\nu}}{(p_1 + p_2)^2} [\bar{v}(\mathbf{p}_2, \sigma_2)(ie\gamma^\mu)u(\mathbf{p}_1, \sigma_1)]$$

# QUANTUM ELECTRODYNAMICS

## QED

- The **vertices** are **interactions** with the electromagnetic field.
- The **straight lines** are **electrons** and the **wiggly ones** are **photons**.
- Between interactions (vertices), particles **propagate** as free particles.
- The **higher** the number of **vertices**, the **less likely** for the interaction to happen.
  - Higher “order”.



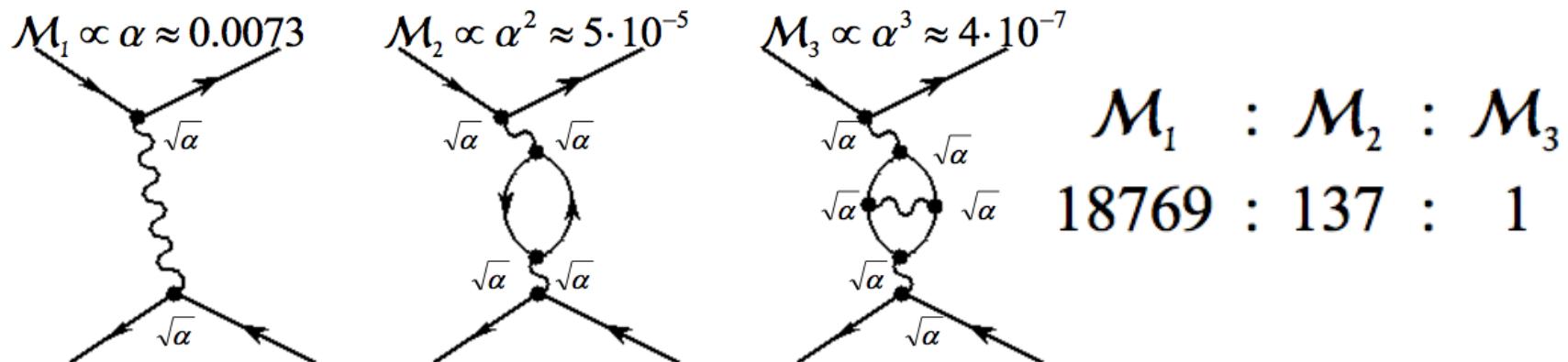
# QUANTUM ELECTRODYNAMICS

## QED

- Each **vertex** contributes a **coupling constant**  $\sqrt{\alpha}$ , where  $\alpha$  is a small dimensionless number:

$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

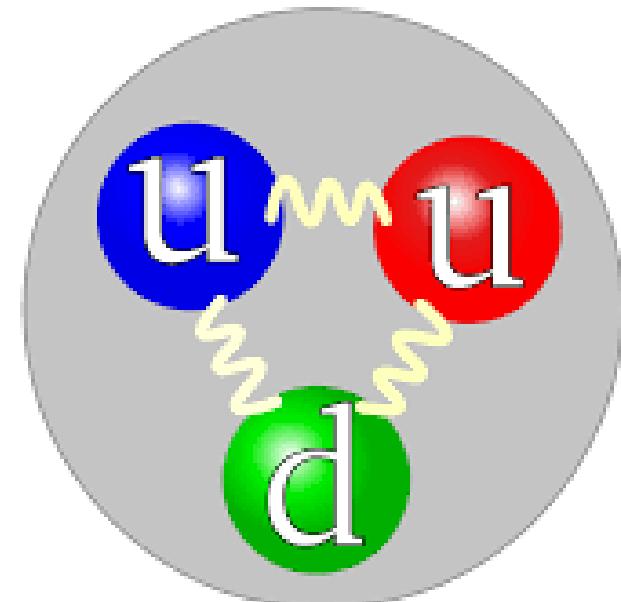
- Hence, **higher-order** diagrams get **suppressed** relative to diagrams with fewer vertices.



# QUANTUM CHROMODYNAMICS

## QCD

- Quarks and **bound states**:
  - Since quarks are **spin-1/2** particles (fermions), they must obey the **Pauli Exclusion Principle**.
- **Pauli Exclusion Principle**: fermions in a bound state (e.g., the quarks inside a hadron) **cannot** have the **same** quantum numbers.
- Then, how can we squeeze **three** quarks into a baryon?
- Give them an **additional charge**, called **color**.
- This **removes** the quantum numbers **degeneracy**.



# QUANTUM CHROMODYNAMICS

## QCD

- Gluons carry a **color** and an **anti-color**.
- There are **9 possible combinations**, but 1 is white, which is not allowed.
  - **No evidence** for colorless particles exchanging gluons.
- This leaves **8 types of gluon**.

A quark changes color by emitting or absorbing gluons.



# PERTURBATION THEORY ASIDE

- Use a **power series** in a parameter  $\varepsilon$  (such that  $\varepsilon \ll 1$ ) - known as perturbation series - as an **approximation** to the full solution.
- For example:

$$A = A_0 + \varepsilon A_1 + \varepsilon^2 A_2 + \dots$$

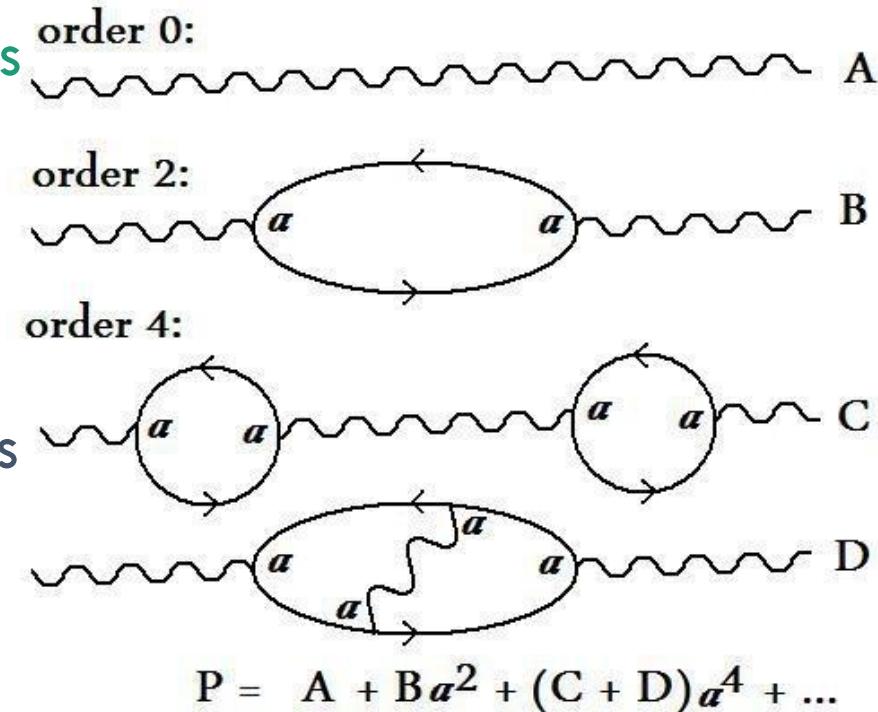
- In this example,  $A_0$  is the “**leading order**” solution, while  $A_1, A_2, \dots$  represent higher order terms.
- **Note:** if  $\varepsilon$  is **small**, the higher-order terms in the series become successively smaller.
- Approximation:

$$A \approx A_0 + \varepsilon A_1$$

# PERTURBATION THEORY IN QFT

## ASIDE

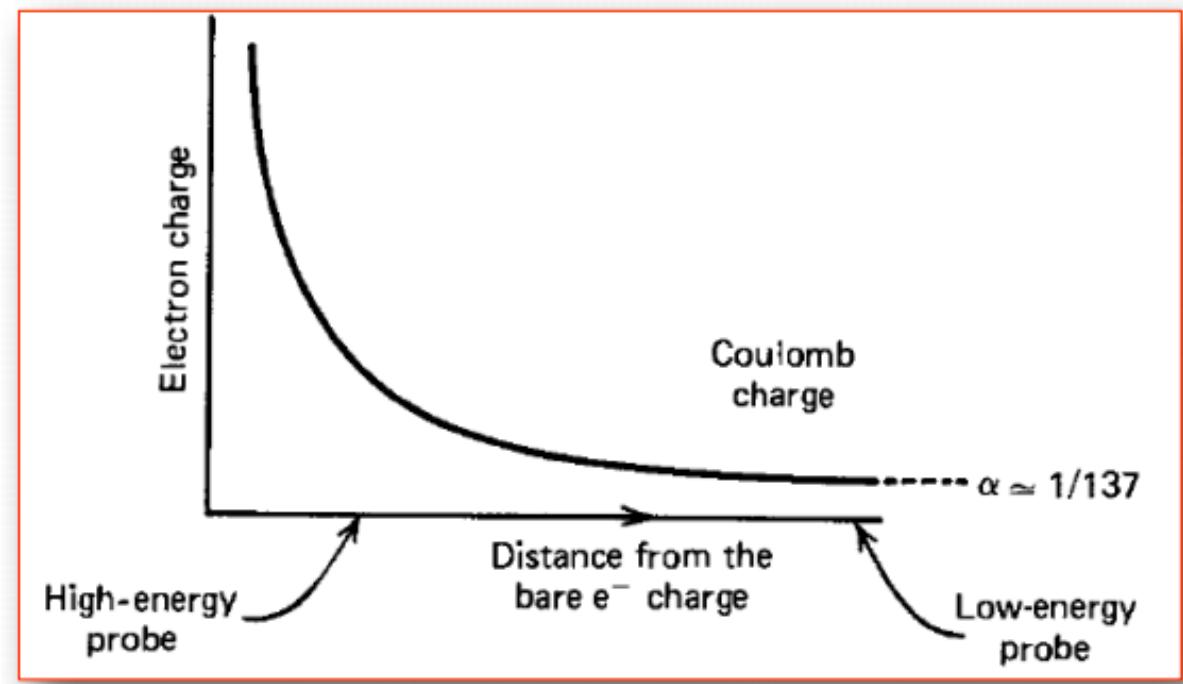
- Perturbation theory allows for well-defined predictions in quantum field theories (as long as they obey certain requirements).
- Quantum electrodynamics is one of those theories.
- Feynman diagrams correspond to the terms in the perturbation series!



Diagrams define a series in  $\alpha$

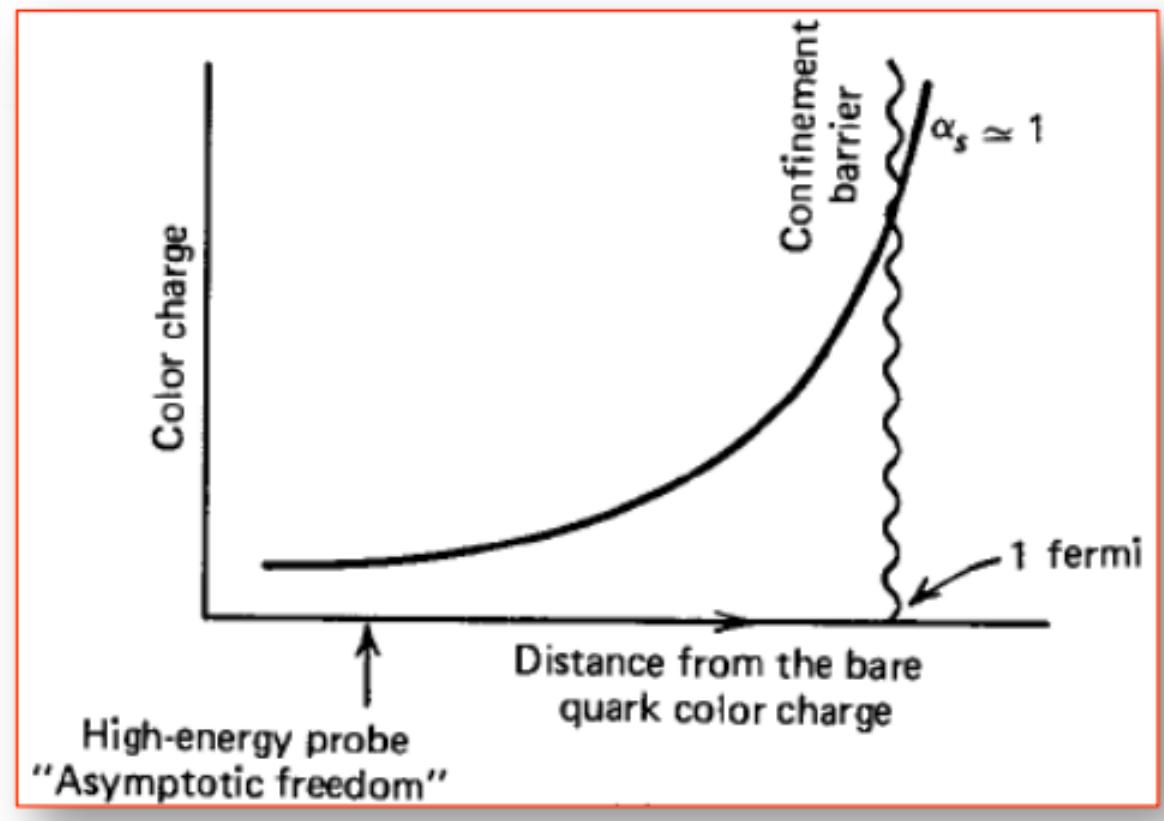
# QCD VS QED

- Recall: In QED, each vertex contributes a **coupling constant**  $\sqrt{\alpha}$ .
- $\alpha$  is not exactly a **constant** though... it “runs” with the scale of the interaction.



# QCD VS QED

- The coupling constant for QCD,  $\alpha_s$ , “runs” in a different way with energy.



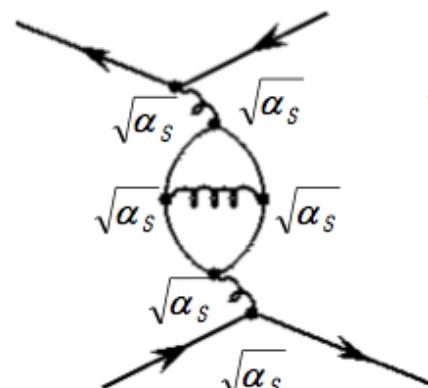
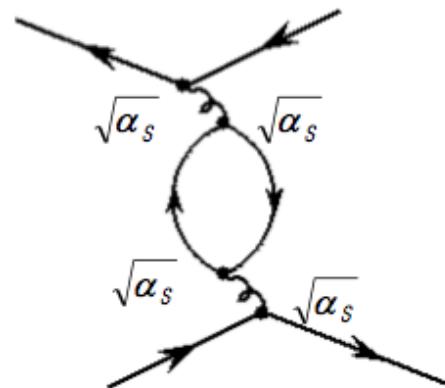
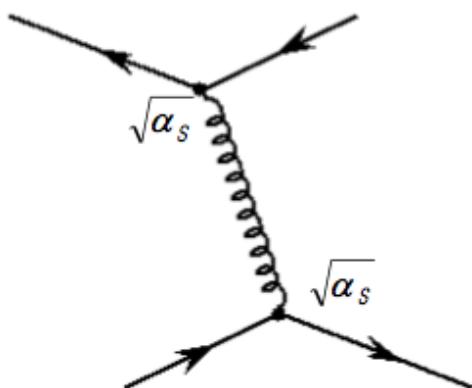
# QCD VS QED

- In QCD, the coupling between quarks and gluons, given by the number  $\alpha_s$ , is much larger than  $1/137$  at low energies.
- In fact, at low energies,  $\alpha_s \gg 1$ , making higher-order diagrams just as important as those with fewer vertices!
- This means we can't truncate the sum over diagrams.
  - Perturbation theory is not a good approximation!
  - Calculations quickly become complicated!

$$\mathcal{M}_1 \propto \alpha_s \approx 1$$

$$\mathcal{M}_2 \propto \alpha_s^2 \approx 1$$

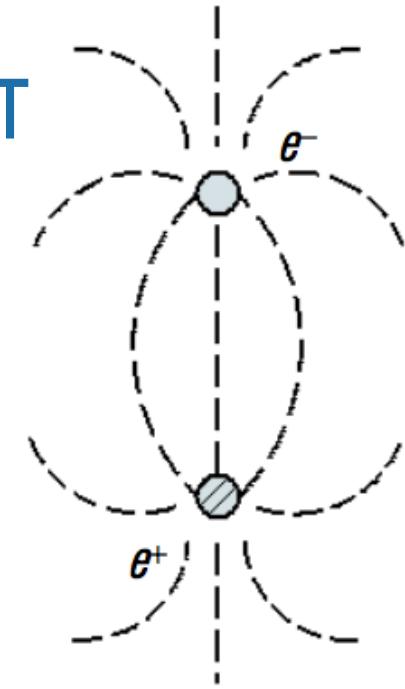
$$\mathcal{M}_3 \propto \alpha_s^3 \approx 1$$



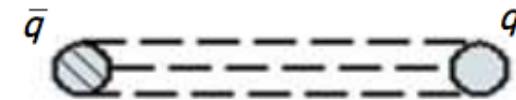
$$\mathcal{M}_1 : \mathcal{M}_2 : \mathcal{M}_3$$
$$1 : 1 : 1$$

# UNDERSTANDING CONFINEMENT

- The **mathematics** of confinement are **complicated**, but we can **understand** them in terms of a very simple picture.
- Recall, the **Coulomb** field between a  $e^+e^-$  pair looks like  $V(r) \sim 1/r$ .
  - As we **pull** the pair **apart**, the **attraction** **weakens**.
- Imagine the **color field** between a quark-antiquark pair like Hooke's Law:  $V(r) \sim r$ .
  - As we **pull** the pair **apart**, the **attraction** between them **increases**.
- So, **separating** two quarks by a **large  $r$**  puts a **large amount of energy** into the **color field**:  
 $V(r) \rightarrow \infty$



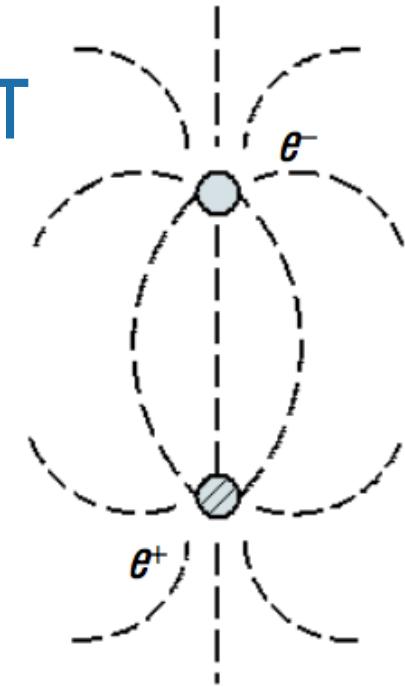
Dipole field for the Coulomb force between opposite electrical charges.



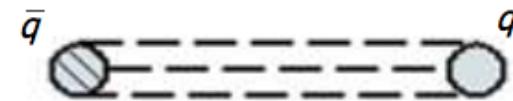
Dipole field between opposite-color quarks.

# UNDERSTANDING CONFINEMENT

- How do we **understand** this picture?
- When a quark and anti-quark **separate**, their color interaction **strengthens** (more gluons appear in the color field).
- Through the interaction of the **gluons with each other**, the color lines of force are **squeezed** into a tube-like region.
- **Contrast** this with the **Coulomb field**: nothing prevents the lines of force from **spreading out**.
  - There is **no self-coupling** of photons to contain them.
- If the color tube has **constant energy density** per unit length  $k$ , the **potential energy** between quark and antiquark will **increase** with separation,  $V(r) \sim kr$ .



Dipole field for the Coulomb force between opposite electrical charges.

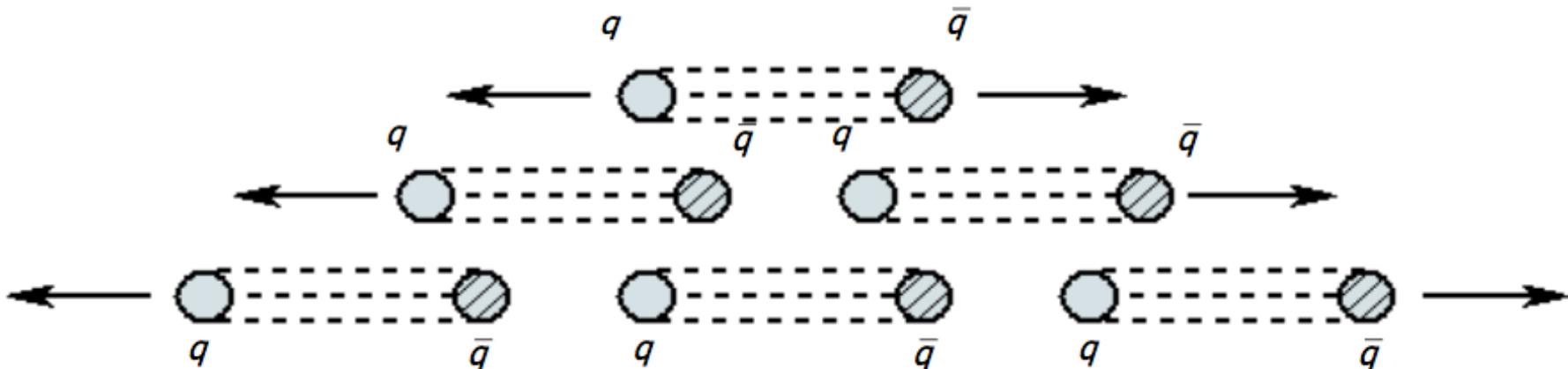


Dipole field between opposite-color quarks.

# COLOR LINES AND HADRON PRODUCTION

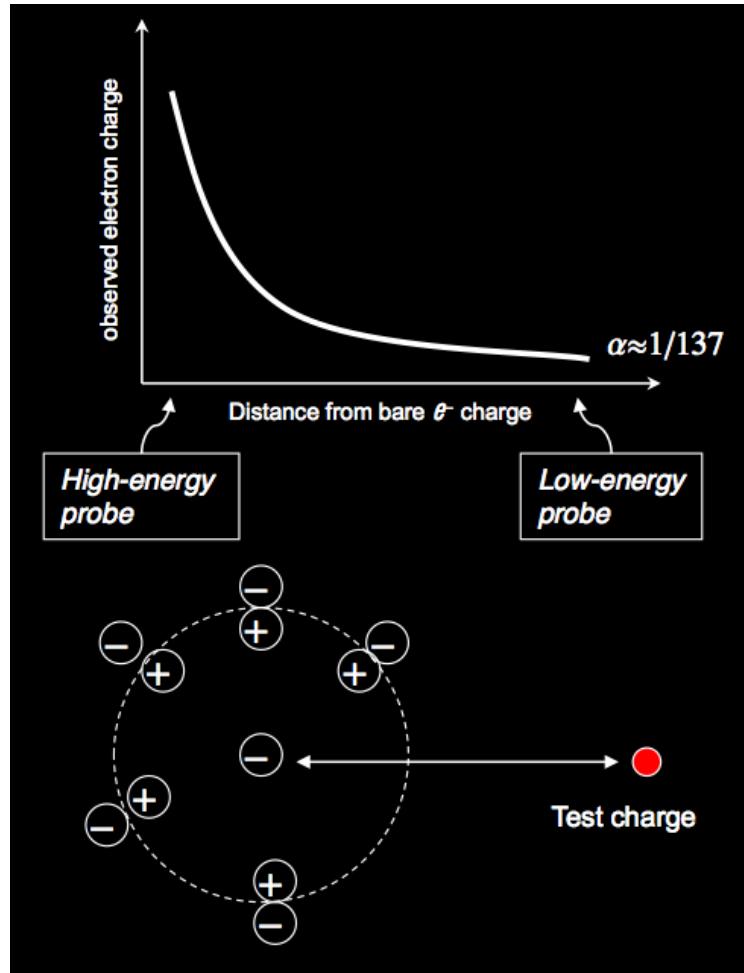
## WHY YOU CAN'T GET FREE QUARKS

- Suppose we have a **meson** and we **try to pull it apart**. The **potential energy** in the quark-antiquark color field starts to **increase**.
- Eventually, the **energy** in the gluon field gets **big enough** that the **gluons** can **pair-produce** another quark-antiquark pair.
  - The new **quarks pair up** with the original quarks to **form mesons**, and thus our four **quarks** remain **confined** in colorless states.
- Experimentally, we see **two particles**!



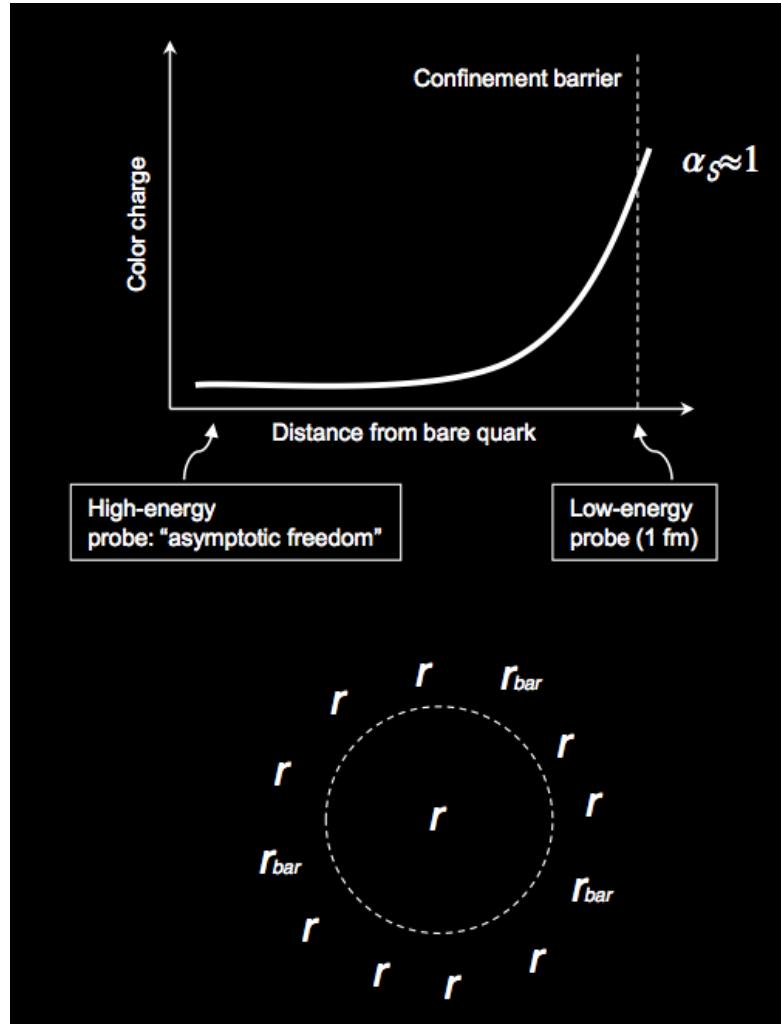
# QED: CHARGE SCREENING

- Now, suppose we want to **measure** the **charge** of the electron by observing the **Coulomb force** experienced by a **test charge**.
- Far **away** from the electron, its charge is screened by a cloud of **virtual positrons** and **electrons**, so the **effective charge** is smaller than its **bare** charge.
- As we move **closer** in, **fewer** positrons are blocking our line of sight to the electron.
- Hence, with **decreasing distance**, the **effective charge** of the electron **increases**.
- We can think of this as  $\alpha$  increasing with energy.



# QCD: CHARGE ANTI-SCREENING

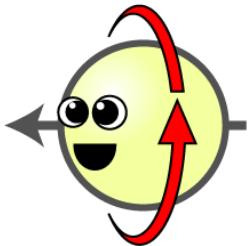
- In QCD, the additional gluon loop diagrams reverse the result of QED:
  - A red charge is preferentially surrounded by other red charges.
- By moving the test probe closer to the original quark, the probe penetrates a sphere of mostly red charge, and the measured red charge decreases.
- This is “antiscreening”.
- We can think of this as  $\alpha_s$  decreasing with energy.



# RUNNING CONSTANTS

- As we probe an electron at increasingly higher energies, its effective charge increases.
- This can be rephrased in the following way: as interactions increase in energy, the QED coupling strength  $\alpha$  between charges and photons also increases.
  - This should not really be a surprise; after all, the coupling strength of EM depends directly on the electron charge.
- Since  $\alpha$  is not a constant, but a (slowly-varying) function of energy, it is called a running coupling constant.
- In QCD, the net effect is that the quark color charge and  $\alpha_s$  decrease as the interaction energy goes up.

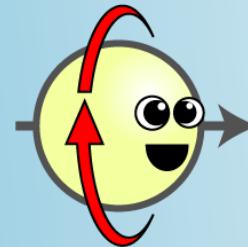
# PARTICLE HELICITY ASIDE



Right-handed:  
Spin in same direction as motion



MIRROR



Left-handed!



- Helicity **flips** when looked at in a mirror
  - Equivalent to inverting one of the space coordinates ( $z \rightarrow -z$ )

# WEAK INTERACTIONS

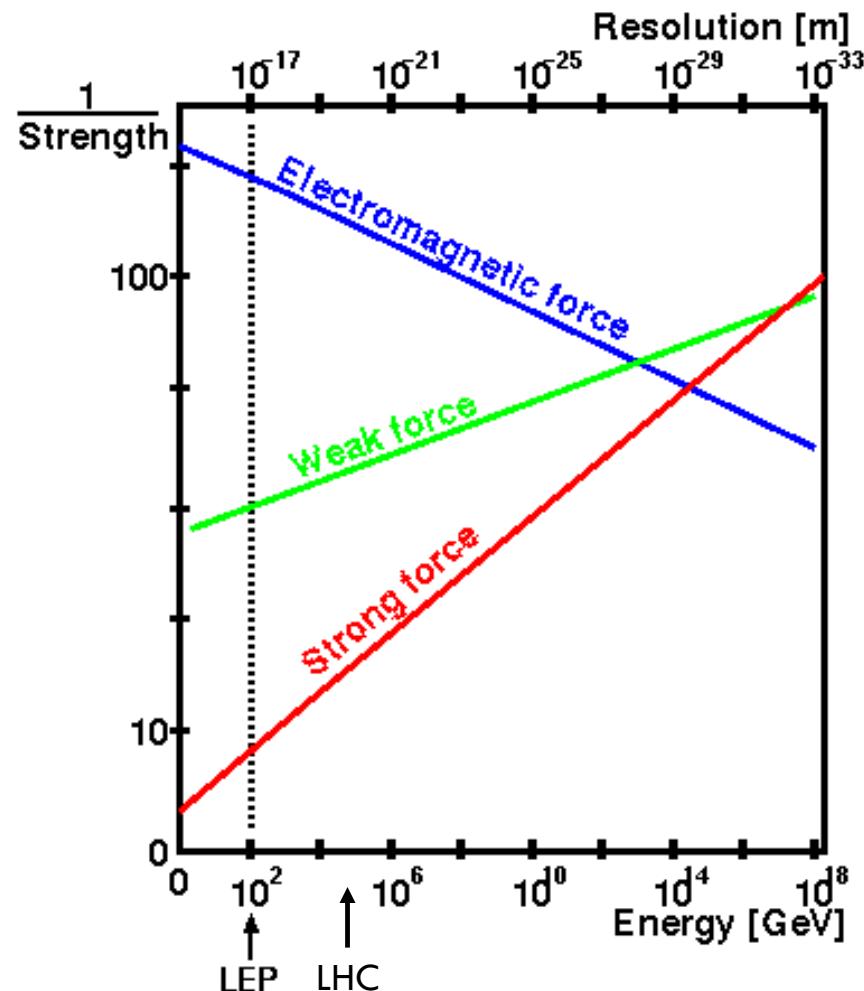
- Unlike electromagnetism and the strong interaction, the weak interaction is mediated by massive bosons:
  - $m_Z = 91.19 \text{ GeV}$
  - $m_W = 80.39 \text{ GeV}$
- This makes it extremely short-range
- And very weak at low energies
- Only left-handed particles\* participate in weak interactions
  - Nature is different if looked at in a mirror!
  - \* and right-handed anti-particles

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$



# FORCE UNIFICATION ASIDE

- At laboratory energies near  $M_w$   $\mathcal{O}(100)$  GeV, the measured values of the **coupling constants** are quite different.
- However, their “running” trends suggest that they approach a common value near  $10^{16}$  GeV.
- This is an insanely high energy!
- The Standard Model provides no explanation for what may happen beyond this **unification scale**, nor why the forces have such different strengths at low energies.



# PARTICLE/FIELD FORMULATION

- How do we **describe** interactions and fields **mathematically**?
- Classically,  
**Lagrangian**  $L = \text{kinetic energy} - \text{potential energy}$
- Particle physics:
  - Same concept, using Dirac equation to describe free spin-1/2 particles:

$$L = \bar{\Psi} (i\gamma^\mu \partial_\mu - m) \Psi$$

↑  
Field

$\Psi$  = wavefunction

$m$  = mass

$\gamma^\mu$  =  $\mu^{\text{th}}$  gamma matrix

$\partial_\mu$  = partial derivative

# STANDARD MODEL LAGRANGIAN

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

Free Fields

Interaction

The diagram illustrates the decomposition of the Standard Model Lagrangian  $\mathcal{L}$  into two parts:  $\mathcal{L}_0$  and  $\mathcal{L}'$ . The equation  $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$  is centered. Above  $\mathcal{L}_0$ , the text "Free Fields" is written in blue, with a blue line pointing downwards to the left of the equation. Above  $\mathcal{L}'$ , the text "Interaction" is written in blue, with a blue line pointing downwards to the right of the equation.

# STANDARD MODEL LAGRANGIAN

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

Free Fields

Interaction

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi$$

Gauge Bosons

Fermions

$$F_{\mu\nu}F^{\mu\nu} = G_{\mu\nu}G^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}$$

# STANDARD MODEL LAGRANGIAN

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

Free Fields

Interaction

$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi$

Gauge Bosons

$\mathcal{L}' = e\bar{\psi}\gamma^\mu A_\mu\psi$

Fermion-Boson Coupling

$eA_\mu = \frac{g_s}{2}\lambda_\nu G_\mu^\nu + \frac{g}{2}\vec{\tau} \vec{W}_\mu + \frac{g'}{2}YB_\mu$

$F_{\mu\nu}F^{\mu\nu} = G_{\mu\nu}G^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}$

# STANDARD MODEL LAGRANGIAN

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

Free Fields

Interaction

Gauge Bosons

Electroweak bosons and interaction

Fermions

Gluons and strong interaction

Fermion-Boson Coupling

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi$$

$$\mathcal{L}' = e\bar{\psi}\gamma^\mu A_\mu\psi$$

$$eA_\mu = \frac{g_s}{2}\lambda_\nu G_\mu^\nu + \frac{g}{2}\vec{\tau} \vec{W}_\mu + \frac{g'}{2}YB_\mu$$

$$F_{\mu\nu}F^{\mu\nu} = G_{\mu\nu}G^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}$$

# STANDARD MODEL LAGRANGIAN

- Encompasses **all** theory.
  - From the Lagrangian to **cross-section predictions**:

$$\sigma \sim \langle f | \mathbf{S} | i \rangle^2$$

Inelastic  
Cross Section  
[for  $|i\rangle \neq |f\rangle$ ]

Time Evolution  
From Schrödinger-Equation  
[Dirac picture]

[Def. :  $|t = +\infty\rangle \equiv \mathbf{S}|t = -\infty\rangle$ ]

$$|t\rangle = |t_0\rangle - i \int_{t_0}^t dt' \mathbf{H}'(t') |t'\rangle$$

$$\mathbf{H}'(t) = - \int \mathcal{L}'(x, t) d^3x$$

Lagrangian  
of Interaction

$$\langle f | \mathbf{S} | i \rangle \cong \delta_{fi} - i \int_{-\infty}^{\infty} dt' \langle f | \mathbf{H}'(t') | i \rangle$$

► Feynman rules

# SYMMETRIES AND INVARIANCE

- Noether's theorem (1915):
  - Every **symmetry** under some operation corresponds to a **conservation law**

symmetry	invariant
Space translation	momentum
Time translation	energy
Rotation	Angular momentum
Global phase; $\Psi \rightarrow e^{i\theta}\Psi$	Electric charge
Local phase; $\Psi \rightarrow e^{i\theta(x,t)}\Psi$	Lagrangian + gauge field ( $\rightarrow$ QED)



# QED FROM LOCAL GAUGE INVARIANCE

- Apply local gauge symmetry to Dirac equation:

$$\bar{\Psi} \rightarrow e^{i\theta(x,t)} \bar{\Psi}, \Psi \rightarrow e^{-i\theta(x,t)} \Psi$$

This type of transformation leaves quantum mechanical amplitudes invariant.

- Consider very small changes in a field:

$$\Psi \rightarrow \Psi + \delta\Psi = \Psi - i\theta(x,t)\Psi \quad \text{ie. } \delta\Psi = -i\theta(x,t)\Psi$$

- The effect on the Lagrangian is:

$$L = \bar{\Psi} (i\gamma^\mu \partial_\mu - m) \Psi \Rightarrow \delta L = \bar{\Psi} \gamma^\mu \partial_\mu \theta(x,t) \Psi$$

# QED FROM LOCAL GAUGE INVARIANCE

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For the Lagrangian to remain invariant:  $\delta L = 0$

# QED FROM LOCAL GAUGE INVARIANCE

- To satisfy  $\delta L=0$ , we “engineer” a mathematical “trick”:
  1. Introduce a **gauge field  $A_\mu$**  to interact with fermions, and  $A_\mu$  transform as:  $A_\mu + \delta A_\mu = A_\mu + 1/e \partial_\mu \theta(x,t)$
  2. In resulting Lagrangian, replace  $\partial_\mu \rightarrow D_\mu = \partial_\mu + ieA_\mu$
- In that case,  $L$  is redefined:

$$L = \bar{\Psi} (i\gamma^\mu D_\mu - m) \Psi$$

The new Lagrangian is invariant under local gauge transformations.

# QED FROM LOCAL GAUGE INVARIANCE

## ONE MORE THING...

- Need to add kinetic term for field (field strength):

Define  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Add term  $-1/4 F_{\mu\nu} F^{\mu\nu}$  (Lorentz invariant, matches Maxwell's equations)

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## Final lagrangian (for QED!):

$$L = -1/4 F_{\mu\nu} F^{\mu\nu} + \bar{\Psi} (i\gamma^\mu D_\mu - m) \Psi$$

- No mass term is allowed for  $A_\mu$ , otherwise the Lagrangian is not gauge invariant
  - The gauge field is massless!

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## Final lagrangian (for QED!):

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- No mass is not gauge invariant
- The gauge

We have mathematically engineered a lagrangian quantum field that couples to fermions, obeys Maxwell's equations and is massless!

# THE HIGGS MECHANISM

# QCD AND WEAK LAGRANGIANS

- Follow similar reasoning, but allow for **self-interaction of gauge bosons**.
  - Jargon: QCD and weak interactions based on **non-abelian** groups.
- In non-abelian groups gauge invariance is achieved by adding  $n^2-1$  massless gauge bosons for  $SU(n)$ .
  - $SU(n)$ : gauge group.
  - $SU(3)$ : 8 massless gluons for QCD ✓
  - $SU(2)$ : 3 massless gauge bosons ( $W_1, W_2, W_3$ ) for weak force
  - If mixing with  $U(1)$ , we get ( $W_1, W_2, W_3$ ) and  $B$ : **electroweak force**.

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By the same mechanism as the **massless photon** arises from QED, a set of **massless bosons** arise when the theory is extended to include the weak nuclear force.

But Nature tells us they have mass!

# HIGGS MECHANISM

- A theoretically proposed mechanism which gives rise to elementary particle masses:  $W^+$ ,  $W^-$  and  $Z$  bosons (and solve other problems...).
- It actually predicts the mass of  $W^+$ ,  $W^-$  and  $Z^0$  bosons:
  - The  $W^+$ ,  $W^-$  bosons should have a mass of  $80.390 \pm 0.018$  GeV
  - The  $Z^0$  boson should have a mass of  $91.1874 \pm 0.0021$  GeV
  - Measurements:
    - ✓  $80.387 \pm 0.019$  GeV
    - ✓  $91.1876 \pm 0.0021$  GeV
- Beware: it ends up also providing a mechanism for fermion masses, but it doesn't make any prediction for those (...).

# HIGGS MECHANISM

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

Higgs Field

$$+ \mathcal{L}_{\phi'}$$

Massless case, from previous slide

# HIGGS MECHANISM

Yukawa Couplings

Higgs Field

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}' + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\phi'}$$

Massless case, from previous slide

# HIGGS MECHANISM

Yukawa Couplings

Higgs Field

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}' + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\phi'}$$

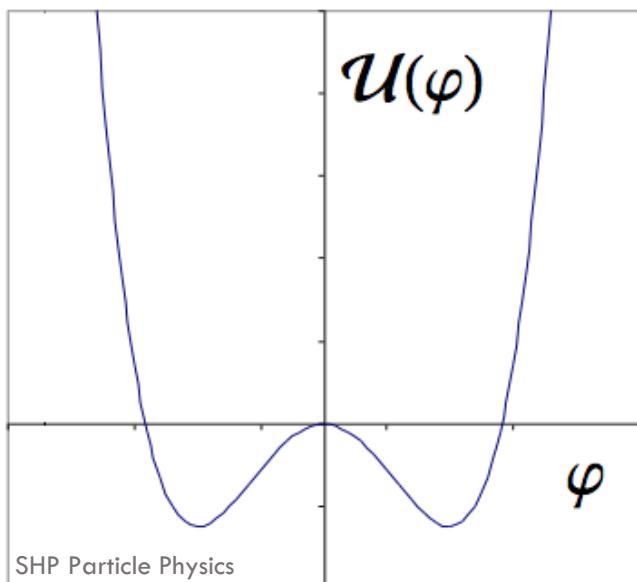
Massless case, from previous slide

$$\mathcal{L}_\phi = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - V(\phi) \quad \text{Higgs Potential}$$

$$\mathcal{L}_{\text{Yuk}} = c_f (\bar{\psi}_L \psi_R \phi + \bar{\psi}_R \psi_L \phi) \quad \text{Higgs Fermion Interaction}$$

# $V(\Phi)$ AND SPONTANEOUS SYMMETRY BREAKING

- When we speak of symmetry in the Standard Model, we usually mean **gauge symmetry**.
- This has to do with the form of the Lagrangian in terms of a given field  $\varphi$ .
- For simplicity, let's consider a simple (and suggestive!) example in which the Lagrangian has reflection symmetry about the  $\varphi=0$  axis:

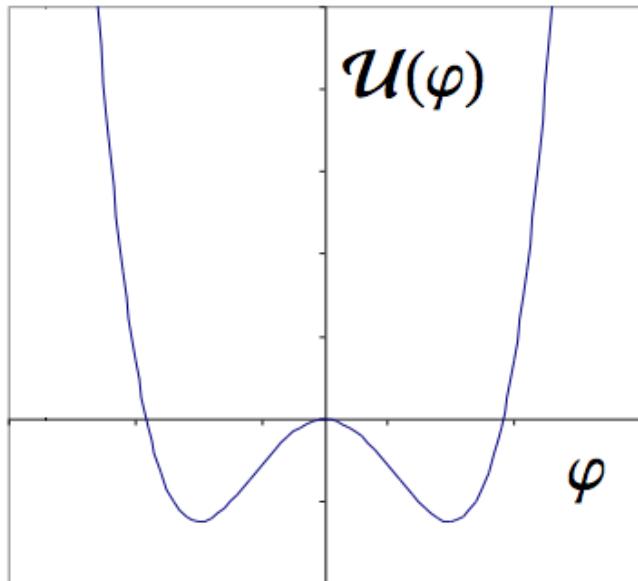


Lagrangian term for Higgs field, over all spacetime:

$$\mathcal{L}(\varphi) = \frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}m^2\varphi^2 - \frac{\lambda^2}{4}\varphi^4$$

“potential energy”: (recall:  $L = T-U$ )  
 $U(\phi) = -1/2 m^2\phi^2 + \lambda^2/4 \phi^4$

# $V(\Phi)$ AND SPONTANEOUS SYMMETRY BREAKING



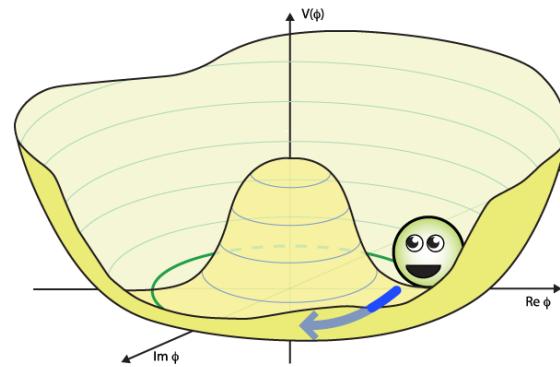
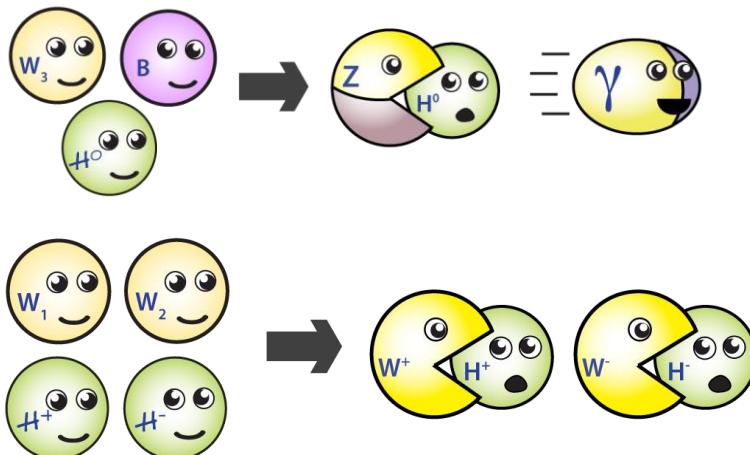
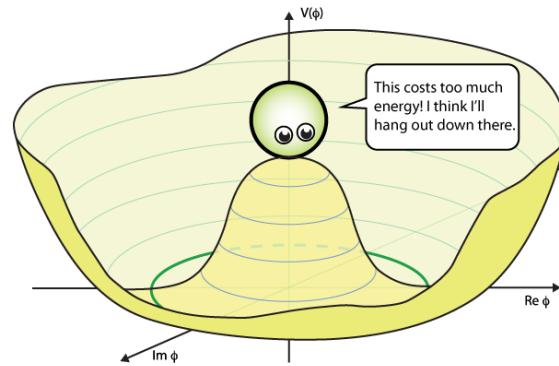
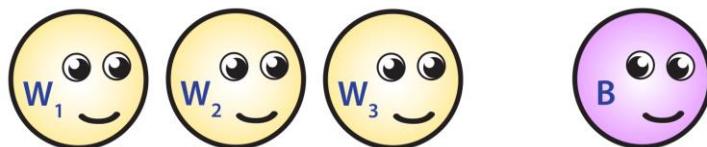
- To get a correct interpretation of the Lagrangian, and to be able to use **perturbative** calculations, we need to find one of its **minima** (a ground state) and look at small **fluctuations** of  $\varphi$  **about** that **minimum**.
- For this Lagrangian, the calculations must be formulated as small fluctuations of the field about **one of two** minima.
- **Switching** to either one of the two minima in  $U$ , effectively **breaks the symmetry** and introduces a new **massive particle**!

Calculation must be formulated in terms of deviations from one or the other of these ground states.

# $V(\Phi)$ AND SPONTANEOUS SYMMETRY BREAKING

- The phenomenon we have just considered is called *spontaneous symmetry breaking*.
- Why symmetry breaking? Our choice of a ground state “breaks” the obvious reflection *symmetry* of the original Lagrangian.
- What about the spontaneous part?
  - The choice of a ground state is *arbitrary* in this system. There is no external agency that favors one over the other, or even forces the choice to begin with.

# $V(\phi)$ AND SPONTANEOUS SYMMETRY BREAKING



# HIGGS MECHANISM

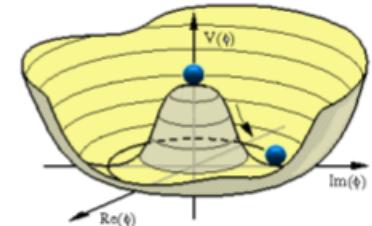


Yukawa Couplings

Higgs Field

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}' + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\phi'}$$

Massless case, from previous slide



$$\mathcal{L}_\phi = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - V(\phi) \quad \text{Higgs Potential}$$

$$\mathcal{L}_{\text{Yuk}} = c_f (\bar{\psi}_L \psi_R \phi + \bar{\psi}_R \psi_L \phi) \quad \text{Higgs Fermion Interaction}$$

- When  $\phi$  “picks a ground state” (i.e. when the gauge symmetry of a Lagrangian is spontaneously broken), all fermion fields and weak bosons become massive!
- Physically, the “massless” bosons acquire mass by interacting with a new “massive” scalar field called the Higgs field. Hence, this process is called the Higgs mechanism.

# ELECTROWEAK UNIFICATION

- In the end, the  $SU(2) \times U(1)$  part of the Standard Model is called the *electroweak theory*, because electromagnetism and the weak force start out **mixed together** in this overall gauge symmetry.
- $SU(2) \times U(1)$  predicts **four** massless bosons, which are not apparent at **everyday** energies.
- Analogous to our simple example, the ground state of the  $SU(2) \times U(1)$  theory (where we live) is one in which this gauge symmetry is **hidden**.
- **Result:** the four **massless** gauge bosons **appear** to us as the **massive**  $W^+$ ,  $W^-$ ,  $Z$  and the massless photon. The explicit  $U(1)$  symmetry of QED is preserved.

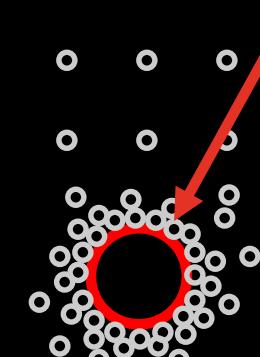
# HIGGS MECHANISM



- A **force field** that permeates the Universe and **slows** particles **down** to below the speed of light. This is the **equivalent** to having **mass**.



Massive Particle



# THE HIGGS PARTICLE

- Spin-0 (scalar) particle which gives **massive** force mediators their **mass**.

To understand the Higgs mechanism, imagine that a room full of physicists chattering quietly is like space filled with the Higgs field ...



# THE HIGGS PARTICLE

- Spin-0 (scalar) particle which gives **massive** force mediators their **mass**.

... a well-known scientist walks in, creating a disturbance as he moves across the room and attracting a cluster of admirers with each step

...



# THE HIGGS PARTICLE

- Spin-0 (scalar) particle which gives **massive** force mediators their **mass**.

... this increases his resistance to movement, in other words, he acquires mass, just like a particle moving through the Higgs field...



# THE STANDARD MODEL HOW SIMPLE, REALLY?

- The Standard Model does not predict:

Determine  
experimentally

3 Couplings	$g_s, e, \sin \theta_W$
4 CKM parameters	$\vartheta_1, \vartheta_2, \vartheta_3, \delta$
2 Boson masses	$m_Z, m_H$
3 Lepton masses	$m_e, m_\mu, m_\tau$
6 Quark masses	$m_u, m_d, m_s, m_c, m_t, m_b$
1 QCD vacuum angle $\theta$	

19 free SM parameters  
no neutrino masses

$$m_W^2 = \frac{1}{2} g^2 \rho_0^2$$

$$m_Z^2 = \frac{1}{2} (g^2 + g'^2) \rho_0^2 \quad g = e / \sin \theta_W$$

$$m_H^2 = 4 \lambda \rho_0^2$$

$$g' = e / \cos \theta_W \quad m_f = c_f \rho_0$$

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$$m_H^2 = 4 \lambda \rho_0^2$$

$$g = e / \sin \theta_W$$

$$g' = e / \cos \theta_W$$

Plus 7 from neutrino mass

$$m_f = c_f \rho_0$$

# THE STANDARD MODEL PREDICTS RELATIONSHIPS

- All observables can be predicted in terms of **26 free parameters** (including neutrino masses, mixing parameters).
- If we have  $> 26$  measurements of those observables, we **overconstrain** the SM.
  - Overconstrain: we don't have any more ad hoc inputs AND we **can test** the consistency of the model.
- In **practice**:
  - Pick **well measured** set of **observables**.
  - **Calculate** other **observables** in terms of these well known quantities.
  - **Test predictions**, measure **observables**, and compare to theory.

# THE HIGGS BOSON DISCOVERY



**Higgs Boson**  
Predicted in 1964 and  
discovered at the LHC  
in 2012!

VOLUME 13, NUMBER 9

PHYSICAL REVIEW LETTERS

31 AUGUST 1964

BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS\*

F. Englert and R. Brout

Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium

(Received 26 June 1964)

VOLUME 13, NUMBER 16

PHYSICAL REVIEW LETTERS

19 OCTOBER 1964

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs

Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland

(Received 31 August 1964)

VOLUME 13, NUMBER 20

PHYSICAL REVIEW LETTERS

16 NOVEMBER 1964

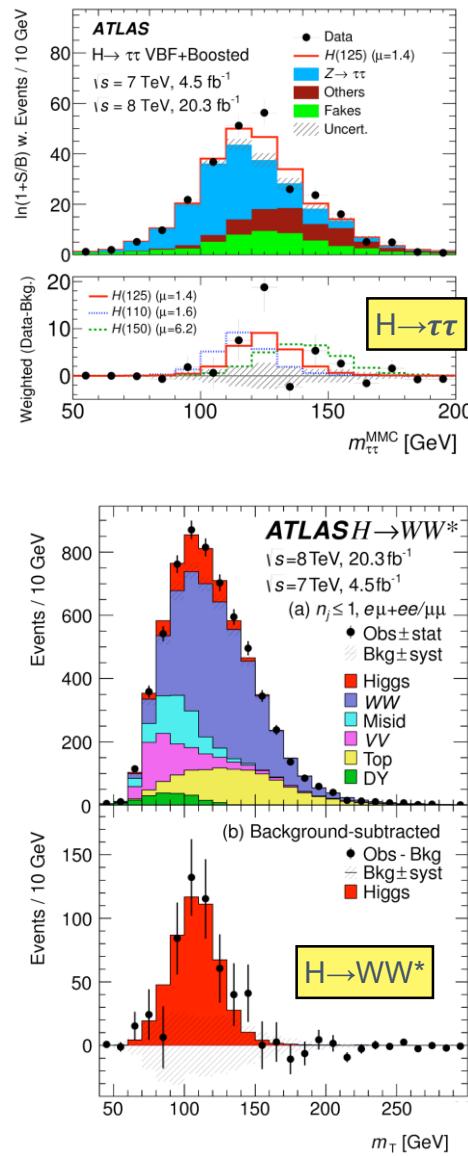
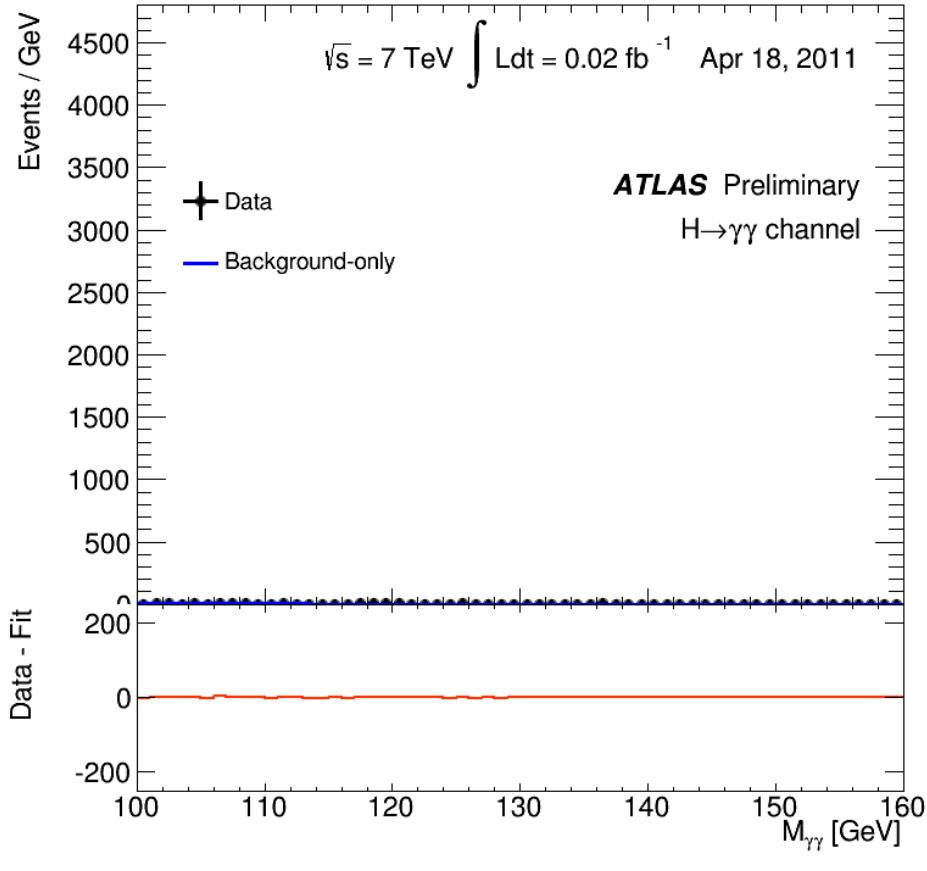
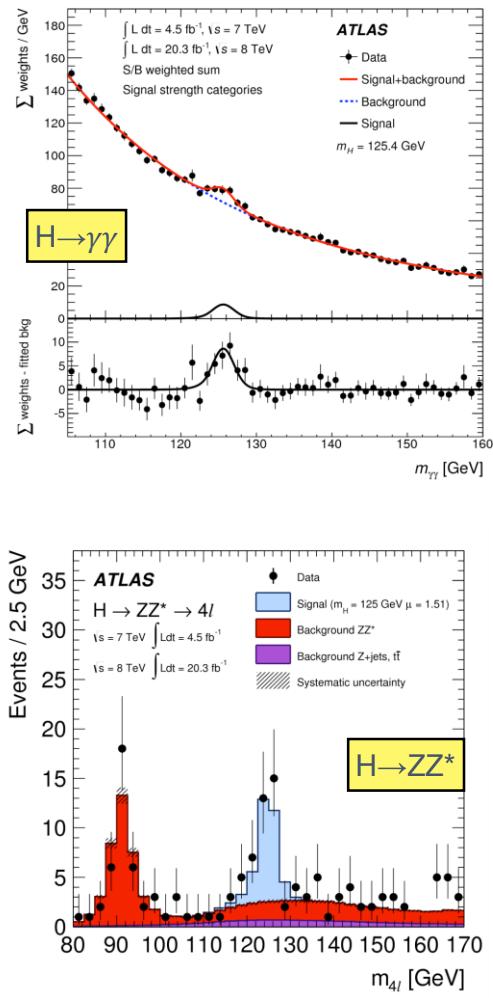
GLOBAL CONSERVATION LAWS AND MASSLESS PARTICLES\*

G. S. Guralnik, † C. R. Hagen, † and T. W. B. Kibble

Department of Physics, Imperial College, London, England

(Received 12 October 1964)

# THE HIGGS DISCOVERY (BY ATLAS + CMS)



# THE MOST PRECISELY TESTED THEORY

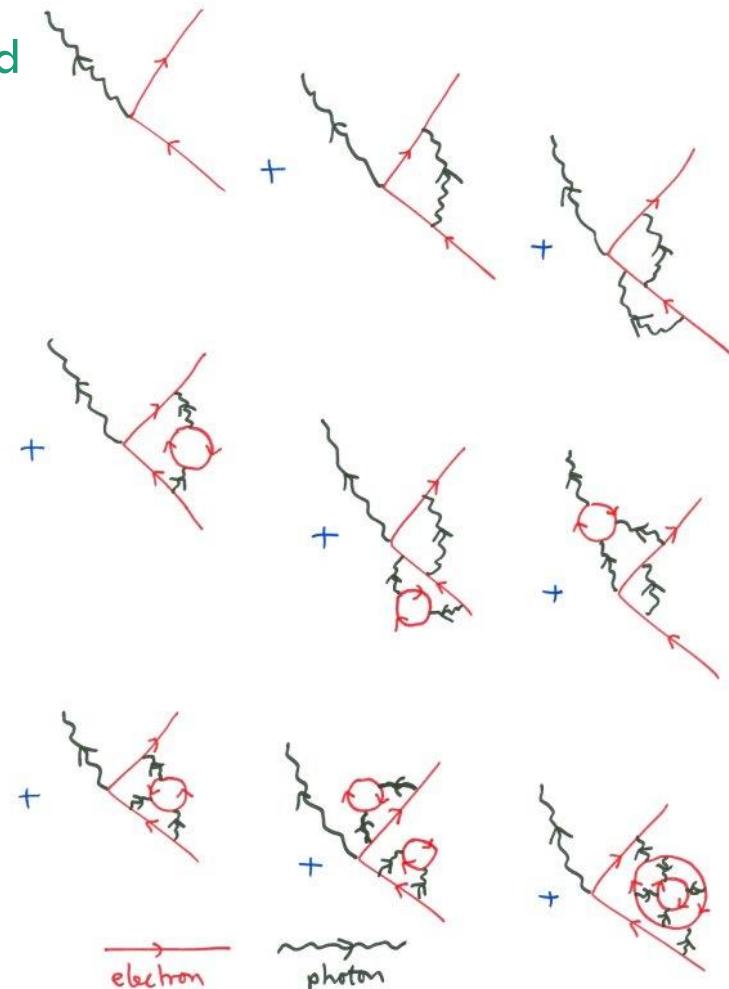
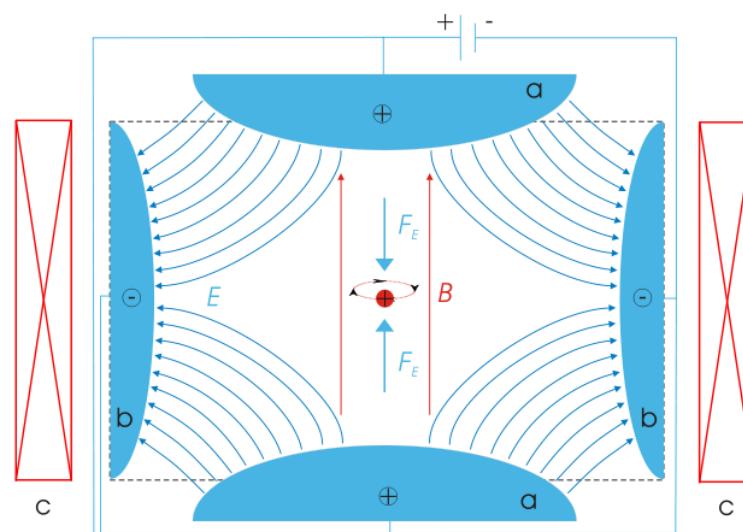
## QED

- Magnetic moment of the electron **calculated** up to order  $\alpha^5$  in QED:

$$a_e = 0.001\ 159\ 652\ 181\ 643(764)$$

- **Measured** in single electron Penning trap experiments:

$$a_e = 0.001\ 159\ 652\ 180\ 73(28)$$



# THE MOST PRECISELY TESTED THEORY

## QED

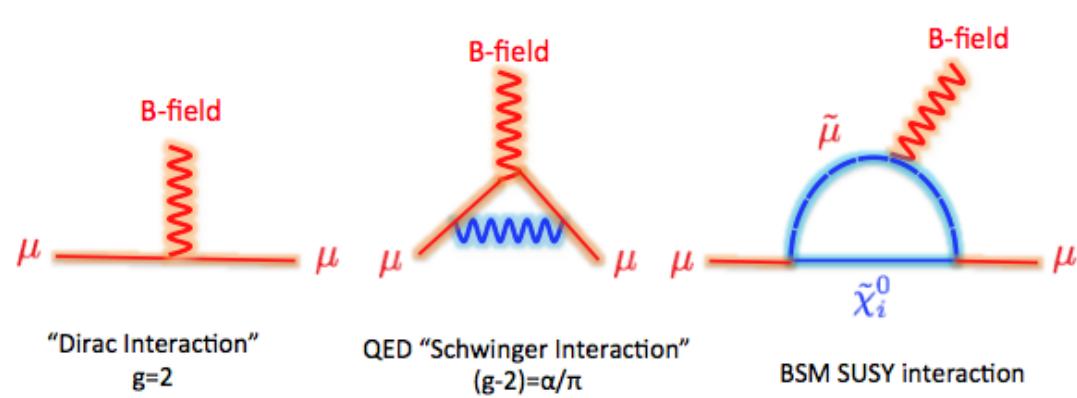
- Equivalent measurements in **muons** are an **active** area of research
- Very small **differences** between prediction:

$$\alpha_{\mu}^{\text{SM}} = \alpha_{\mu}^{\text{QED}} + \alpha_{\mu}^{\text{EW}} + \alpha_{\mu}^{\text{Hadron}} = 0.001\ 165\ 918\ 04(51)$$

- And **measurement**:

$$a_{\mu} = 0.001\ 165\ 920\ 91(54)(33),$$

- Might indicate **new** physics **beyond** the **Standard Model**



# The Standard Model of Particle Physics

## Spin 0 (Higgs Boson)

Hypercharge  $\rightarrow Y$   
Weak Isospin  $\rightarrow T_3$   
Gauge boson coupling

$mass \text{ (GeV)}$   
 $H_Q$   
 $Electric \text{ Charge}$   
 $Q = Y + T_3$

## Spin 1/2 (Fermions)

Hypercharge (L)  $\rightarrow Y$   
Weak Isospin (L)  $\rightarrow T_3$   
Gauge boson coupling

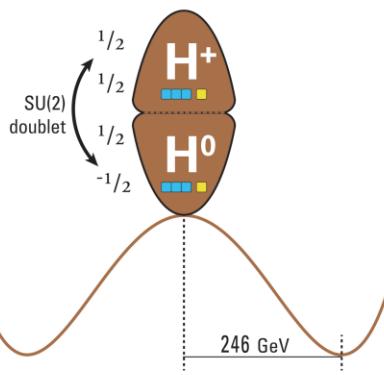
Generation  
flavor

$mass \text{ (GeV)}$   
 $Y$   
 $T_3$   
 $Q$   
 $Q = Y + T_3$

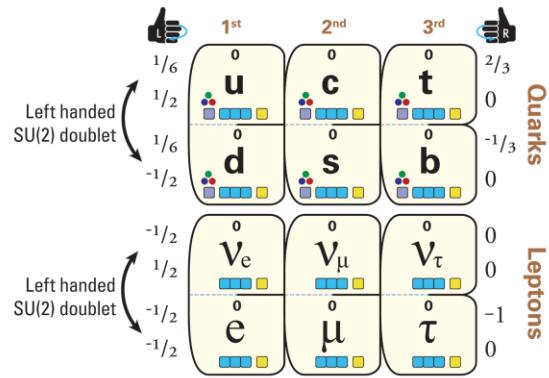
## Spin 1 (Gauge Bosons)

Fermion coupling

$mass \text{ (GeV)}$   
 $symbol$



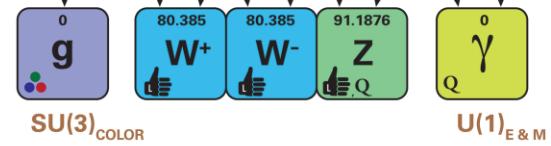
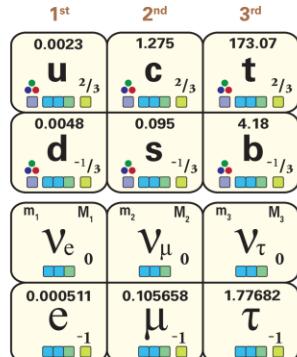
Unbroken Symmetry  
Broken Symmetry



$$W^\pm = (W^1 \pm iW^2)/\sqrt{2}$$

$$Z = \cos \theta_w W^3 - \sin \theta_w B$$

$$\gamma = \sin \theta_w W^3 + \cos \theta_w B$$



# THE STANDARD MODEL LAGRANGIAN

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{gauge/}\psi} . \quad (1)$$

Here,

$$\mathcal{L}_{\text{Dirac}} = i\bar{e}_L^i \partial^\mu e_L^i + i\bar{\nu}_L^i \partial^\mu \nu_L^i + i\bar{e}_R^i \partial^\mu e_R^i + i\bar{u}_L^i \partial^\mu u_L^i + i\bar{d}_L^i \partial^\mu d_L^i + i\bar{u}_R^i \partial^\mu u_R^i + i\bar{d}_R^i \partial^\mu d_R^i ; \quad (2)$$

$$\mathcal{L}_{\text{mass}} = -v \left( \lambda_e^i \bar{e}_L^i e_R^i + \lambda_u^i \bar{u}_L^i u_R^i + \lambda_d^i \bar{d}_L^i d_R^i + \text{h.c.} \right) - M_W^2 W_\mu^+ W^{-\mu} - \frac{M_W^2}{2 \cos^2 \theta_W} Z_\mu Z^\mu ; \quad (3)$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} (G_{\mu\nu}^a)^2 - \frac{1}{2} W_{\mu\nu}^+ W^{-\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{WZA} , \quad (4)$$

where

$$\begin{aligned} G_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_3 f^{abc} A_\mu^b A_\nu^c \\ W_{\mu\nu}^\pm &= \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm \\ Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu , \end{aligned} \quad (5)$$

and

$$\begin{aligned} \mathcal{L}_{WZA} = & ig_2 \cos \theta_W \left[ (W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+) \partial^\mu Z^\nu + W_{\mu\nu}^+ W^{-\mu} Z^\nu - W_{\mu\nu}^- W^{+\mu} Z^\nu \right] \\ & + ie \left[ (W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+) \partial^\mu A^\nu + W_{\mu\nu}^+ W^{-\mu} A^\nu - W_{\mu\nu}^- W^{+\mu} A^\nu \right] \\ & + g_2^2 \cos^2 \theta_W (W_\mu^+ W_\nu^- Z^\mu Z^\nu - W_\mu^+ W^{-\mu} Z_\nu Z^\nu) \\ & + g_2^2 (W_\mu^+ W_\nu^- A^\mu A^\nu - W_\mu^+ W^{-\mu} A_\nu A^\nu) \\ & + g_2 e \cos \theta_W \left[ W_\mu^+ W_\nu^- (Z^\mu A^\nu + Z^\nu A^\mu) - 2W_\mu^+ W^{-\mu} Z_\nu A^\nu \right] \\ & + \frac{1}{2} g_2^2 (W_\mu^+ W_\nu^-) (W^{+\mu} W^{-\nu} - W^{+\nu} W^{-\mu}) ; \end{aligned} \quad (6)$$

and

$$\mathcal{L}_{\text{gauge/}\psi} = -g_3 A_\mu^a J_{(3)}^{\mu a} - g_2 \left( W_\mu^+ J_{W^+}^\mu + W_\mu^- J_{W^-}^\mu + Z_\mu J_Z^\mu \right) - e A_\mu J_A^\mu , \quad (7)$$

where

$$\begin{aligned} J_{(3)}^{\mu a} &= \bar{u}^i \gamma^\mu T_{(3)}^a u^i + \bar{d}^i \gamma^\mu T_{(3)}^a d^i \\ J_{W^+}^\mu &= \frac{1}{\sqrt{2}} (\bar{\nu}_L^i \gamma^\mu e_L^i + V^{ij} \bar{u}_L^i \gamma^\mu d_L^j) \\ J_{W^-}^\mu &= (J_{W^+}^\mu)^* \\ J_Z^\mu &= \frac{1}{\cos \theta_W} \left[ \frac{1}{2} \bar{\nu}_L^i \gamma^\mu \nu_L^i + \left( -\frac{1}{2} + \sin^2 \theta_W \right) \bar{e}_L^i \gamma^\mu e_L^i + (\sin^2 \theta_W) \bar{e}_R^i \gamma^\mu e_R^i \right. \\ &+ \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \bar{u}_L^i \gamma^\mu u_L^i + \left( -\frac{2}{3} \sin^2 \theta_W \right) \bar{u}_R^i \gamma^\mu u_R^i \\ &+ \left. \left( -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right) \bar{d}_L^i \gamma^\mu d_L^i + \left( \frac{1}{3} \sin^2 \theta_W \right) \bar{d}_R^i \gamma^\mu d_R^i \right] \\ J_A^\mu &= (-1) \bar{e}^i \gamma^\mu e^i + \left( \frac{2}{3} \right) \bar{u}^i \gamma^\mu u^i + \left( -\frac{1}{3} \right) \bar{d}^i \gamma^\mu d^i . \end{aligned} \quad (8)$$

# THE STANDARD MODEL LAGRANGIAN



# SUMMARY

- The **Standard Model** is a framework that describes the **elementary particles** and their **electromagnetic, weak** and **strong** interactions.
  - Furthermore, it provides an explanation for the electromagnetic and weak interactions in terms of a **spontaneously broken electroweak symmetry**.
- The Standard Model predicts **cross-sections**, couplings.
- Latest success: the **discovery** of the Higgs boson!
- However, the Standard Model is **not** completely satisfactory:
  - 26 free parameters...
  - We know it is **incomplete**: it does not fully describe nature.

# STANDARD MODEL LIMITATIONS

# STANDARD MODEL LIMITATIONS OBSERVED DISCREPANCIES

- How can we describe **gravity** and the three Standard Model forces consistently?
  - We know gravity exists, but **incompatible** with Gauge theories...
- What is the **mass** of **neutrinos**, and what is its origin?
  - We **know** neutrino mass is finite.
  - Simple extensions to the Standard Model **may** be able to accommodate neutrino mass.
- What are **dark matter** and **dark energy**?
  - We know they exist but we **don't know** what they are. Theories beyond the Standard Model might offer solutions...
- How did we end up in a **matter** dominated universe?
  - The Standard Model might explain this...

# STANDARD MODEL LIMITATIONS

## PHILOSOPHICAL NUISANCES (“WHY” RATHER THAN “WHAT/HOW”)

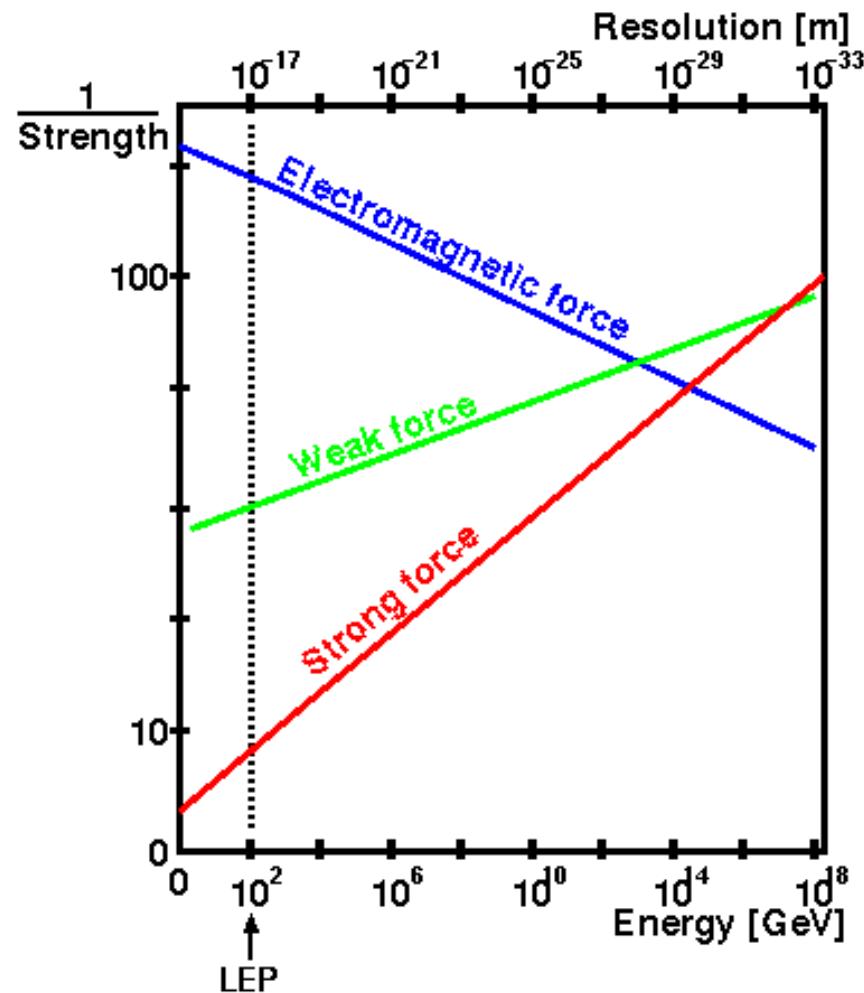
- Why are there **three** generations of matter?
  - “Because” this is required for CP violation? And CP violation required for matter-dominated universe: us?
    - Anthropic principle – perhaps not great Science...
- Why are some of the Standard Model parameters **“unnaturally”** large / small?
  - Parameters that look like “1” give theorists a warm and fuzzy feeling
- Why are the **masses** what they are?
  - And why are they so **different**?!  $m_t/m_\nu \sim 10^{14}$

# FACING THE STANDARD MODEL LIMITATIONS

- Over the history of particle physics, a great deal of time has been spent by both **theorists** and **experimentalists** to try to **resolve** the limitations of the Standard Model.
- Generally, the strategy has been to **extend** the Standard Model in a way or another:
  - **Grand Unified Theories.**
    - Predicts **proton decay**.
  - **Supersymmetry.**
    - Might explain dark matter.
- A longer term, and more **ambitious** project has been to formulate a **Theory of Everything**, that includes gravity.
  - So far these theories have **lacked predictive power**...

# GUT SCALE

- Grand unification scale:  
 $10^{16}$  GeV
- The Standard Model provides **no explanation** for what may happen beyond this unification scale, nor why the forces have such different strengths at low energies.



# GRAND UNIFIED THEORIES

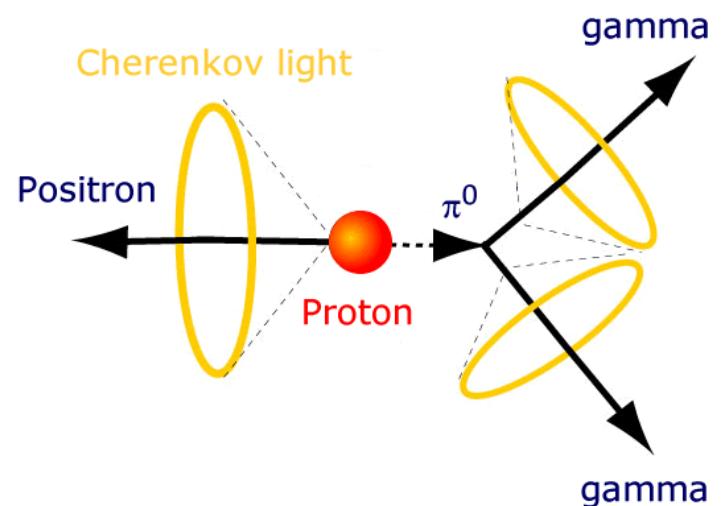
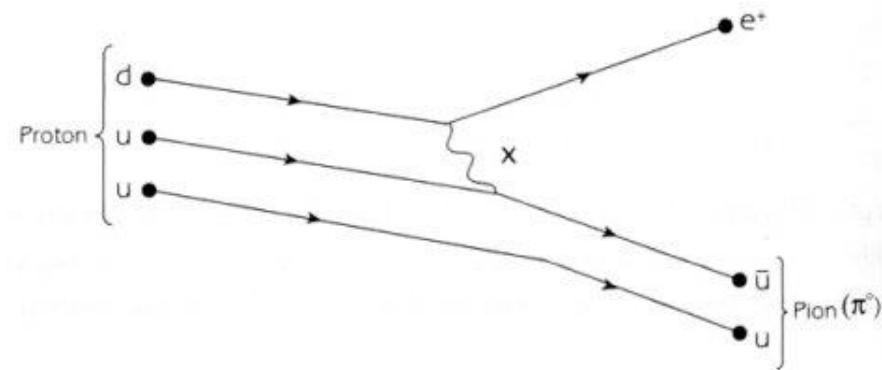
- In the 1970's, people started to think a lot about how to combine the  $SU(3)$ ,  $SU(2)$ , and  $U(1)$  gauge symmetries of the Standard Model into a more **fundamental, global** symmetry.
- The first such Grand Unified Theory is a 1974 model based on  $SU(5)$  symmetry.
- This model groups **all** of the known fermions – i.e., the leptons and quarks – into multiplets.
- Inside the multiplets, quarks and leptons can **couple** to each other and **transform** into one another.
- In essence, this theory imposes a grand **symmetry**: all fermions, whether **quarks** or **leptons**, are fundamentally the **same**.

# THE SU(5) GUT

- In this early model, interactions between the quarks and leptons are mediated by two **new** massive **bosons**, called the X and Y.
- To **conserve** electric and color charge, the X and Y have electric charges of  $-4e/3$  and  $-e/3$ , and one of three possible colors. They are also incredibly **massive**, close to the grand unification scale of  $10^{16}$  GeV.
- Hence, including both particles and antiparticles, the model predicts **12** types of X and Y.
- In addition to these 12, there are also 8 gluons, 3 weak bosons, and 1 photon, for a total of **24**. This makes sense, for recall that a theory exhibiting **SU(n)** gauge symmetry requires the existence of  **$n^2-1$**  gauge bosons.

# SU(5) GUT OBSERVATIONAL CONSEQUENCES

- The SU(5) GUT implies that it would take **huge** energies to even hope to **see** an X or Y particle “in the wild”.
- However, even at “**normal**” (i.e. low) energies, virtual X and Y exchanges **can** take place.
- This is major: if **quarks** can **decay** into **leptons** via **virtual** X and Y exchange, then “stable” particles might actually be unstable!
- Example: the **proton** could possibly decay via exchange of a virtual X.



# PROTON DECAY

- The **instability** of the proton is one of the few **tests** of GUT physics that would be manifest at everyday energies.
- Computations show that relative to most elementary particles, the proton is very stable; its lifetime according to the SU(5) GUT is  $10^{30}$  years!
- How can we detect such an effect?
- Put **many** protons together – e.g., in a huge tank of water – and **wait** for some to decay...
- The **Super-Kamiokande** water Cherenkov detector, was built to **search** for proton decay, although it has made major contributions to understanding neutrino oscillations!



# PROTON DECAY



Neil did not see a proton decay, but he was very excited about neutrino oscillations!

# PROTON DECAY

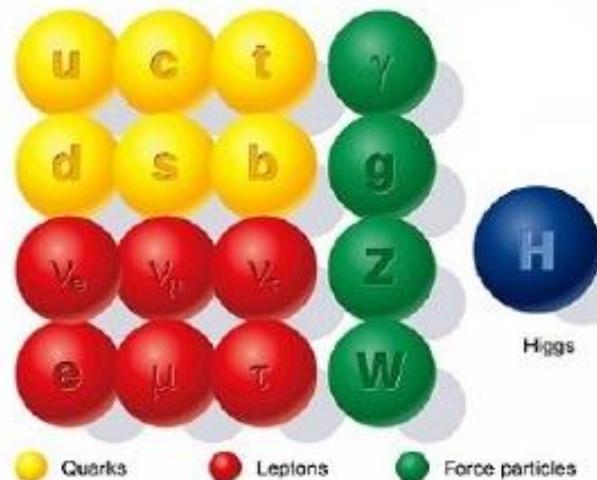
- Even though the proton lifetime is **very long**, a kiloton of material contains about  $10^{32}$  protons, so about **one decay per day** should occur in such a sample.
- The Super-Kamiokande detector holds **50 kilotons** of water viewed by 11000 photomultiplier tubes. It is located **underground**, shielded from background noise due to cosmic radiation at the Earth's surface.
- **No proton decays have been observed** despite more than **two decades** of searching, leading to lower limits on the proton lifetime of about  $10^{34}$  years.
- Is this a disaster for the theory? Not necessarily. GUT physics can be saved if we introduce **supersymmetry**...

# SUPERSYMMETRY

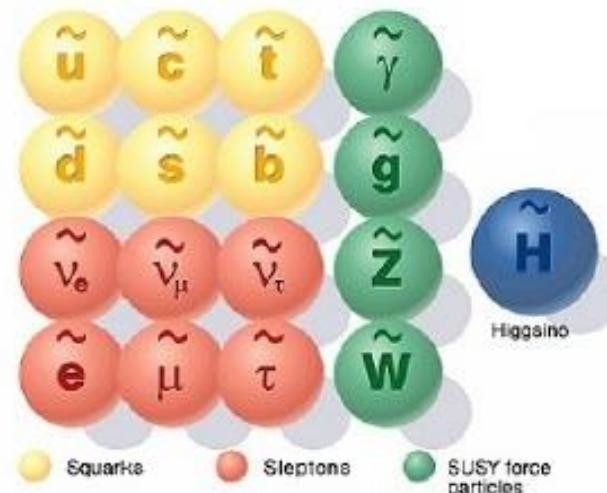
- The idea behind GUTs like  $SU(5)$  symmetry is to present different particles as **transformed versions** of each other.
- The  $SU(5)$  GUT treats quarks and leptons as the same underlying “something”, **unifying the fermions** in the Standard Model.
- However, these early approaches to grand unification **failed** to incorporate the gauge bosons and Higgs scalars of the Standard Model into a unified scheme.
- Hence, some eventually suggested that the **bosons** and **fermions** should be **united** somehow by invoking a new kind of symmetry: **supersymmetry** (or **SUSY** for short).
- Aside: theorists did not develop supersymmetry explicitly for the Standard Model; the field started obscurely in the early 1970's during investigations of the spacetime symmetries in quantum field theory.

# SUPERSYMMETRY

- So what, exactly, does supersymmetry **predict**?
- It says that for each known **boson**, there should be a **fermion** of **identical mass** and **charge**, and similarly for each known **fermion**, there exists a **boson** of identical mass and charge.



Standard particles



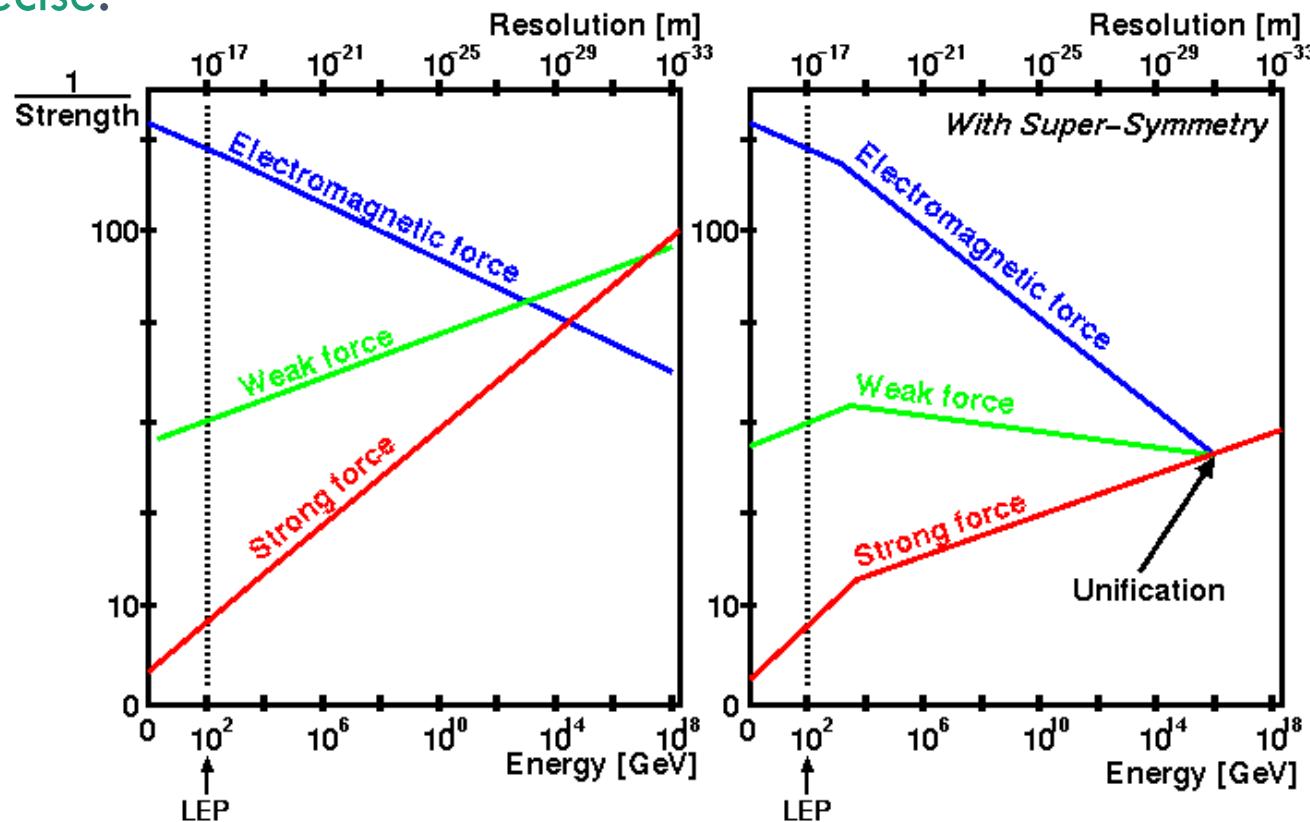
SUSY particles

# SUPERSYMMETRY

- So what, exactly, does supersymmetry **predict**?
- It says that for each known **boson**, there should be a **fermion of identical mass and charge**, and similarly for each known **fermion**, there exists a **boson of identical mass and charge**.
- If this is the case, why haven't we observed a **selectron** of mass **0.511 MeV**?
  - Supersymmetry is a **broken symmetry** at our energy scale.
  - **All sparticles** are expected to be **more massive** than their Standard Model partners.

# SUPERSYMMETRY AND THE GUT SCALE

- If instead of the Standard Model, we start from the Minimal Supersymmetric Standard Model, the alignment of the three coupling constants at the GUT scale becomes much **more precise**.



# SUPERSYMMETRY AT THE LHC

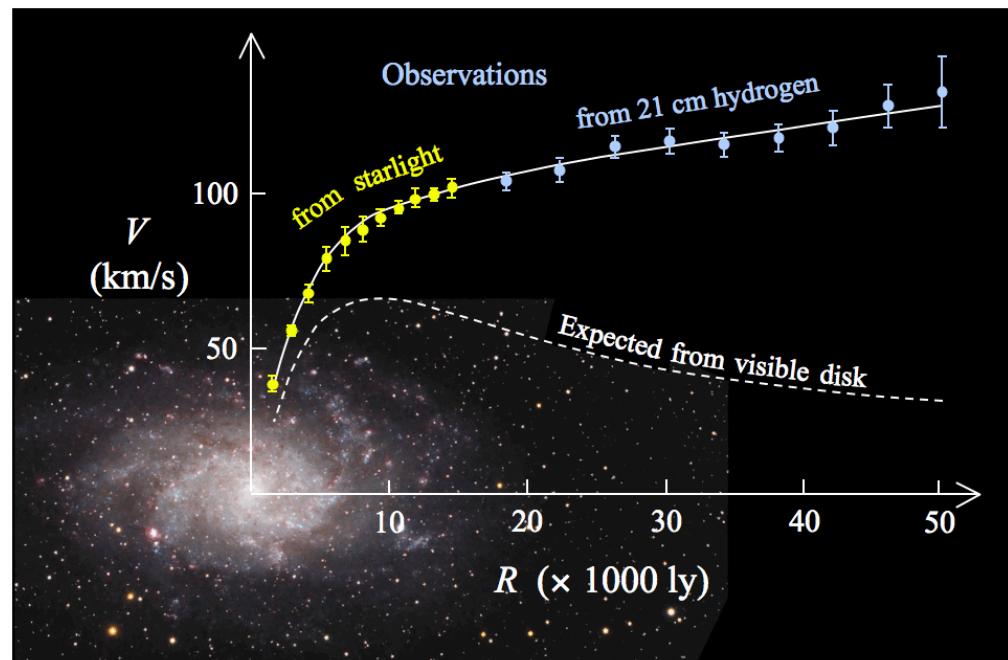
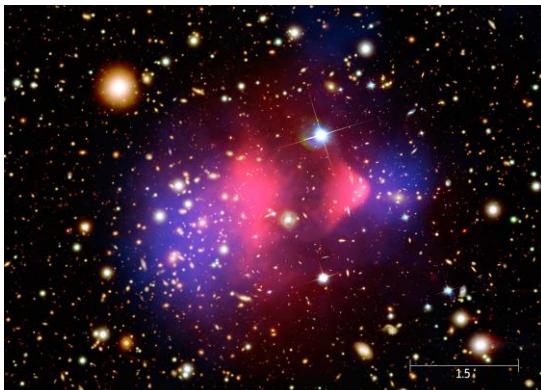
- If SUSY is a symmetry of nature, some of the sparticles might have a mass around 1000 GeV.
- We hope to see evidence for SUSY at the LHC.
  - But so far no candidates have been found...



Excited LHC physicists still haven't found SUSY.

# DARK MATTER

- Galaxies rotate **faster** than expected, implying **more mass** than what is **seen**.
- A host of **other observations** point in the same direction...
- Alternatively, **general relativity** might need **modification**.
  - Probably not the solution...
- Scientific consensus leaning towards the weakly interacting particle hypothesis.
  - We need to observe it in the lab!

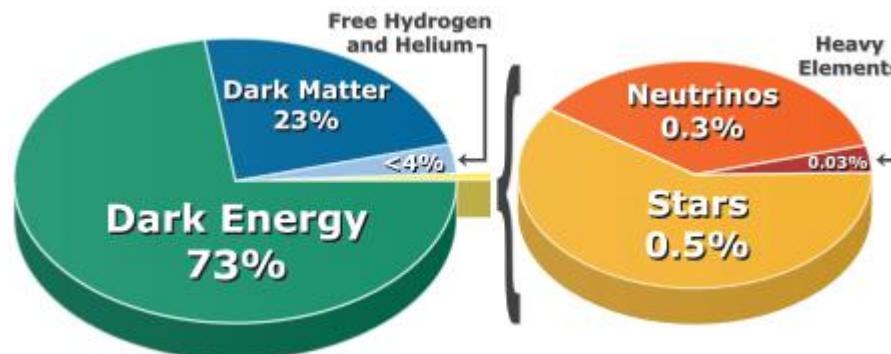


# SUPERSYMMETRY AND DARK MATTER

- The neutrinos that we know are not good candidates to explain dark matter.
  - They are “dark” but also too light – moving too fast to cluster around galaxies.
- To account for dark matter, we need a particle that looks a lot like a neutrino, but more massive.
  - No electric charge or color.
- Lightest Supersymmetric Particle
  - We haven’t seen proton decay so, if SUSY exists, decays of sparticles into particles must be highly suppressed.
    - We call this R-parity.
  - Hence the LSP is stable, much heavier than neutrinos, and in several SUSY models, charge neutral.
    - A dark matter candidate!

# ...AND THERE'S ALSO DARK ENERGY

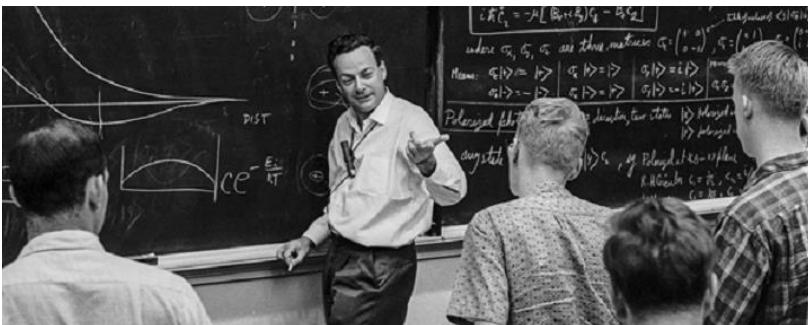
- The universe is **expanding** at an **accelerating** rate!
  - This can be accommodated in **general relativity** by having a non-zero **cosmological constant** ( $\Lambda$ ), also called **vacuum energy**.
- What is the **origin** of this vacuum energy? Quantum field theories tend to predict vacuum energies that are **far too large**...



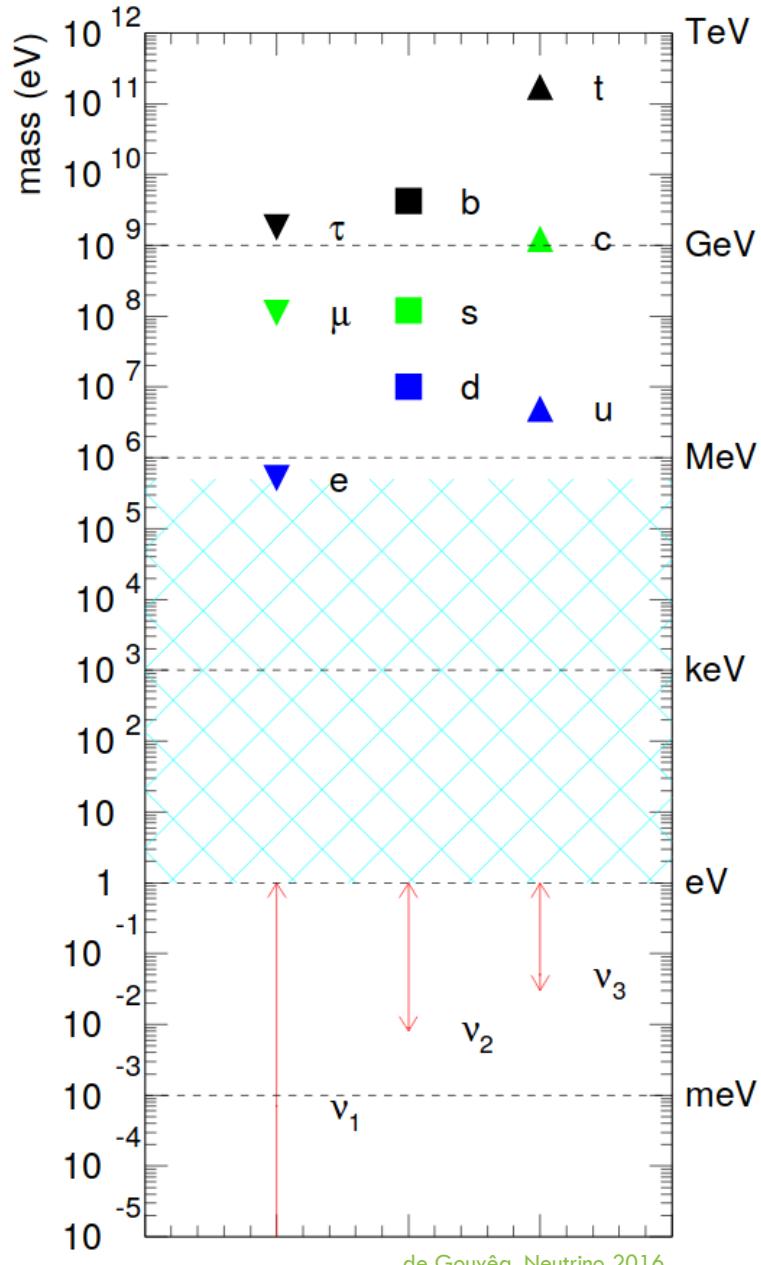
# PARTICLE MASSES

- “There remains one especially unsatisfactory feature [of the Standard Model of particle physics]: the observed masses of the particles,  $m$ . There is no theory that adequately explains these numbers. We use the numbers in all our theories, but we do not understand them – what they are, or where they come from. I believe that from a fundamental point of view, this is a very interesting and serious problem.”

- R. P. Feynman



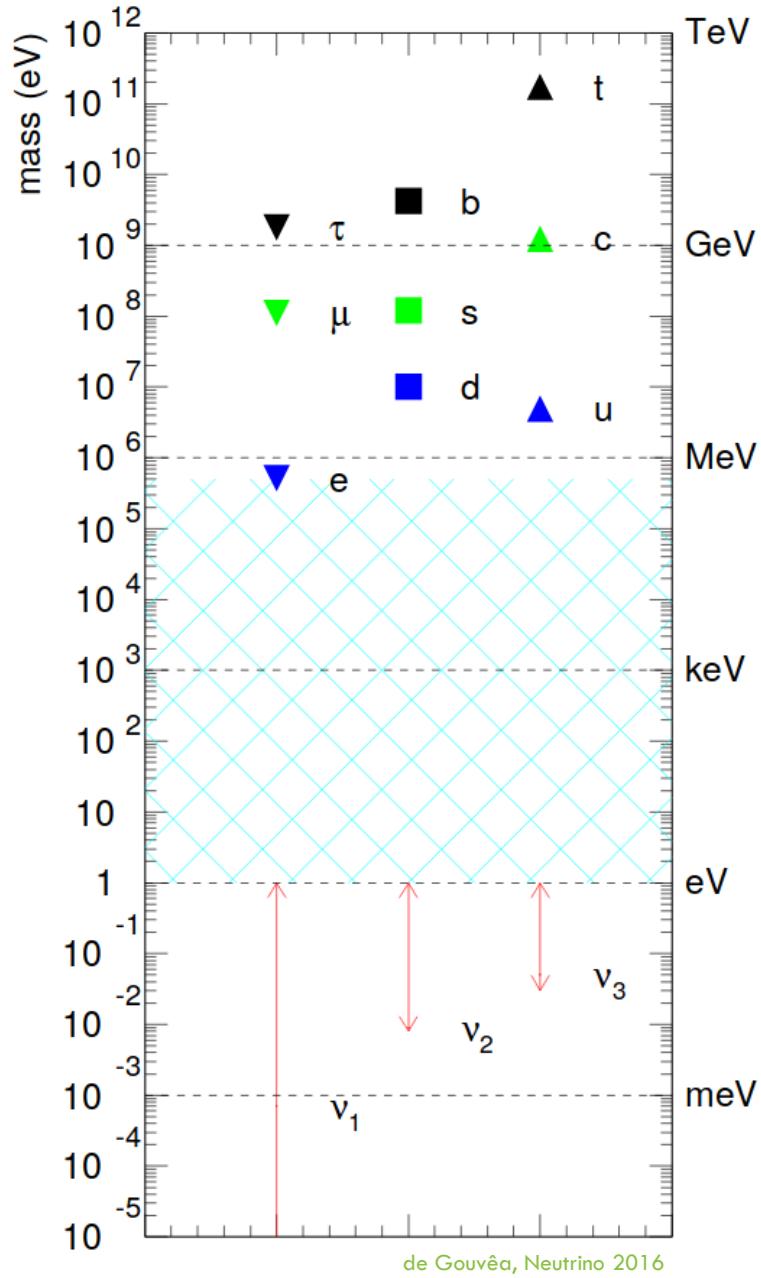
SHP Particle Physics



de Gouv  a, Neutrino 2016

# PARTICLE MASSES

- Is there a theory that can accurately predict the masses of the fermions?
  - None so far...
  - Would it elucidate the nature of the three generations?
- Why are neutrino masses so much smaller than the other fermion masses?
  - Is the mechanism that generates neutrino masses qualitatively different from the mechanism that generates other fermion masses?

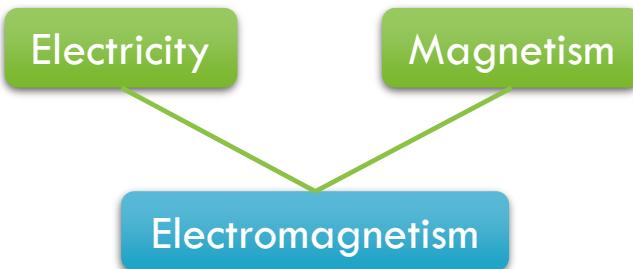


# A THEORY OF EVERYTHING?

Electricity

Magnetism

# A THEORY OF EVERYTHING?



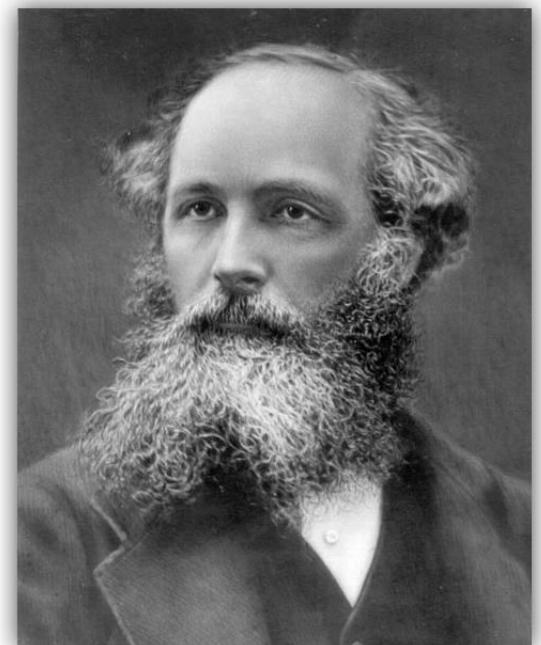
$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

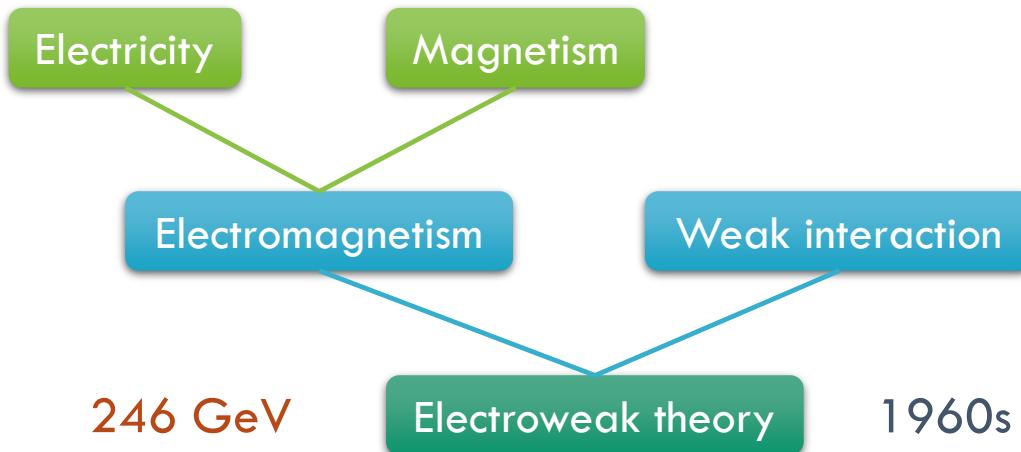
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

James Clerk Maxwell  
1800s



# A THEORY OF EVERYTHING?



Sheldon Lee  
Glashow

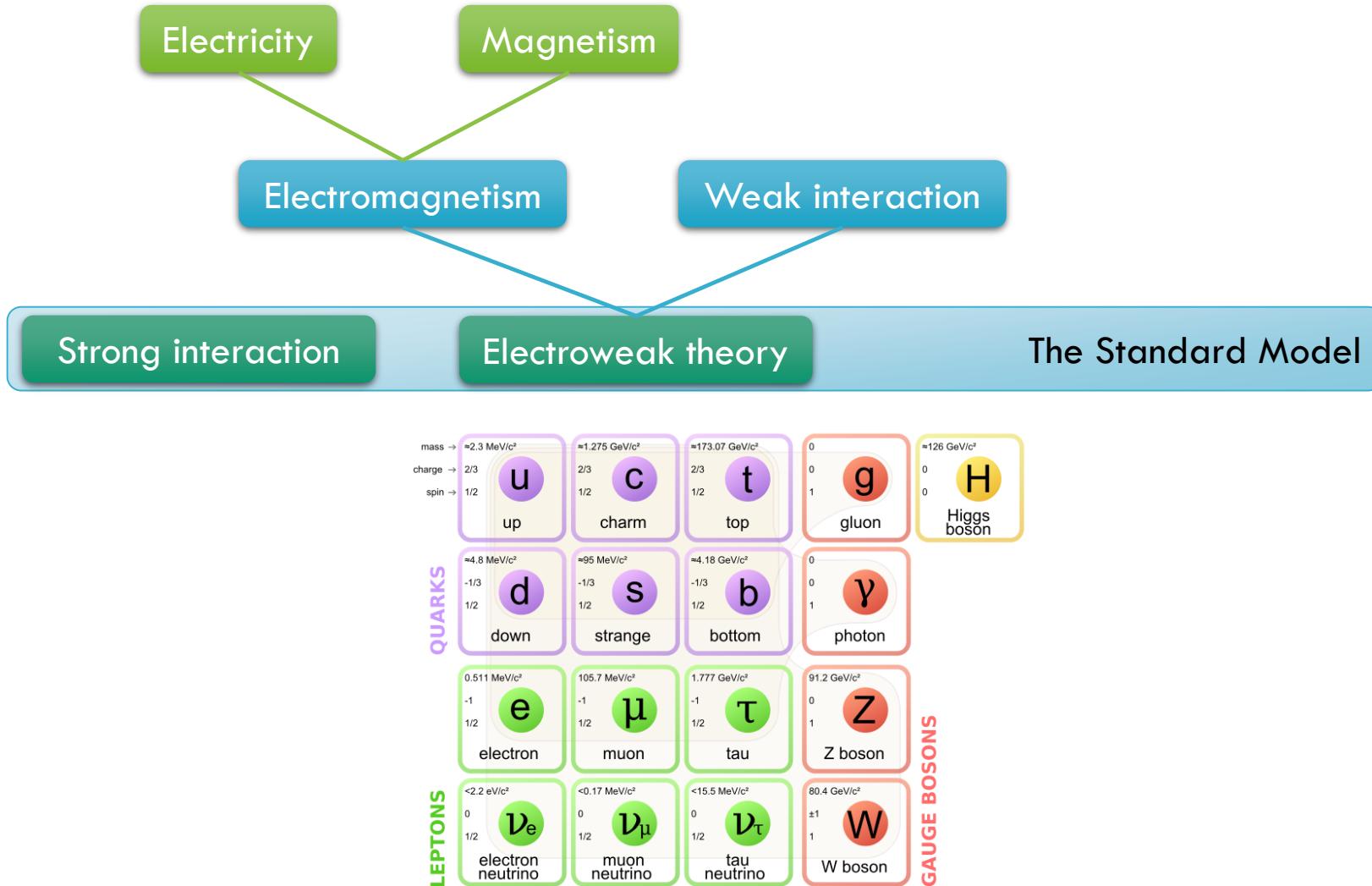


Abdus Salam

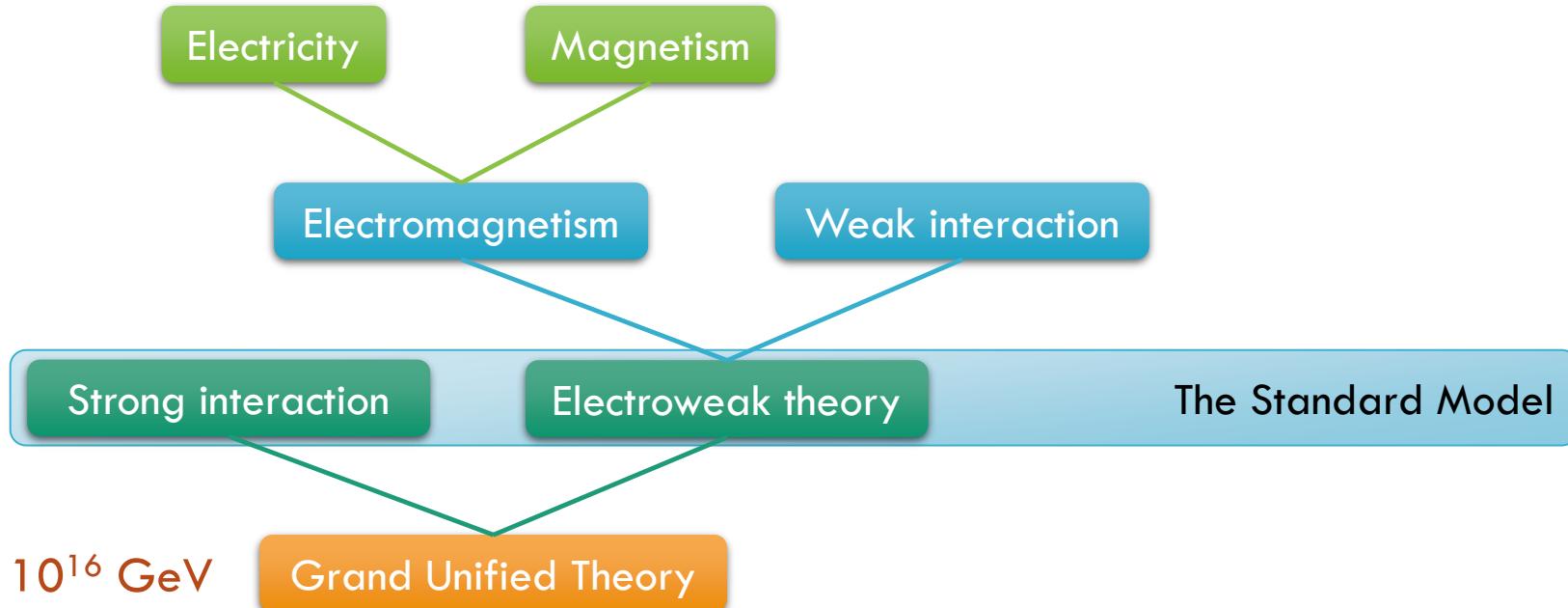


Steven Weinberg

# A THEORY OF EVERYTHING?

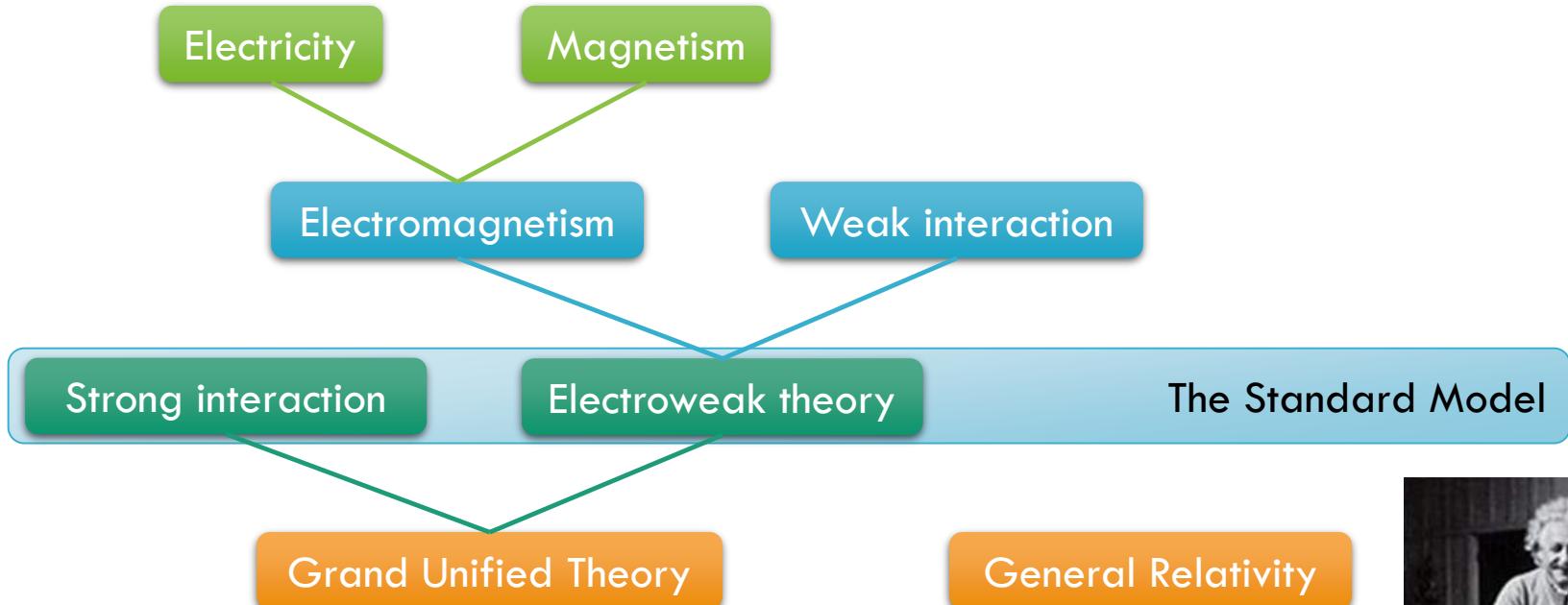


# A THEORY OF EVERYTHING?



SU(5)? SO(10)? SU(8)? O(16)?  
1970s – today...

# A THEORY OF EVERYTHING?



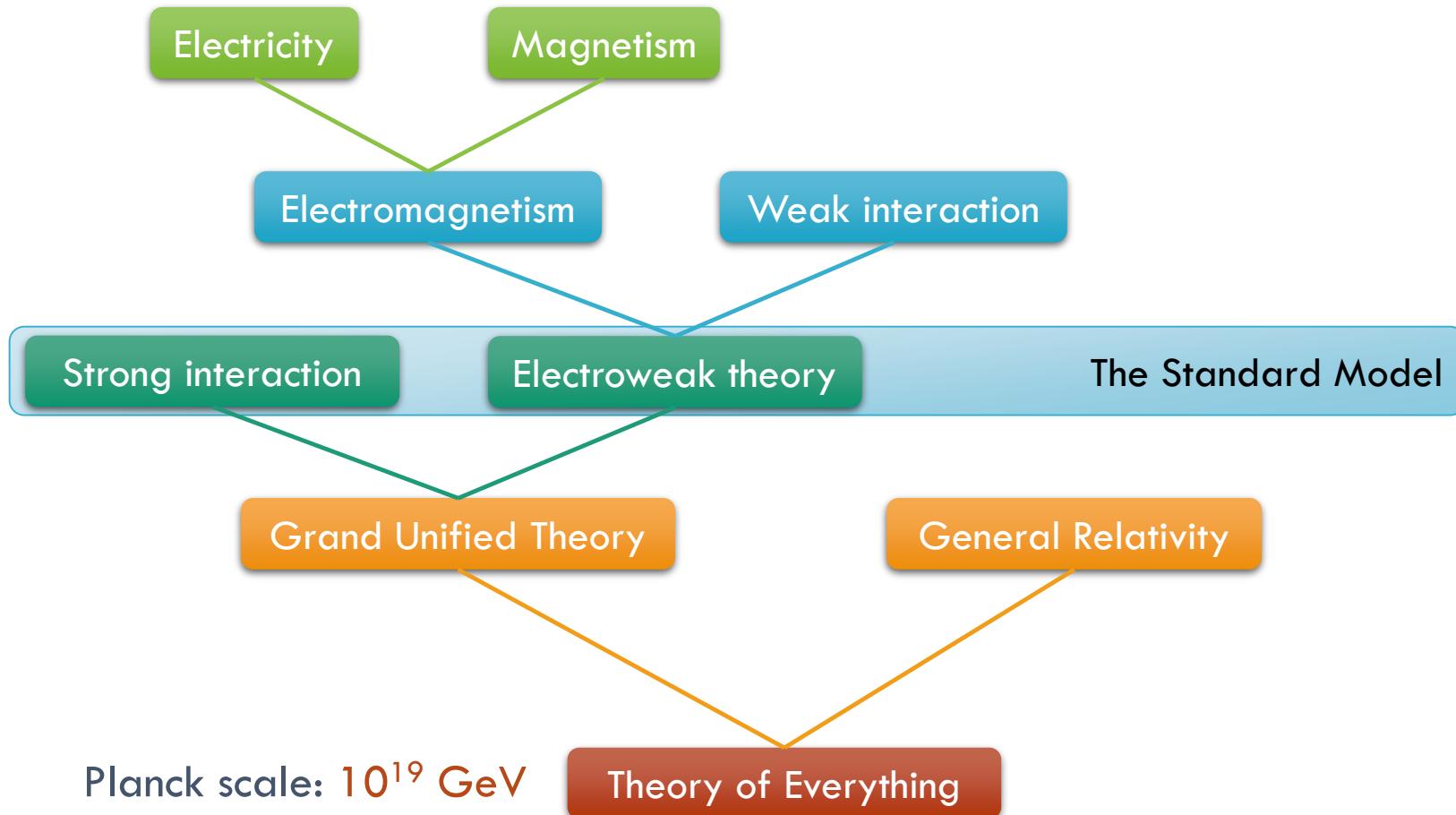
SO(5)? SO(10)? SU(8)? O(16)?  
1970s – today...

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

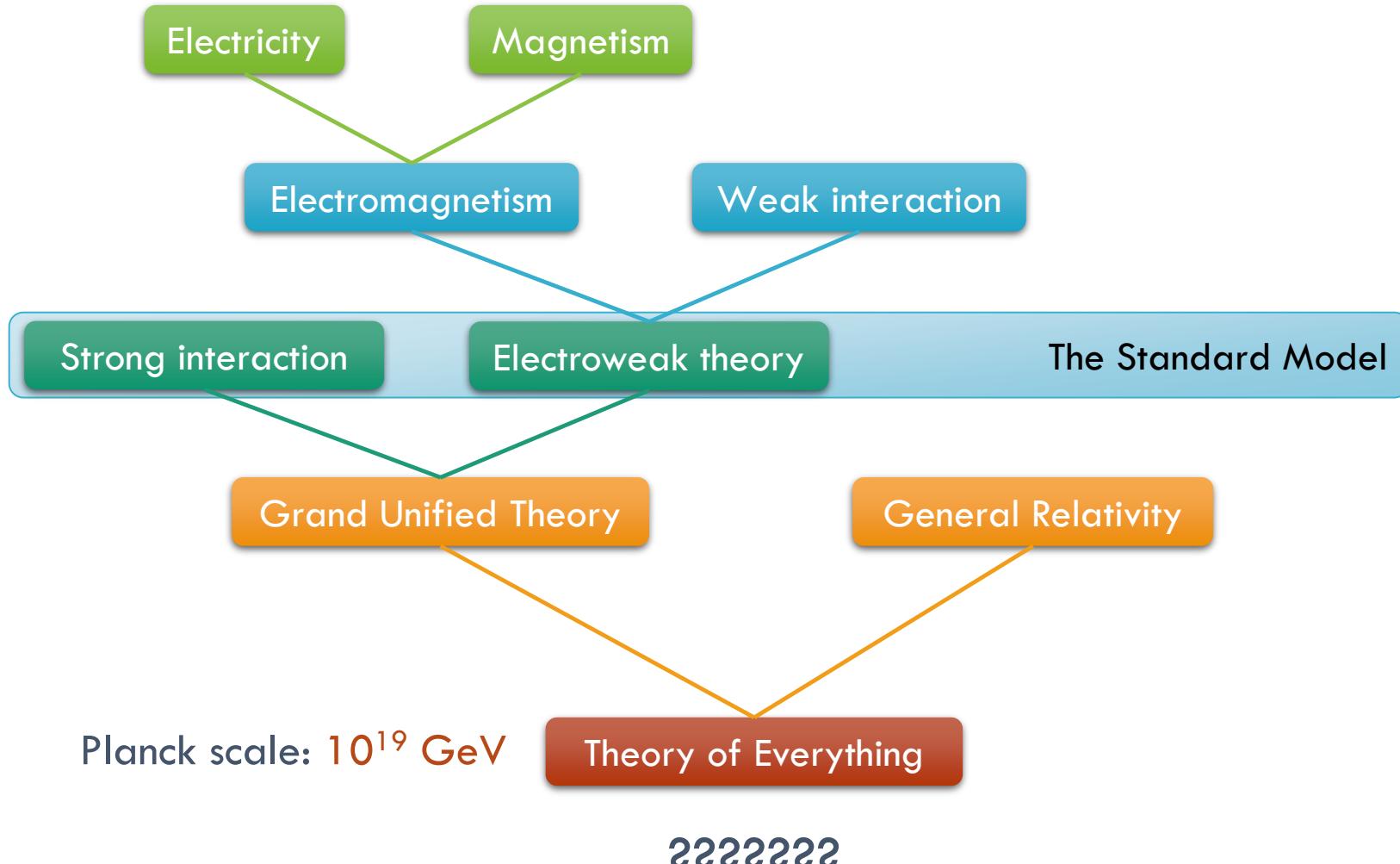
Albert Einstein  
1925



# A THEORY OF EVERYTHING?



# A THEORY OF EVERYTHING?



???????

# A THEORY OF EVERYTHING?

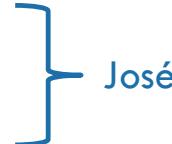
- No one knows how to build a theory of everything.
  - General relativity and quantum mechanics are fundamentally different:
    - GR predicts smooth spacetime; QM predicts vacuum fluctuations.
    - A Grand Unified Theory is probably an intermediate step.
- String theory / M-theory
  - Particles are replaced by one-dimensional “strings”.
    - Different vibration modes give rise to different particle properties.
  - Needs extra dimensions to work! (9+1; 10+1)
  - Looks like supersymmetry at low energies.
- Loop quantum gravity
  - Granular description of space – quantization!
  - Needs only three space dimensions...

# STANDARD MODEL LIMITATIONS SUMMARY

- We know that the Standard Model is **incomplete**: it does not fully account for experimental **observations**.
  - It doesn't describe **gravity** – and it seems **very difficult** to extend it in order to do so.
  - There isn't a unique way of describing **neutrino mass** in the SM context.
  - It doesn't provide a candidate for **dark matter**.
- Offers **unsatisfactory** answers to some **fundamental** questions:
  - Doesn't **predict** particle **masses**.
  - Or why there are **three** generations of matter.
  - Very large differences in the **scale** of some parameters remain unexplained.
    - Including some “lucky” **cancelations**.

# SCHEDULE

1. ~~Introduction~~
2. ~~History of Particle Physics~~
3. ~~Special Relativity~~
4. ~~Quantum Mechanics~~
5. ~~Experimental Methods~~
6. ~~The Standard Model - Overview~~
7. **The Standard Model - Limitations**
8. Neutrino Theory
9. Neutrino Experiment
10. LHC and Experiments
11. The Higgs Boson and Beyond
12. Particle Cosmology

 José

 Inês

 Cris

NO CLASS NEXT WEEK  
NEXT CLASS: MARCH 25<sup>th</sup>



# INTEGRAL OVER “ALL POSSIBLE PATHS”

- Where does the integral notion come from?
- Recall, QM picture of free particle motion: there is some amplitude for a free electron to travel along any path from the source to some point  $p$ . Not just the straight, classical trajectory!
  - The word “path” here doesn’t only refer to a  $x(y)$  path in space, but also the time at which it passes each point in space.
  - In 3D, a path (sometimes called wordline) is defined by three functions  $x(t)$ ,  $y(t)$  and  $z(t)$ . An electron has an amplitude associated with a given path.
- The total amplitude for the electron to arrive at some final point is the sum of the amplitudes of all possible paths. Since there are an infinite number of paths, the sum turns into an integral.

# THE QUANTUM MECHANICAL AMPLITUDE

- Feynman: Each path has a corresponding probability amplitude. The amplitude  $\psi$  for a system to travel along a given path  $x(t)$  is:

$$\psi[x(t)] = \text{const. } e^{iS[x(t)]/\hbar}$$

where the object  $S[x(t)]$  is called the **action** corresponding to  $x(t)$ .

- The total amplitude is the sum of contributions from each path:

$$\sum_{\text{over all paths}} \psi[x(t)]$$

# THE QUANTUM MECHANICAL AMPLITUDE

$$\psi[x(t)] = \text{const. } e^{iS[x(t)]/\hbar}$$

1. What is  $e^{iS[x(t)]/\hbar}$ ?
2. What is the action  $S[x(t)]$ ?

# UNDERSTANDING THE PHASE

- You may not have seen numbers like  $e^{i\theta}$ , so let's review.
- Basically,  $e^{i\theta}$  is just a fancy way of writing sinusoidal functions; from Euler's famous formula:

$$e^{i\theta} = \cos \theta + i \sin \theta$$

- Note: those of you familiar with complex numbers (of the form  $z=x+iy$ ) know that  $e^{i\theta}$  is the phase of the so-called polar form of  $z$ , in which  $z=re^{i\theta}$ , with:

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

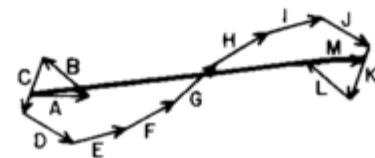
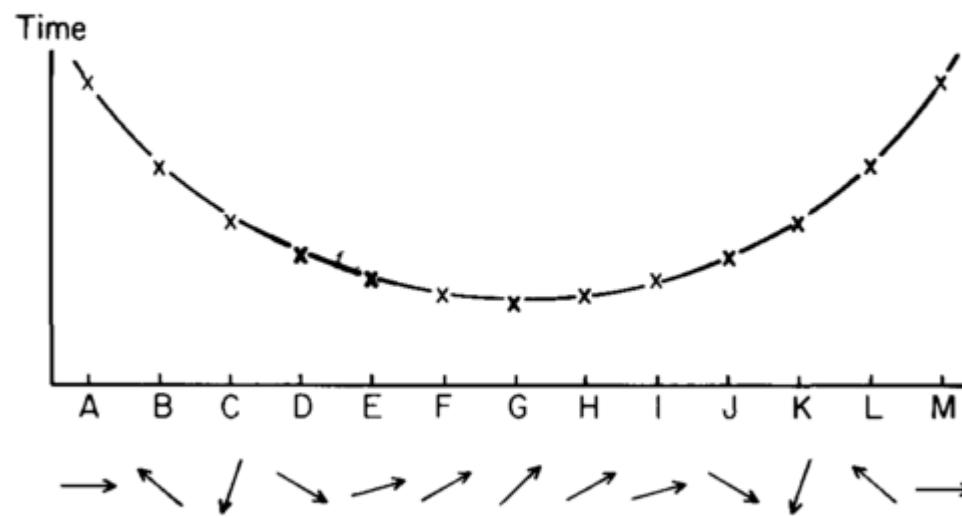
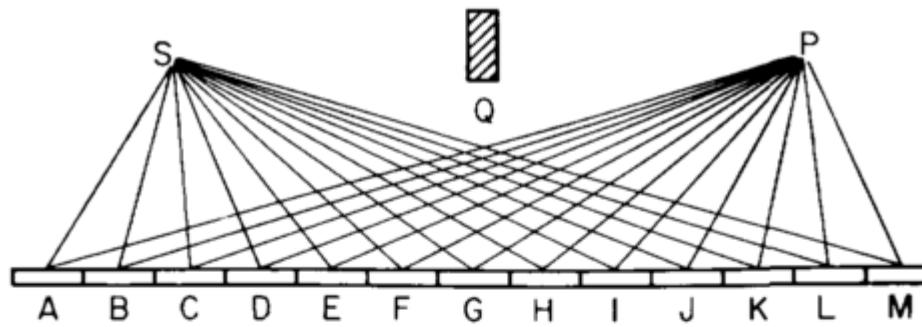
# COMMENTS ON THE AMPLITUDE

- Now we can understand the probability amplitude  $\psi[x(t)] \sim e^{iS[x(t)]/\hbar}$  a little better.
- The amplitude is a sinusoidal function – a wave – that oscillates along the worldline  $x(t)$ . The frequency of oscillation is determined by how rapidly the action  $S$  changes along the path.
- The probability that a particle will take a given path (up to some overall multiplication constant) is:

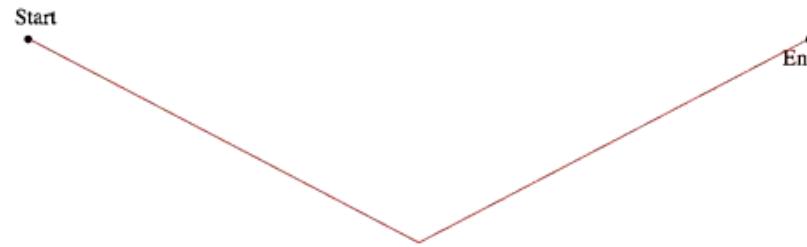
$$\begin{aligned} P &\propto |\psi|^2 = \psi^* \psi \\ &\propto e^{-iS[x(t)]/\hbar} e^{iS[x(t)]/\hbar} \\ &= e^{iS[x(t)]/\hbar - iS[x(t)]/\hbar} = e^0 \\ &= 1 \end{aligned}$$

- This is the same for every worldline. According to Feynman, the particle is equally likely to take any path through space and time!
  - Contributions from “crazy” paths will likely be suppressed by interference!

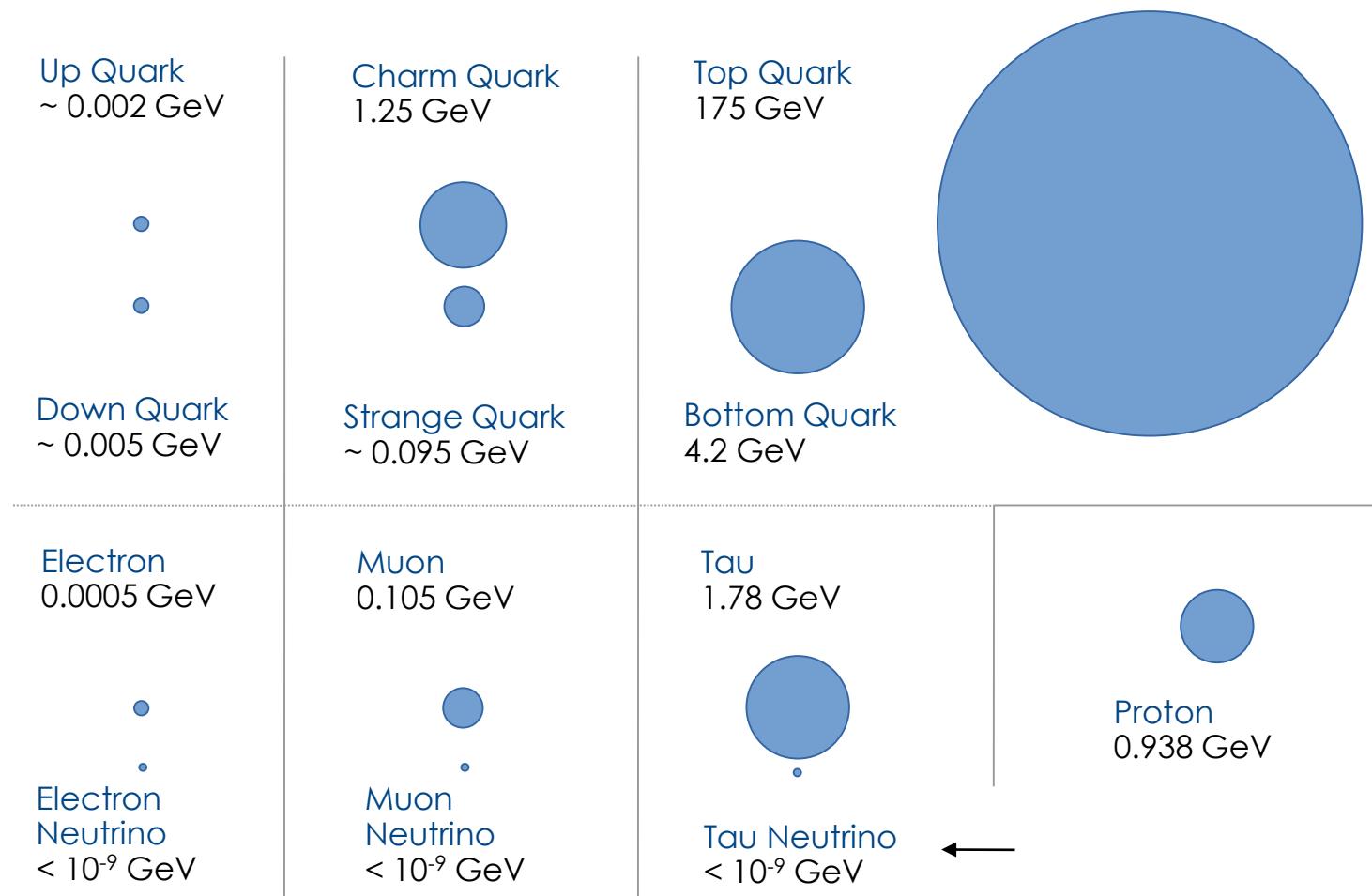
# COMMENTS ON THE AMPLITUDE



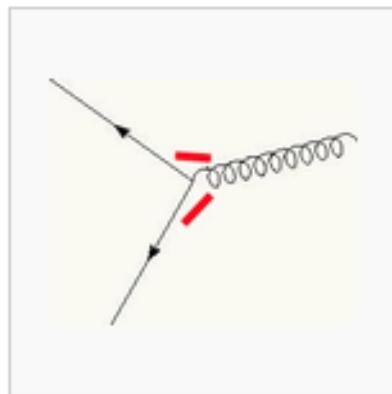
# COMMENTS ON THE AMPLITUDE



# MASSES OF SM FERMIONS



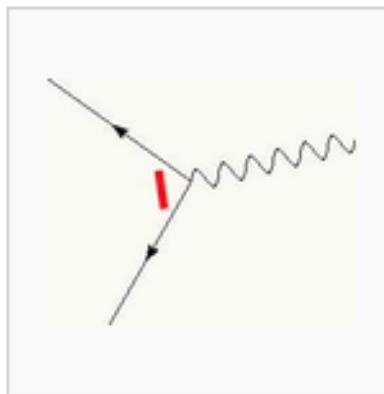
# GLUON DISCOVERY AT TASSO, PETRA



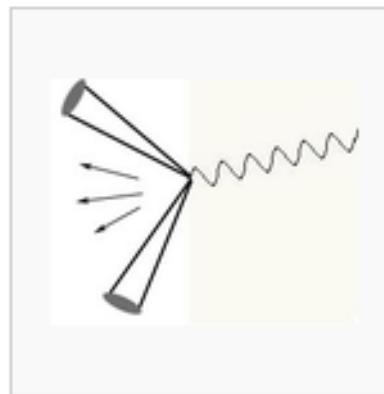
Two quarks (solid lines) and a gluon (curly line) fly apart, with the strings (red bars) primarily between the gluon and each quark.



As a result, three jets (cones) form, with extra hadrons (arrows) found where the strings formed.



For comparison, physicists looked at events with two quarks and a photon (wavy line). Here the string forms only between quarks.



Therefore extra hadrons are found only between the two jets, which is inconsistent with observations.