

PARTICLE PHYSICS

COLUMBIA SCIENCE HONORS PROGRAM

WEEK 7

OVERVIEW OF THE STANDARD MODEL (CONT'ED)

LIMITATIONS OF THE STANDARD MODEL

COURSE POLICIES

- Attendance
 - Up to four excused absences
 - Two with notes from parent/guardian
 - shpattendance@columbia.edu
 - Valid excuses:
 - Illness, family emergency, tests or athletic/academic competitions, mass transit breakdowns
 - Invalid excuses:
 - Sleeping in, missing the train...
 - I will take attendance during class
- No cell phones
- Ask questions!

LECTURE MATERIALS

- <https://twiki.nevis.columbia.edu/twiki/bin/view/Main/ScienceHonorsProgram>
- Questions: cristovao.vilela@stonybrook.edu

LAST WEEK...

AN OVERVIEW OF THE STANDARD MODEL

THE WORLD, ACCORDING TO A PARTICLE PHYSICIST

mass →	$\approx 2.3 \text{ MeV}/c^2$	$\approx 1.275 \text{ GeV}/c^2$	$\approx 173.07 \text{ GeV}/c^2$	0	$\approx 126 \text{ GeV}/c^2$
charge →	$2/3$	$2/3$	$2/3$	0	0
spin →	$1/2$	$1/2$	$1/2$	1	0
	u up	c charm	t top	g gluon	H Higgs boson
QUARKS	$\approx 4.8 \text{ MeV}/c^2$	$\approx 95 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	0	
	$-1/3$	$-1/3$	$-1/3$	0	
	$1/2$	$1/2$	$1/2$	1	
	d down	s strange	b bottom	γ photon	
LEPTONS	$0.511 \text{ MeV}/c^2$	$105.7 \text{ MeV}/c^2$	$1.777 \text{ GeV}/c^2$	$91.2 \text{ GeV}/c^2$	
	-1	-1	-1	0	
	$1/2$	$1/2$	$1/2$	1	
	e electron	μ muon	τ tau	Z Z boson	
	$< 2.2 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 15.5 \text{ MeV}/c^2$	$80.4 \text{ GeV}/c^2$	
	0	0	0	± 1	
	$1/2$	$1/2$	$1/2$	1	
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	
				GAUGE BOSONS	

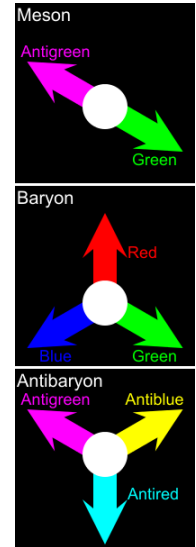
PARTICLE CHARGES

Electric Weak Color

Quarks ✓ ✓ ✓

- Quarks:

- There are **no** free quarks.
- They form **colorless** composite objects, hadrons;
 - Baryons: $qqq, \bar{q}\bar{q}\bar{q}$
 - Mesons: $q\bar{q}, q\bar{q}, q\bar{q}$



Name	Symbol	Mass (MeV/c ²) [*]	<i>J</i>	<i>B</i>	<i>Q</i>	<i>I</i> ₃	<i>C</i>	<i>S</i>	<i>T</i>	<i>B'</i>	Antiparticle	Antiparticle symbol
First generation												
Up	u	1.7 to 3.3	1/2	+1/3	+2/3	+1/2	0	0	0	0	Antiup	\bar{u}
Down	d	4.1 to 5.8	1/2	+1/3	-1/3	-1/2	0	0	0	0	Antidown	\bar{d}
Second generation												
Charm	c	1,270 ⁺⁷⁰ ₋₉₀	1/2	+1/3	+2/3	0	+1	0	0	0	Anticharm	\bar{c}
Strange	s	101 ⁺²⁹ ₋₂₁	1/2	+1/3	-1/3	0	0	-1	0	0	Antistrange	\bar{s}
Third generation												
Top	t	172,000 ± 900 ± 1,300	1/2	+1/3	+2/3	0	0	0	+1	0	Antitop	\bar{t}
Bottom	b	4,190 ⁺¹⁸⁰ ₋₆₀	1/2	+1/3	-1/3	0	0	0	0	-1	Antibottom	\bar{b}

J = total angular momentum, *B* = baryon number, *Q* = electric charge, *I*₃ = isospin, *C* = charm, *S* = strangeness, *T* = topness, *B'* = bottomness.

PARTICLE CHARGES

- Leptons

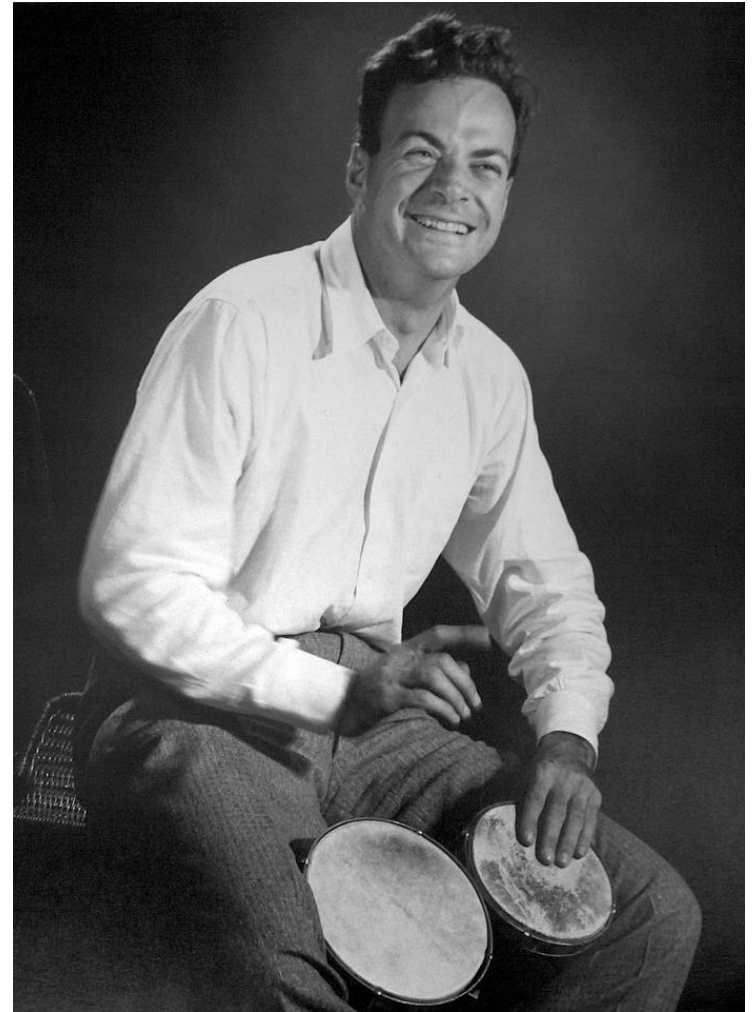
- Exist as **free** particles

	Electric	Weak	Color
Quarks	✓	✓	✓
Leptons			
Charged	✓	✓	✗
Neutral	✗	✓	✗

Particle/Antiparticle Name	Symbol	Q (e)	S	L _e	L _μ	L _τ	Mass (MeV/c ²)	Lifetime (s)
Electron / Antielectron ^[17]	e^-/e^+	-1/+1	1/2	+1/-1	0	0	0.510 998 910(13)	Stable
Muon / Antimuon ^[18]	μ^-/μ^+	-1/+1	1/2	0	+1/-1	0	105.658 3668(38)	$2.197\,019(21) \times 10^{-6}$
Tau / Antitau ^[20]	τ^-/τ^+	-1/+1	1/2	0	0	+1/-1	1,776.84(17)	$2.906(10) \times 10^{-13}$
Electron neutrino / Electron antineutrino ^[33]	$\nu_e/\bar{\nu}_e$	0	1/2	+1/-1	0	0	$< 0.000\,0022^{[35]}$	Unknown
Muon neutrino / Muon antineutrino ^[33]	$\nu_\mu/\bar{\nu}_\mu$	0	1/2	0	+1/-1	0	$< 0.17^{[35]}$	Unknown
Tau neutrino / Tau antineutrino ^[33]	$\nu_\tau/\bar{\nu}_\tau$	0	1/2	0	0	+1/-1	$< 15.5^{[35]}$	Unknown

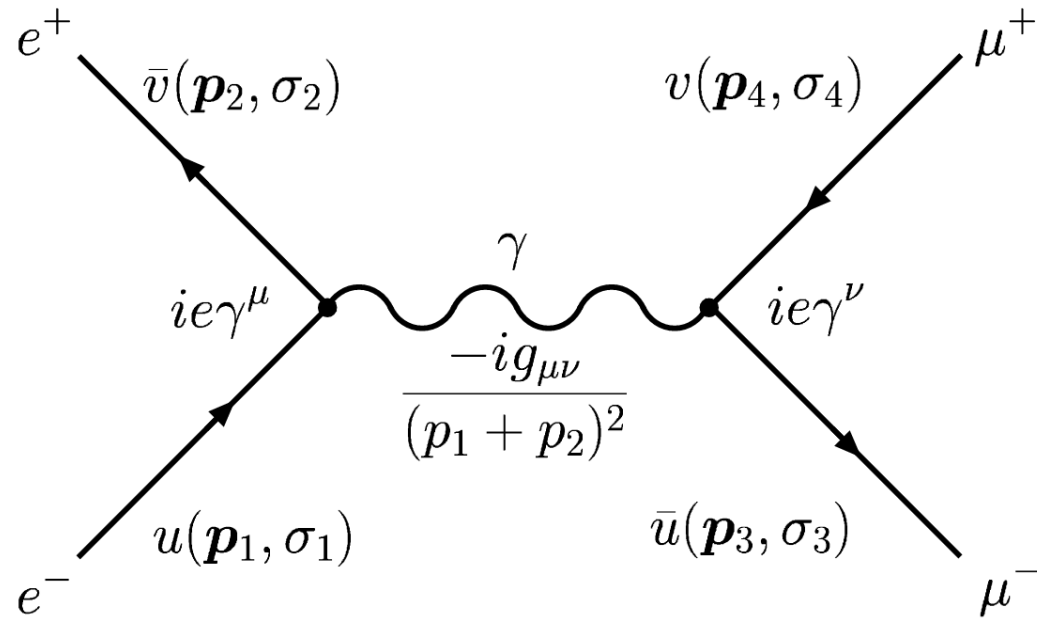
PARTICLE DYNAMICS: FEYNMAN DIAGRAMS

- Feynman Rules!
- 1948: introduced pictorial representation scheme for the mathematical expressions governing the behavior of subatomic particles.
 - Can be used to easily calculate probability amplitudes
 - Other options: cumbersome mathematical derivations



FEYNMAN DIAGRAMS

AN EXAMPLE CALCULATION

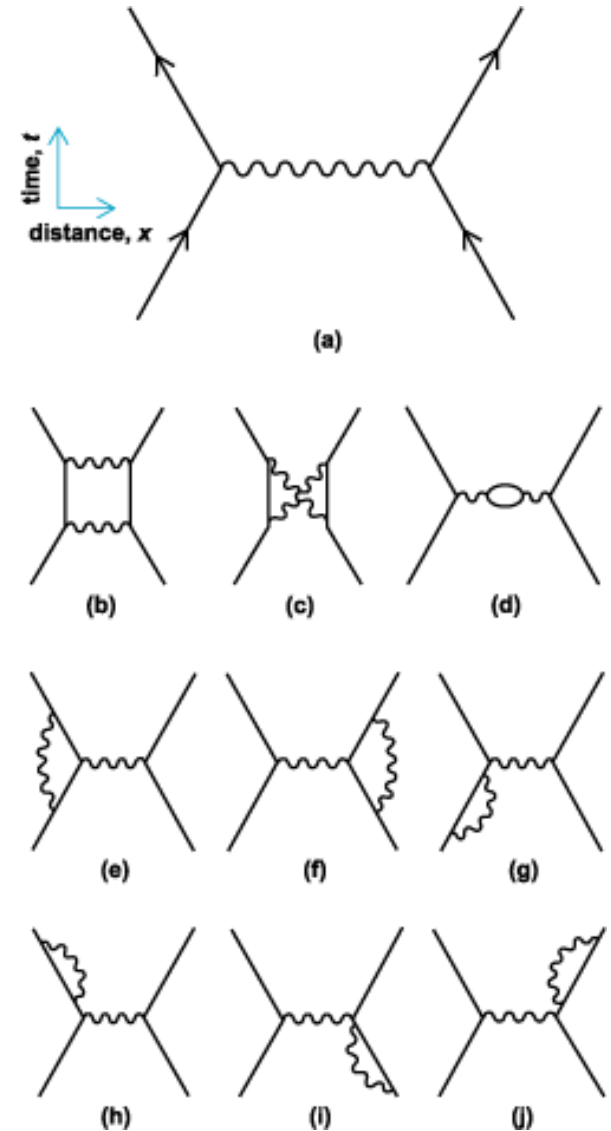


$$-i\mathcal{M} = [\bar{u}(\mathbf{p}_3, \sigma_3)(ie\gamma^\nu)v(\mathbf{p}_4, \sigma_4)] \frac{-ig_{\mu\nu}}{(p_1 + p_2)^2} [\bar{v}(\mathbf{p}_2, \sigma_2)(ie\gamma^\mu)u(\mathbf{p}_1, \sigma_1)]$$

QUANTUM ELECTRODYNAMICS

QED

- The **vertices** are **interactions** with the electromagnetic field.
- The **straight lines** are **electrons** and the **wiggly** ones are **photons**.
- Between interactions (vertices), particles **propagate** as free particles.
- The **higher** the number of **vertices**, the **less likely** for the interaction to happen.
 - Higher “order”.



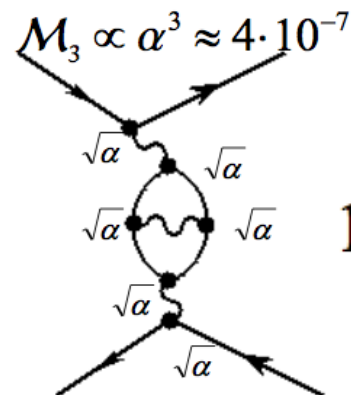
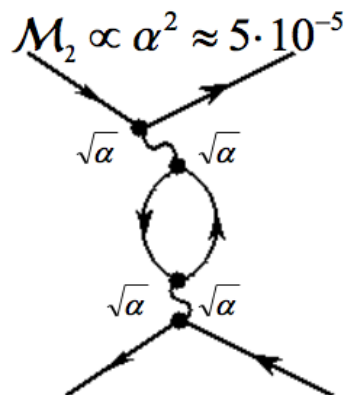
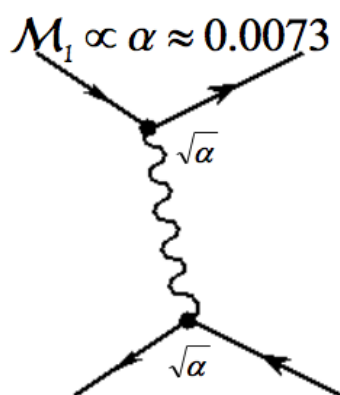
QUANTUM ELECTRODYNAMICS

QED

- Each **vertex** contributes a **coupling constant** $\sqrt{\alpha}$, where α is a small dimensionless number:

$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

- Hence, **higher-order** diagrams get **suppressed** relative to diagrams with fewer vertices.

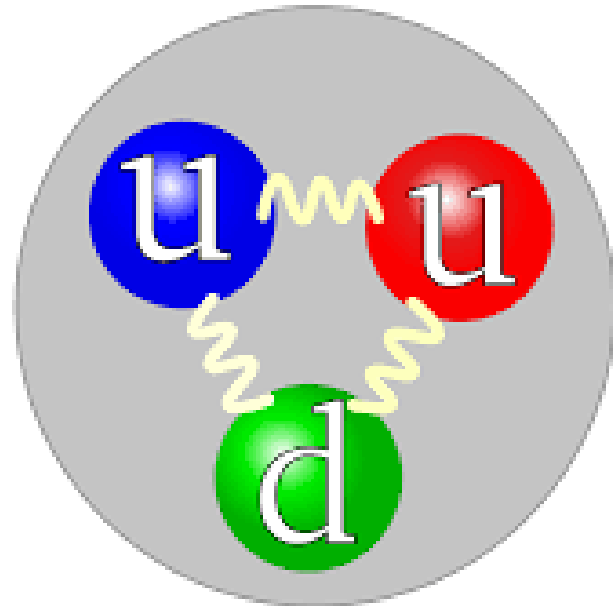


$$\mathcal{M}_1 : \mathcal{M}_2 : \mathcal{M}_3 \\ 18769 : 137 : 1$$

QUANTUM CHROMODYNAMICS

QCD

- Quarks and **bound states**:
 - Since quarks are **spin-1/2** particles (fermions), they must obey the **Pauli Exclusion Principle**.
- **Pauli Exclusion Principle**: fermions in a bound state (e.g., the quarks inside a hadron) **cannot** have the **same quantum numbers**.
- Then, how can we squeeze **three** quarks into a baryon?
- Give them an **additional charge**, called color.
- This **removes** the quantum numbers **degeneracy**.



QUANTUM CHROMODYNAMICS

QCD

- Gluons carry a **color** and an **anti-color**.
- There are **9 possible combinations**, but 1 is white, which is not allowed.
 - **No evidence** for colorless particles exchanging gluons.
- This leaves **8 types of gluon**.

A quark changes color by emitting or absorbing gluons.



PERTURBATION THEORY

ASIDE

- Use a **power series** in a parameter ε (such that $\varepsilon \ll 1$) - known as perturbation series - as an **approximation** to the full solution.
- For example:

$$A = A_0 + \epsilon A_1 + \epsilon^2 A_2 + \dots$$

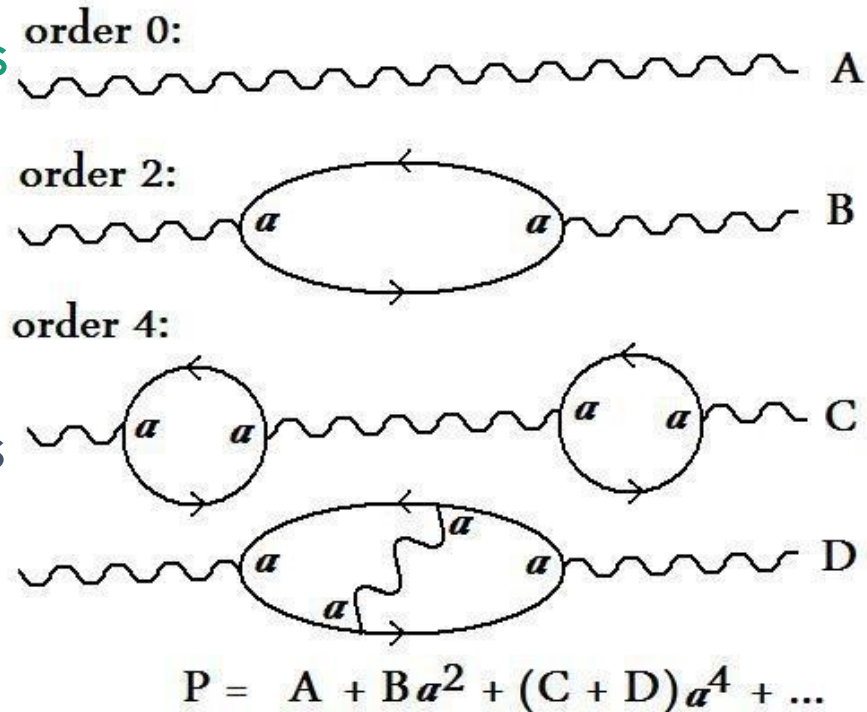
- In this example, A_0 is the “**leading order**” solution, while A_1, A_2, \dots represent higher order terms.
- **Note:** if ε is **small**, the higher-order terms in the series become successively smaller.
- Approximation:

$$A \approx A_0 + \epsilon A_1$$

PERTURBATION THEORY IN QFT

ASIDE

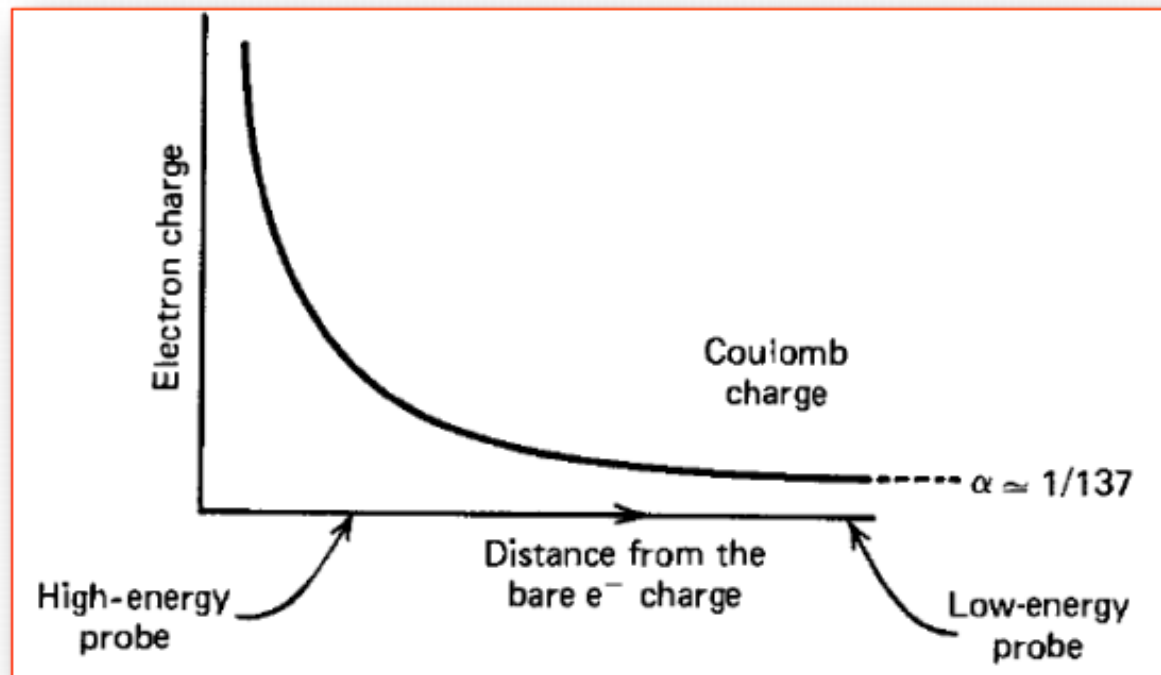
- Perturbation theory allows for well-defined predictions in quantum field theories (as long as they obey certain requirements).
- Quantum electrodynamics is one of those theories.
- Feynman diagrams correspond to the terms in the perturbation series!



Diagrams define a series in α

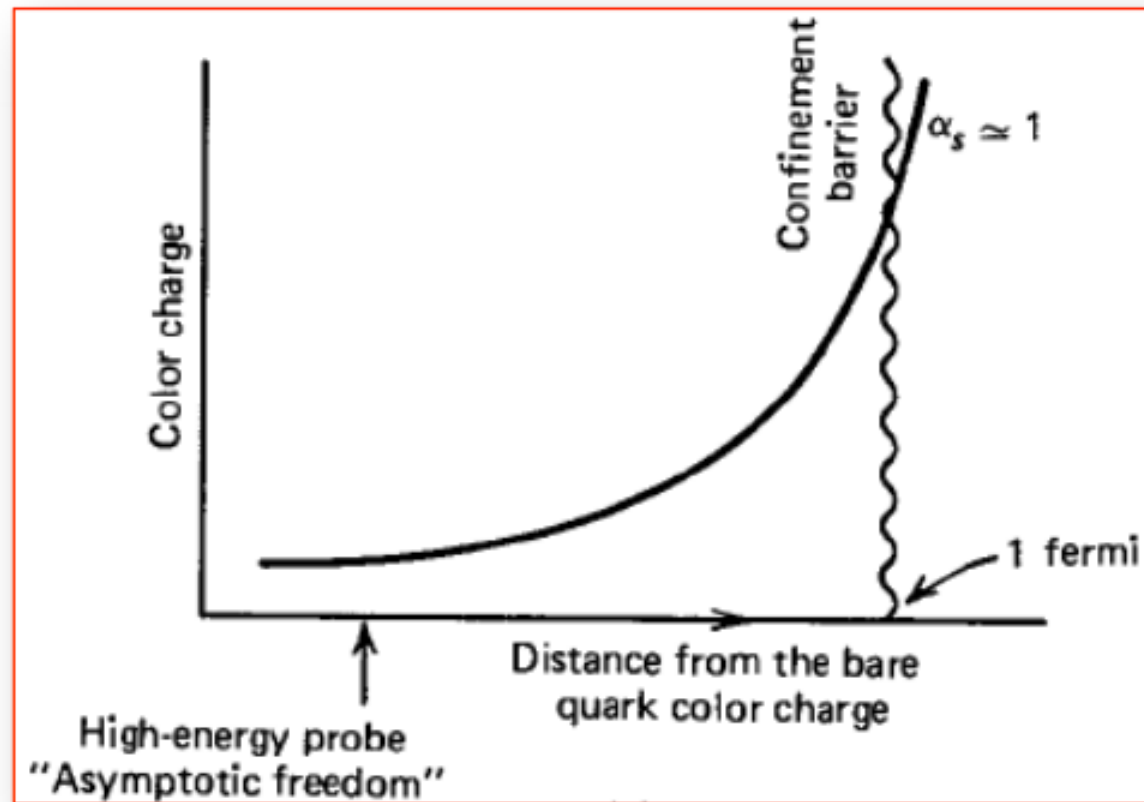
QCD VS QED

- Recall: In QED, each vertex contributes a **coupling constant** $\sqrt{\alpha}$.
- α is not exactly a **constant** though... it “**runs**” with the scale of the interaction.



QCD VS QED

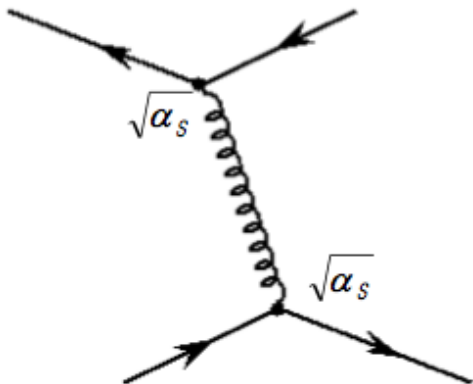
- The coupling constant for QCD, α_s , “runs” in a different way with energy.



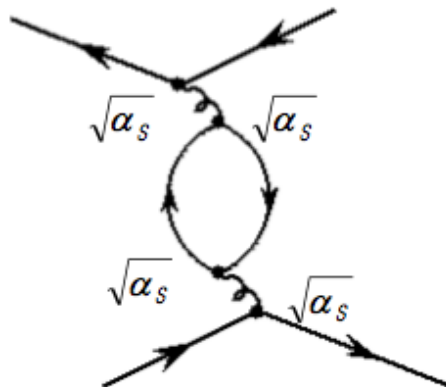
QCD VS QED

- In QCD, the coupling between quarks and gluons, given by the number α_s , is **much larger** than $1/137$ at **low energies**.
- In fact, at low energies, $\alpha_s \gg 1$, making **higher-order** diagrams just as **important** as those with fewer vertices!
- This means we can't **truncate** the sum over diagrams.
 - **Perturbation** theory is **not** a good approximation!
- Calculations quickly become **complicated**!

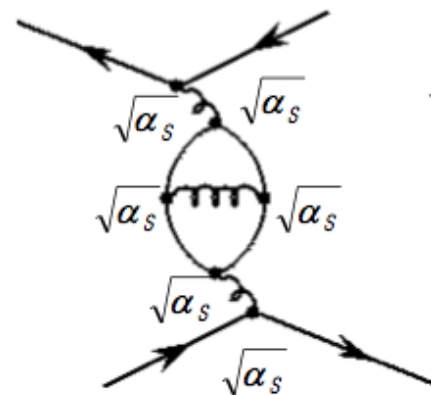
$$\mathcal{M}_1 \propto \alpha_s \approx 1$$



$$\mathcal{M}_2 \propto \alpha_s^2 \approx 1$$



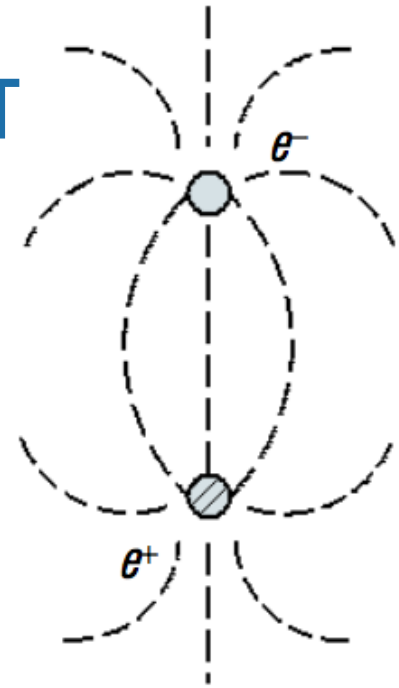
$$\mathcal{M}_3 \propto \alpha_s^3 \approx 1$$



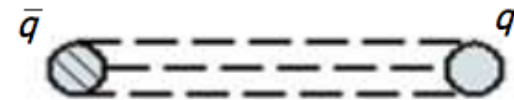
$$\mathcal{M}_1 : \mathcal{M}_2 : \mathcal{M}_3 \\ 1 : 1 : 1$$

UNDERSTANDING CONFINEMENT

- The **mathematics** of confinement are **complicated**, but we can **understand** them in terms of a very simple picture.
- Recall, the **Coulomb** field between a e^+e^- pair looks like $V(r) \sim 1/r$.
 - As we **pull** the pair **apart**, the **attraction weakens**.
- Imagine the **color field** between a quark-antiquark pair like Hooke's Law: $V(r) \sim r$.
 - As we **pull** the pair **apart**, the **attraction** between them **increases**.
- So, **separating** two quarks by a **large r** puts a **large amount of energy** into the color field:
 $V(r) \rightarrow \infty$



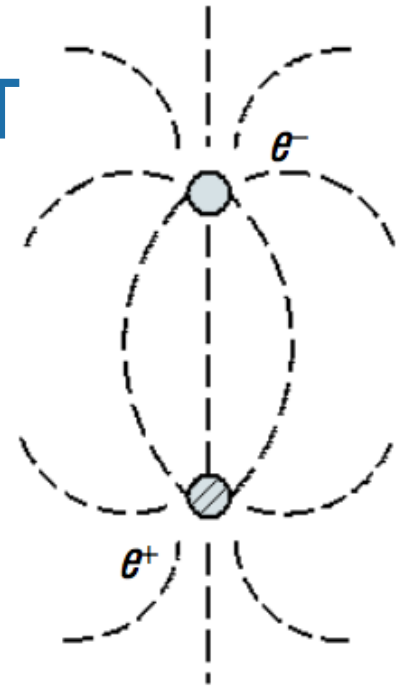
Dipole field for the Coulomb force between opposite electrical charges.



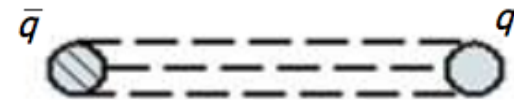
Dipole field between opposite-color quarks.

UNDERSTANDING CONFINEMENT

- How do we **understand** this picture?
- When a quark and anti-quark **separate**, their color interaction **strengthens** (more gluons appear in the color field).
- Through the interaction of the **gluons with each other**, the color lines of force are **squeezed** into a tube-like region.
- **Contrast** this with the **Coulomb field**: nothing prevents the lines of force from **spreading out**.
 - There is **no self-coupling** of photons to contain them.
- If the color tube has **constant energy density** per unit length k , the **potential energy** between quark and antiquark will **increase** with separation, $V(r) \sim kr$.



Dipole field for the Coulomb force between opposite electrical charges.

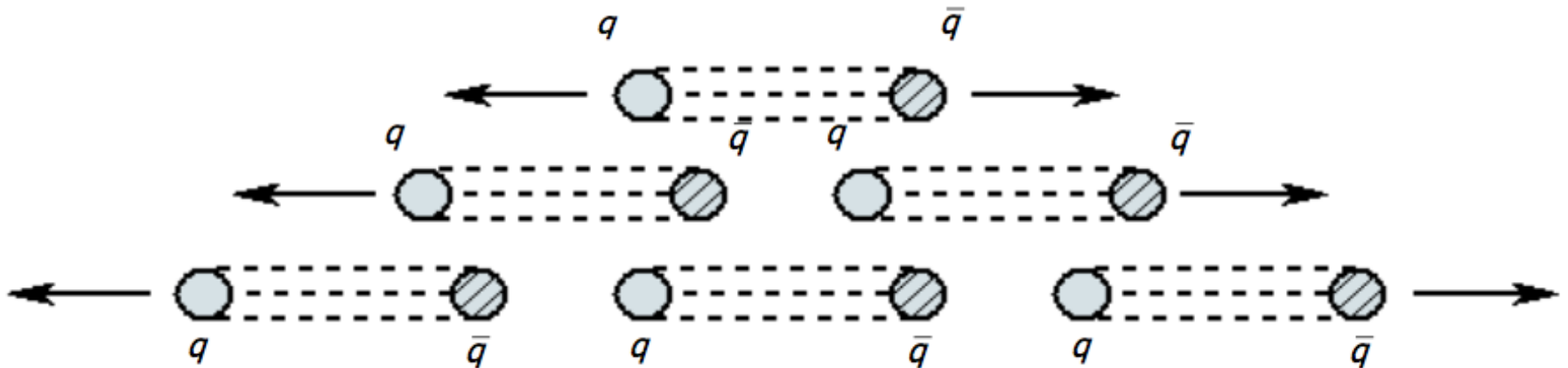


Dipole field between opposite-color quarks.

COLOR LINES AND HADRON PRODUCTION

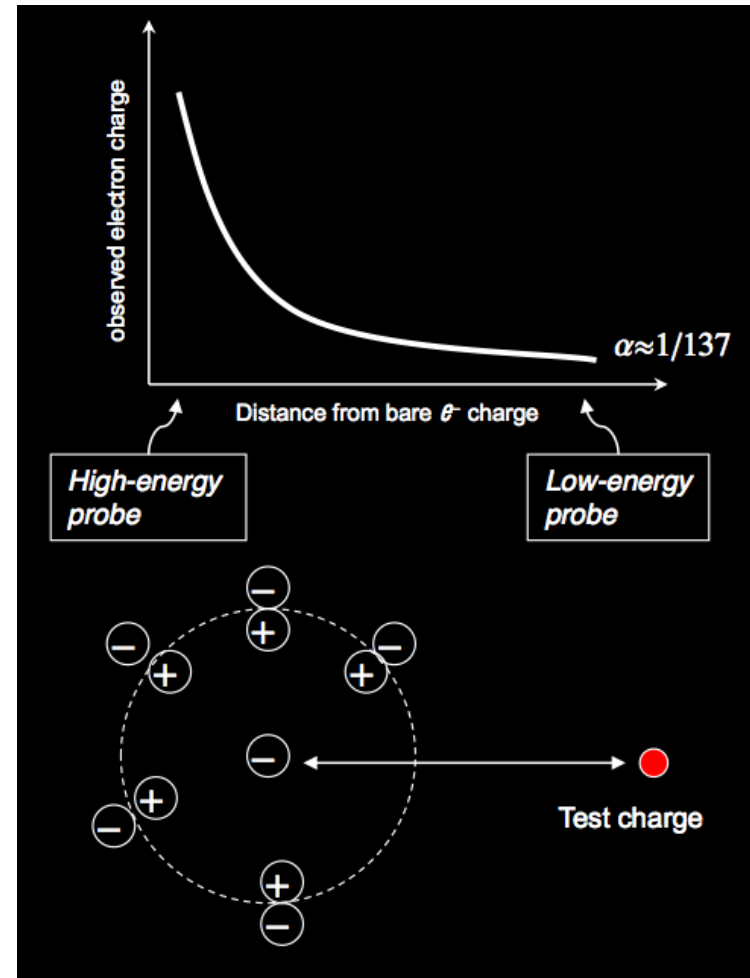
WHY YOU CAN'T GET FREE QUARKS

- Suppose we have a meson and we try to pull it apart. The potential energy in the quark-antiquark color field starts to increase.
- Eventually, the energy in the gluon field gets big enough that the gluons can pair-produce another quark-antiquark pair.
 - The new quarks pair up with the original quarks to form mesons, and thus our four quarks remain confined in colorless states.
- Experimentally, we see two particles!



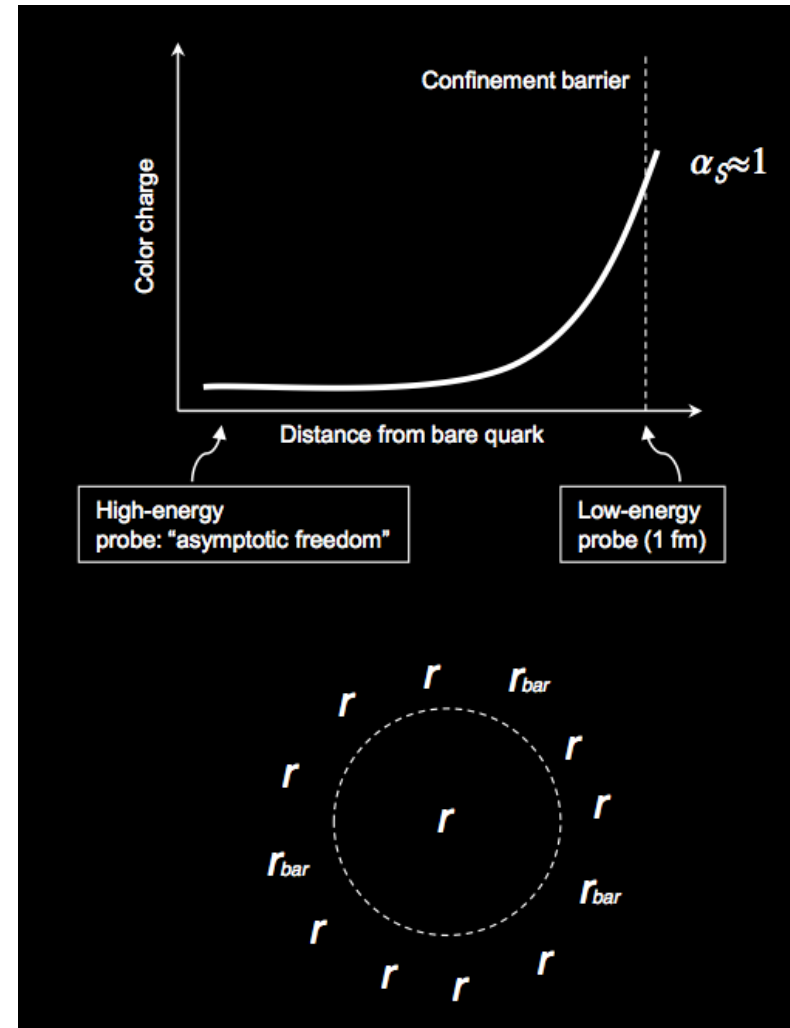
QED: CHARGE SCREENING

- Now, suppose we want to **measure** the **charge** of the electron by observing the **Coulomb** force experienced by a **test charge**.
- Far away from the electron, its charge is **screened** by a cloud of **virtual positrons and electrons**, so the **effective** charge is **smaller** than its **bare** charge.
- As we move **closer** in, **fewer** positrons are blocking our line of sight to the electron.
- Hence, with **decreasing distance**, the **effective charge** of the electron **increases**.
- **We can think of this as α increasing with energy.**



QCD: CHARGE ANTI-SCREENING

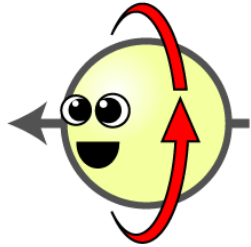
- In QCD, the additional gluon loop diagrams reverse the result of QED:
 - A red charge is preferentially surrounded by other red charges.
- By moving the test probe closer to the original quark, the probe penetrates a sphere of mostly red charge, and the measured red charge decreases.
- This is “antiscreening”.
- We can think of this as α_s decreasing with energy.



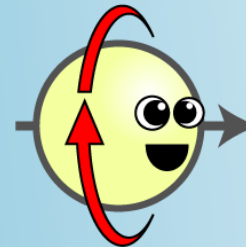
RUNNING CONSTANTS

- As we probe an **electron** at **increasingly higher** energies, its effective **charge increases**.
- This can be **rephrased** in the following way: as **interactions increase** in **energy**, the QED coupling strength α between charges and photons **also increases**.
 - This should not really be a surprise; after all, the coupling strength of EM depends directly on the electron charge.
- Since α is not a constant, but a (slowly-varying) function of energy, it is called a **running coupling constant**.
- In **QCD**, the net effect is that the quark color charge and α_s **decrease** as the interaction **energy** goes **up**.

PARTICLE HELICITY ASIDE



Right-handed:
Spin in same direction as motion



Left-handed!



MIRROR



- Helicity **flips** when looked at in a mirror
 - Equivalent to inverting one of the space coordinates ($z \rightarrow -z$)

WEAK INTERACTIONS

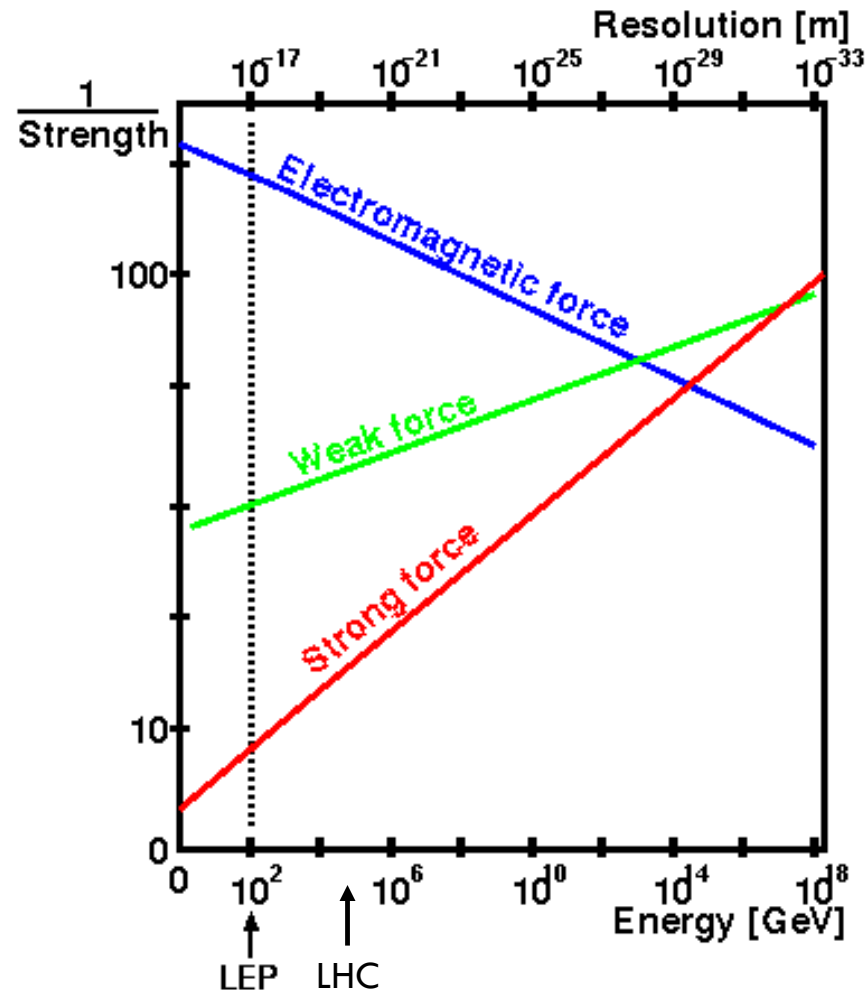
- Unlike electromagnetism and the strong interaction, the weak interaction is mediated by massive bosons:
 - $m_Z = 91.19 \text{ GeV}$
 - $m_W = 80.39 \text{ GeV}$
- This makes it extremely short-range
- And very weak at low energies
- Only left-handed particles* participate in weak interactions
 - Nature is different if looked at in a mirror!
 - * and right-handed anti-particles

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$



FORCE UNIFICATION ASIDE

- At laboratory energies near M_W $O(100)$ GeV, the measured values of the coupling constants are quite different.
- However, their “running” trends suggest that they approach a common value near 10^{16} GeV.
- This is an insanely high energy!
- The Standard Model provides no explanation for what may happen beyond this unification scale, nor why the forces have such different strengths at low energies.



PARTICLE/FIELD FORMULATION

- How do we **describe** interactions and fields **mathematically**?
- Classically,

Lagrangian L = kinetic energy - potential energy

- Particle physics:
 - Same concept, using Dirac equation to describe free spin-1/2 particles:

$$L = \bar{\Psi}(i\gamma^\mu\partial_\mu - m)\Psi$$

↑
Field

Ψ = wavefunction

m = mass

γ^μ = μ^{th} gamma matrix

∂_μ = partial derivative

STANDARD MODEL LAGRANGIAN

Free Fields

Interaction

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

STANDARD MODEL LAGRANGIAN

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

Free Fields

Interaction

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi$$

Gauge Bosons

Fermions

$$F_{\mu\nu}F^{\mu\nu} = G_{\mu\nu}G^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}$$

STANDARD MODEL LAGRANGIAN

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

Free Fields
Interaction

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi$$

Gauge Bosons
Fermions

$$\mathcal{L}' = e\bar{\psi}\gamma^\mu A_\mu\psi$$

Fermion-Boson
Coupling

$$eA_\mu = \frac{g_s}{2}\lambda_\nu G_\mu^\nu + \frac{g}{2}\vec{\tau} \cdot \vec{W}_\mu + \frac{g'}{2}YB_\mu$$

$$F_{\mu\nu}F^{\mu\nu} = G_{\mu\nu}G^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}$$

STANDARD MODEL LAGRANGIAN

Free Fields

Interaction

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

Gauge Bosons

Fermions

Electroweak bosons and interaction

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi$$

Fermion-Boson Coupling

Gluons and strong interaction

$$\mathcal{L}' = e\bar{\psi}\gamma^\mu A_\mu\psi$$

$$eA_\mu = \frac{g_s}{2}\lambda_\nu G_\mu^\nu + \frac{g}{2}\vec{\tau} \cdot \vec{W}_\mu + \frac{g'}{2}Y B_\mu$$

$$F_{\mu\nu}F^{\mu\nu} = G_{\mu\nu}G^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}$$

STANDARD MODEL LAGRANGIAN

- Encompasses **all** theory.
 - From the Lagrangian to **cross-section predictions**:

$$\sigma \sim \langle f | \mathbf{S} | i \rangle^2$$

Inelastic
Cross Section
[for $|i\rangle \neq |f\rangle$]

[Def. : $|t = +\infty\rangle \equiv \mathbf{S}|t = -\infty\rangle$]

Time Evolution

From Schrödinger-Equation
[Dirac picture]

$$|t\rangle = |t_0\rangle - i \int_{t_0}^t dt' \mathbf{H}'(t') |t'\rangle$$

$$\mathbf{H}'(t) = - \int \mathcal{L}'(x, t) d^3x$$

Lagrangian
of Interaction

$$\langle f | \mathbf{S} | i \rangle \cong \delta_{fi} - i \int_{-\infty}^{\infty} dt' \langle f | \mathbf{H}'(t') | i \rangle$$

→ Feynman rules

SYMMETRIES AND INVARIANCE

- Noether's theorem (1915):
 - Every **symmetry** under some operation corresponds to a **conservation law**

symmetry	invariant
Space translation	momentum
Time translation	energy
Rotation	Angular momentum
Global phase; $\Psi \rightarrow e^{i\theta} \Psi$	Electric charge
Local phase; $\Psi \rightarrow e^{i\theta(x,t)} \Psi$	Lagrangian + gauge field (\rightarrow QED)



QED FROM LOCAL GAUGE INVARIANCE

- Apply **local** gauge symmetry to Dirac equation:

$$\bar{\Psi} \rightarrow e^{i\theta(x,t)} \bar{\Psi}, \quad \Psi \rightarrow e^{-i\theta(x,t)} \Psi$$

This type of transformation leaves quantum mechanical amplitudes invariant.

- Consider very small changes in a field:

$$\Psi \rightarrow \Psi + \delta\Psi = \Psi - i\theta(x,t)\Psi \quad \text{ie. } \delta\Psi = -i\theta(x,t)\Psi$$

- The effect on the Lagrangian is:

$$L = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi \Rightarrow \delta L = \bar{\Psi} \gamma^\mu \partial_\mu \theta(x,t) \Psi$$

QED FROM LOCAL GAUGE INVARIANCE

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- The effect on the Lagrangian is:

$$L = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi \Rightarrow \delta L = \bar{\Psi} \gamma^\mu \partial_\mu \theta(x,t) \Psi$$

For the Lagrangian to remain invariant: $\delta L = 0$

QED FROM LOCAL GAUGE INVARIANCE

- To satisfy $\delta L=0$, we “engineer” a mathematical “trick”:

1. Introduce a **gauge field** A_μ to interact with fermions,
and A_μ transform as: $A_\mu + \delta A_\mu = A_\mu + 1/e \partial_\mu \theta(x,t)$
2. In resulting Lagrangian, replace $\partial_\mu \rightarrow D_\mu = \partial_\mu + ieA_\mu$

- In that case, L is redefined:

$$L = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

The new Lagrangian is invariant under local gauge transformations.

QED FROM LOCAL GAUGE INVARIANCE

ONE MORE THING...

- Need to add kinetic term for field (field strength):

Define $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Add term $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ (Lorentz invariant, matches
Maxwell's equations)

QED FROM LOCAL GAUGE INVARIANCE

ONE MORE THING...

- Need to add kinetic term for field (field strength):

Define $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Add term $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ (Lorentz invariant, **matches**
Maxwell's equations)

Final lagrangian (for QED!):

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

- **No mass** term is allowed for A_μ , otherwise the Lagrangian is **not** gauge **invariant**
 - The gauge field is **massless**!

QED FROM LOCAL GAUGE INVARIANCE

ONE MORE THING...

- Need to add kinetic term for field (field strength):

Define $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Add term $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ (Lorentz invariant, **matches Maxwell's equations**)

Final lagrangian (for QED!):

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

- No mass term for photon in lagrangian
is not gauge invariant
- The photon is massless

We have mathematically engineered a quantum field that couples to fermions, obeys Maxwell's equations and is massless!

THE HIGGS MECHANISM

QCD AND WEAK LAGRANGIANS

- Follow similar reasoning, but allow for self-interaction of gauge bosons.
 - Jargon: QCD and weak interactions based on non-abelian groups.
- In non-abelian groups gauge invariance is achieved by adding n^2-1 massless gauge bosons for $SU(n)$.
 - $SU(n)$: gauge group.
 - $SU(3)$: 8 massless gluons for QCD ✓
 - $SU(2)$: 3 massless gauge bosons (W_1, W_2, W_3) for weak force
 - If mixing with $U(1)$, we get (W_1, W_2, W_3) and B: electroweak force.

QCD AND WEAK LAGRANGIANS

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 - If mixing with $U(1)$, we get (W_1, W_2, W_3) and B: **electroweak force**.

By the same mechanism as the **massless photon** arises from QED, a set of **massless bosons** arise when the theory is extended to include the weak nuclear force.

But Nature tells us they have mass!

HIGGS MECHANISM

- A theoretically proposed mechanism which gives rise to elementary particle masses: W^+ , W^- and Z bosons (and solve other problems...).
- It actually predicts the mass of W^+ , W^- and Z^0 bosons:
 - The W^+ , W^- bosons should have a mass of 80.390 ± 0.018 GeV
 - The Z^0 boson should have a mass of 91.1874 ± 0.0021 GeV
 - Measurements:
 - ✓ 80.387 ± 0.019 GeV
 - ✓ 91.1876 ± 0.0021 GeV
- Beware: it ends up also providing a mechanism for fermion masses, but it doesn't make any prediction for those (...).

HIGGS MECHANISM

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

Massless case, from previous slide

$$+ \mathcal{L}_{\phi'}$$

Higgs Field

HIGGS MECHANISM

Yukawa Couplings

Higgs Field

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}' + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\phi'}$$

Massless case, from previous slide

HIGGS MECHANISM

Yukawa Couplings

Higgs Field

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}' + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\phi'}$$

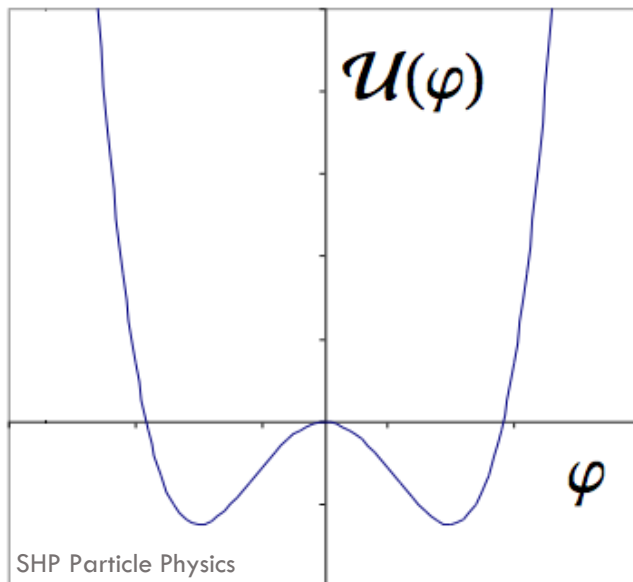
Massless case, from previous slide

$$\mathcal{L}_{\phi} = (\partial_{\mu} \phi^{\dagger})(\partial^{\mu} \phi) - V(\phi) \quad \text{Higgs Potential}$$

$$\mathcal{L}_{\text{Yuk}} = c_f (\bar{\psi}_L \psi_R \phi + \bar{\psi}_R \psi_L \phi) \quad \text{Higgs Fermion Interaction}$$

$V(\Phi)$ AND SPONTANEOUS SYMMETRY BREAKING

- When we speak of symmetry in the Standard Model, we usually mean **gauge symmetry**.
- This has to do with the form of the Lagrangian in terms of a given field φ .
- For simplicity, let's consider a simple (and suggestive!) example in which the Lagrangian has reflection symmetry about the $\varphi=0$ axis:



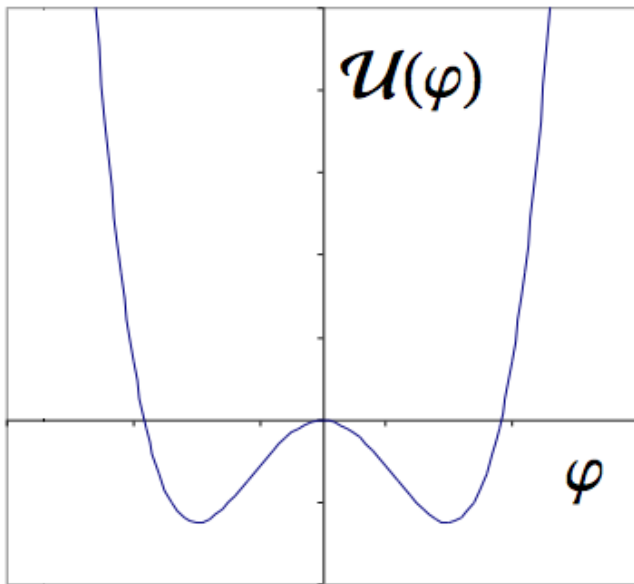
Lagrangian term for Higgs field, over all spacetime:

$$\mathcal{L}(\varphi) = \frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}m^2\varphi^2 - \frac{\lambda^2}{4}\varphi^4$$

“potential energy”: (recall: $L = T - U$)

$$U(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{\lambda^2}{4}\phi^4$$

$V(\Phi)$ AND SPONTANEOUS SYMMETRY BREAKING



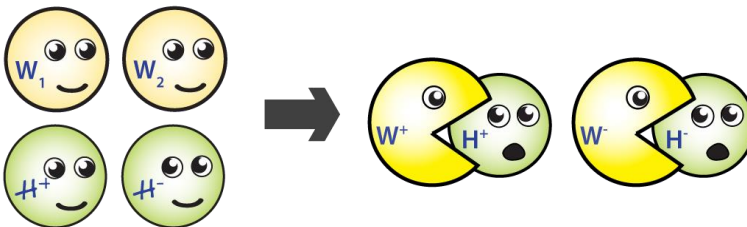
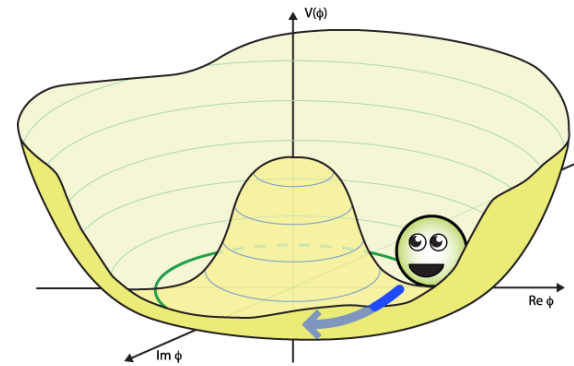
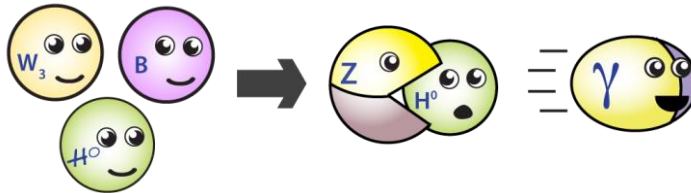
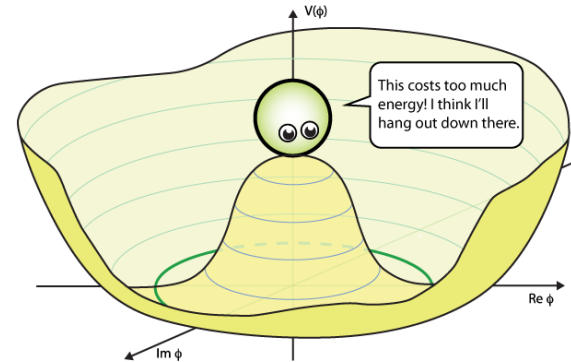
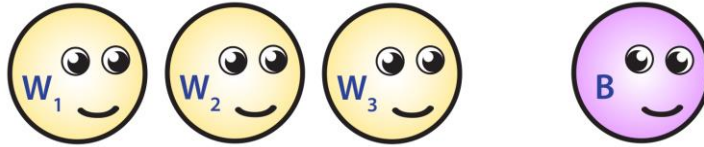
Calculation must be formulated in terms of deviations from one or the other of these ground states.

- To get a correct interpretation of the Lagrangian, and to be able to use **perturbative** calculations, we need to find one of its **minima** (a ground state) and look at small **fluctuations** of φ **about** that **minimum**.
- For this Lagrangian, the calculations must be formulated as small fluctuations of the field about **one of two** minima.
- **Switching** to **either** one of the two minima in U , effectively **breaks the symmetry** and introduces a new **massive particle**!

$V(\Phi)$ AND SPONTANEOUS SYMMETRY BREAKING

- The phenomenon we have just considered is called *spontaneous symmetry breaking*.
- Why symmetry breaking? Our choice of a ground state “breaks” the obvious reflection *symmetry* of the original Lagrangian.
- What about the spontaneous part?
 - The choice of a ground state is *arbitrary* in this system. There is no external agency that favors one over the other, or even forces the choice to begin with.

$V(\Phi)$ AND SPONTANEOUS SYMMETRY BREAKING



HIGGS MECHANISM

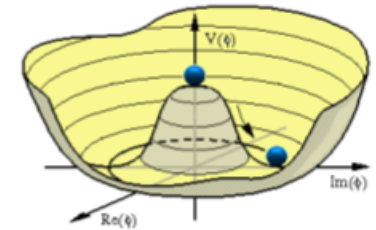


Yukawa Couplings

Higgs Field

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}' + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\phi'}$$

Massless case, from previous slide



$$\mathcal{L}_{\phi} = (\partial_{\mu} \phi^{\dagger})(\partial^{\mu} \phi) - V(\phi)$$

Higgs Potential

$$\mathcal{L}_{\text{Yuk}} = c_f (\bar{\psi}_L \psi_R \phi + \bar{\psi}_R \psi_L \phi)$$

Higgs Fermion Interaction

- When ϕ “picks a ground state” (i.e. when the gauge symmetry of a Lagrangian is spontaneously broken), all fermion fields and weak bosons become massive!
- Physically, the “massless” bosons acquire mass by interacting with a new “massive” scalar field called the Higgs field. Hence, this process is called the Higgs mechanism.

ELECTROWEAK UNIFICATION

- In the end, the $SU(2) \times U(1)$ part of the Standard Model is called the *electroweak theory*, because electromagnetism and the weak force start out **mixed together** in this overall gauge symmetry.
- $SU(2) \times U(1)$ predicts **four** massless bosons, which are not apparent at **everyday** energies.
- Analogous to our simple example, the ground state of the $SU(2) \times U(1)$ theory (where we live) is one in which this gauge symmetry is **hidden**.
- **Result:** the four **massless** gauge bosons **appear** to us as the **massive** W^+ , W^- , Z and the massless photon. The explicit $U(1)$ symmetry of QED is preserved.

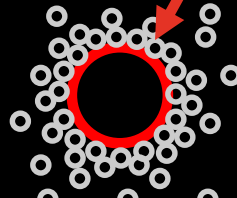
HIGGS MECHANISM



- A **force field** that permeates the Universe and **slows** particles **down** to below the speed of light. This is the **equivalent** to having **mass**.



Massive Particle



THE HIGGS PARTICLE

- Spin-0 (scalar) particle which gives **massive** force mediators their **mass**.

To understand the Higgs mechanism, imagine that a room full of physicists chattering quietly is like space filled with the Higgs field ...



THE HIGGS PARTICLE

- Spin-0 (scalar) particle which gives **massive** force mediators their **mass**.

... a well-known scientist walks in, creating a disturbance as he moves across the room and attracting a cluster of admirers with each step ...



THE HIGGS PARTICLE

- Spin-0 (scalar) particle which gives **massive** force mediators their **mass**.

... this increases his resistance to movement, in other words, he acquires mass, just like a particle moving through the Higgs field...



THE STANDARD MODEL

HOW SIMPLE, REALLY?

- The Standard Model does not predict:

Determine
experimentally

3 Couplings	$g_s, e, \sin \theta_W$
4 CKM parameters	$\vartheta_1, \vartheta_2, \vartheta_3, \delta$
2 Boson masses	m_Z, m_H
3 Lepton masses	m_e, m_μ, m_τ
6 Quark masses	$m_u, m_d, m_s, m_c, m_t, m_b$
1 QCD vacuum angle θ	

19 free SM parameters
no neutrino masses

$$m_W^2 = \frac{1}{2} g^2 \rho_0^2$$

$$m_Z^2 = \frac{1}{2} (g^2 + g'^2) \rho_0^2$$

$$m_H^2 = 4 \lambda \rho_0^2$$

$$g = e / \sin \theta_W$$

$$g' = e / \cos \theta_W$$

$$m_f = c_f \rho_0$$

THE STANDARD MODEL

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- The Standard Model does not predict:

Determine
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3 Couplings	$g_s, e, \sin \theta_W$
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6 Quark masses	$m_u, m_d, m_s, m_c, m_t, m_b$
1 QCD vacuum angle θ	

19 free SM parameters

Plus 7 from neutrino mass

$$m_W^2 = \frac{1}{2} g^2 \rho_0^2$$

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$$g = e / \sin \theta_W$$

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$$m_f = c_f \rho_0$$

THE STANDARD MODEL

PREDICTS RELATIONSHIPS

- All observables can be predicted in terms of **26 free parameters** (including neutrino masses, mixing parameters).
- If we have > 26 measurements of those observables, we **overconstrain** the SM.
 - Overconstrain: we don't have any more ad hoc inputs AND we **can test** the consistency of the model.
- In **practice**:
 - Pick **well measured** set of **observables**.
 - **Calculate** other **observables** in terms of these well known quantities.
 - **Test predictions**, measure observables, and compare to theory.

THE HIGGS BOSON DISCOVERY



Higgs Boson
Predicted in 1964 and
discovered at the LHC
in 2012!

VOLUME 13, NUMBER 9

PHYSICAL REVIEW LETTERS

31 AUGUST 1964

BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

F. Englert and R. Brout

Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium
(Received 26 June 1964)

VOLUME 13, NUMBER 16

PHYSICAL REVIEW LETTERS

19 OCTOBER 1964

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs

Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

VOLUME 13, NUMBER 20

PHYSICAL REVIEW LETTERS

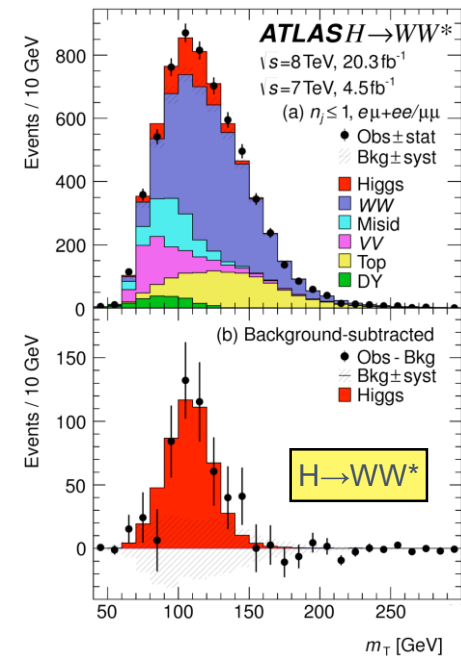
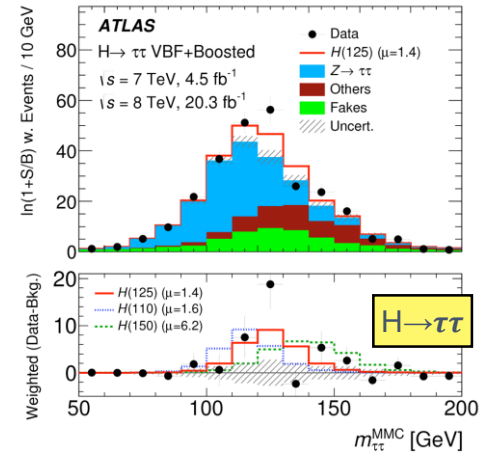
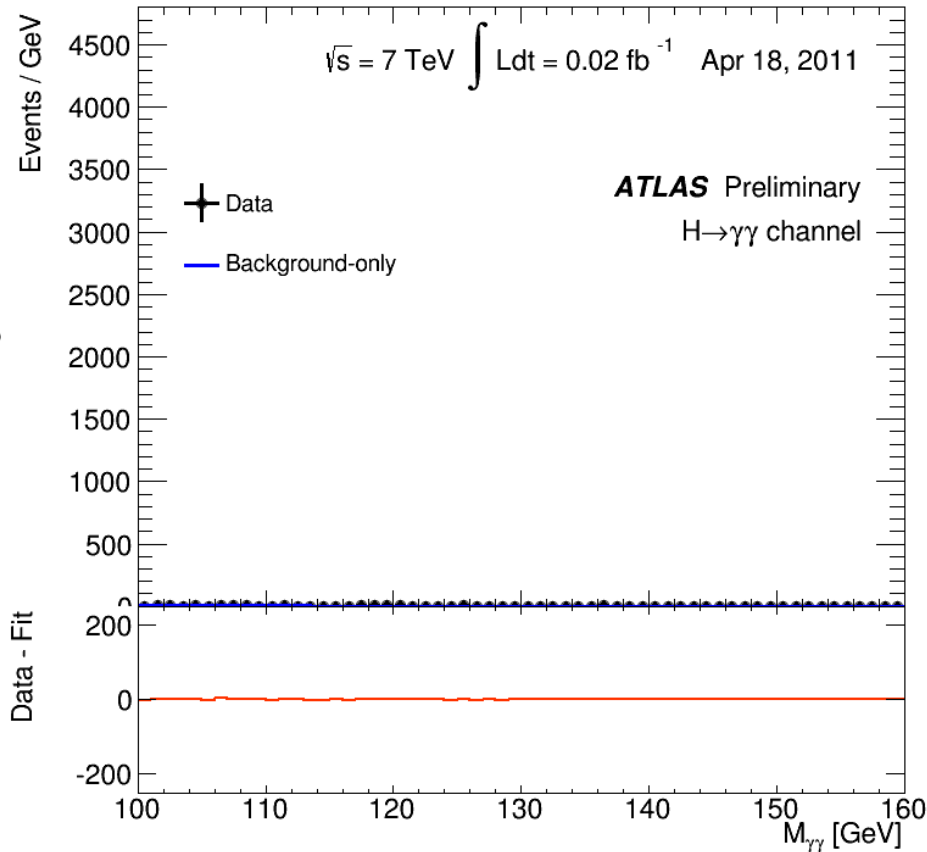
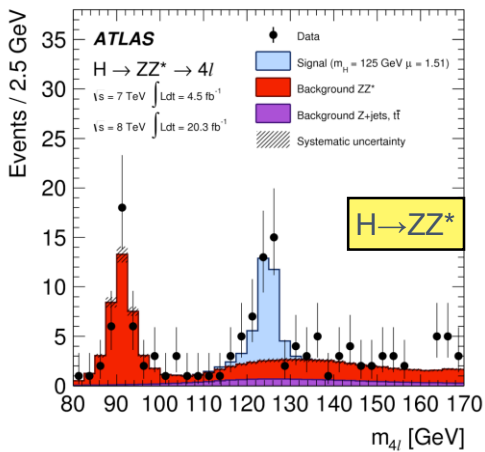
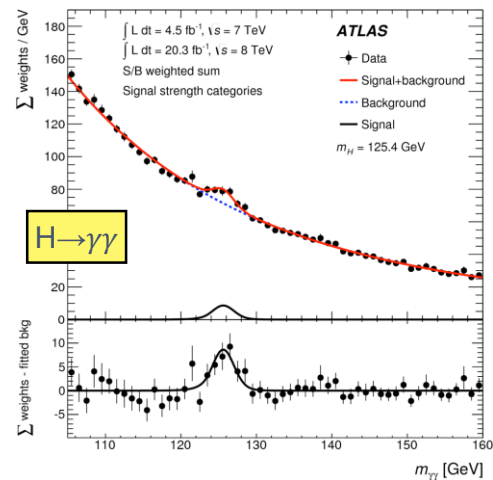
16 NOVEMBER 1964

GLOBAL CONSERVATION LAWS AND MASSLESS PARTICLES*

G. S. Guralnik,† C. R. Hagen,‡ and T. W. B. Kibble

Department of Physics, Imperial College, London, England
(Received 12 October 1964)

THE HIGGS DISCOVERY (BY ATLAS + CMS)



THE MOST PRECISELY TESTED THEORY

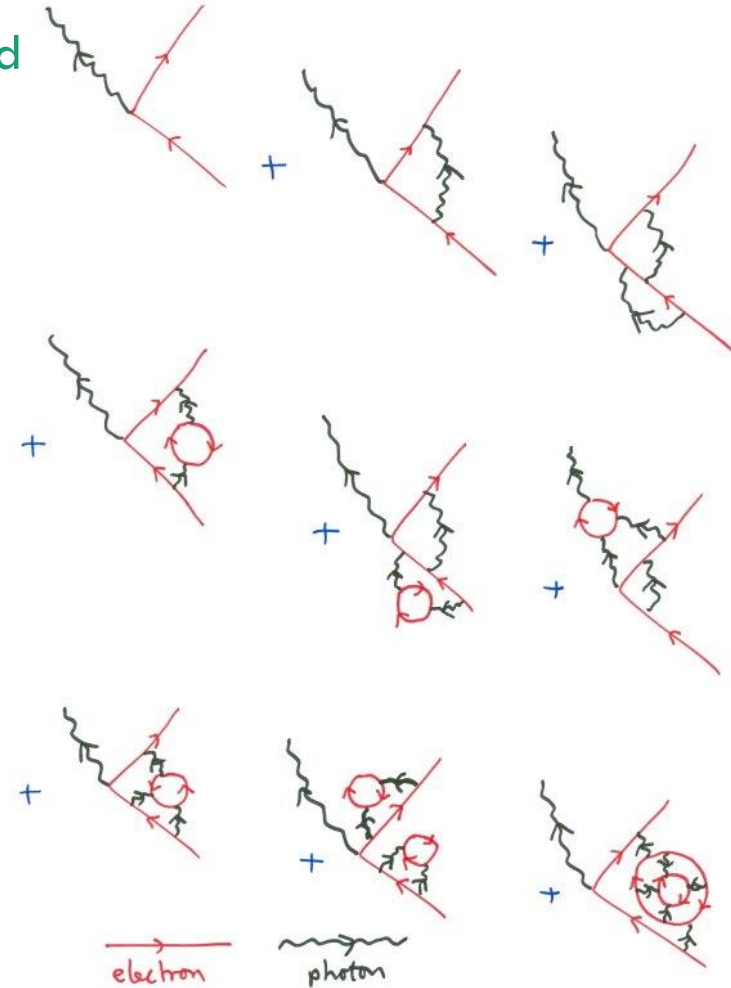
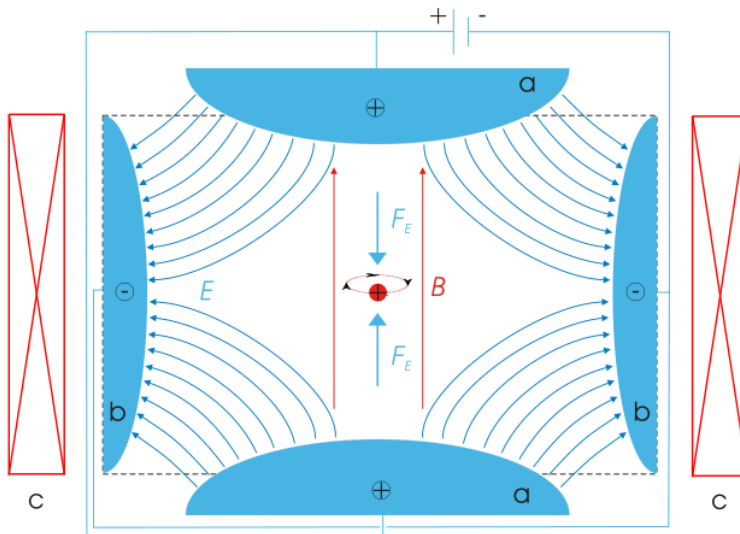
QED

- Magnetic moment of the electron **calculated** up to order α^5 in QED:

$$a_e = 0.001\,159\,652\,181\,643(764)$$

- Measured** in single electron Penning trap experiments:

$$a_e = 0.001\,159\,652\,180\,73(28)$$



THE MOST PRECISELY TESTED THEORY

QED

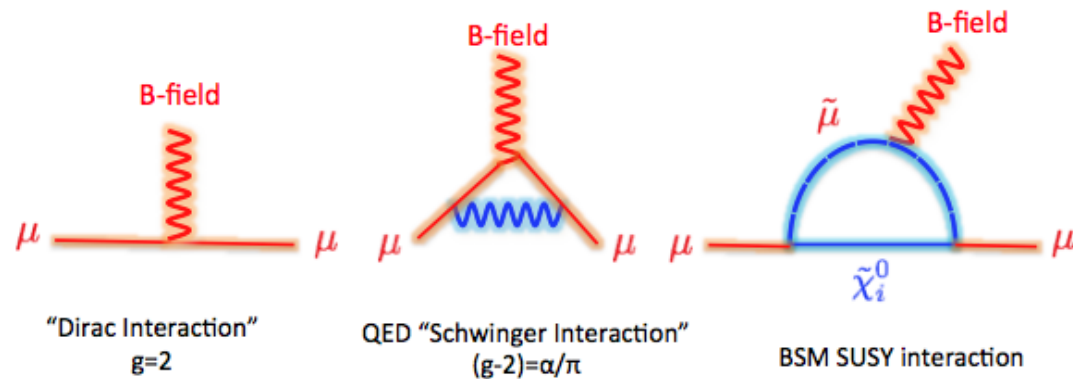
- Equivalent measurements in **muons** are an **active** area of research
- Very small **differences** between prediction:

$$\alpha_{\mu}^{\text{SM}} = \alpha_{\mu}^{\text{QED}} + \alpha_{\mu}^{\text{EW}} + \alpha_{\mu}^{\text{Hadron}} = 0.001\,165\,918\,04(51)$$

- And **measurement**:

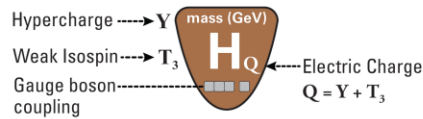
$$a_{\mu} = 0.001\,165\,920\,91(54)(33),$$

- Might indicate **new** physics **beyond** the **Standard Model**

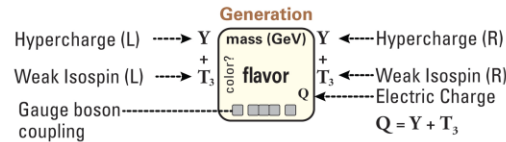


The Standard Model of Particle Physics

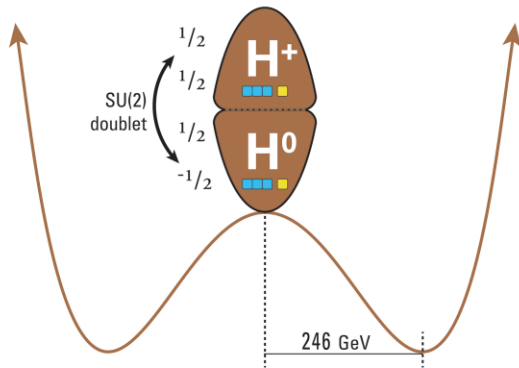
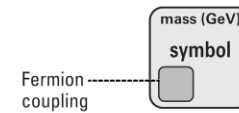
Spin 0 (Higgs Boson)



Spin 1/2 (Fermions)



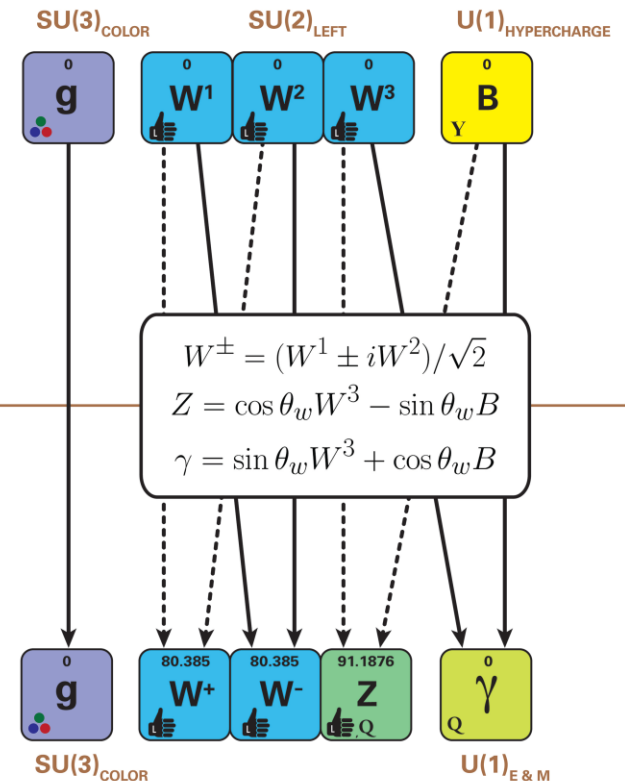
Spin 1 (Gauge Bosons)



Unbroken Symmetry
Broken Symmetry

	1st	2nd	3rd	
Left handed SU(2) doublet	$\begin{matrix} 1/6 \\ 1/2 \\ 1/6 \\ -1/2 \end{matrix}$	$\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}$	$\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}$	$\begin{matrix} 2/3 \\ 0 \\ -1/3 \\ 0 \end{matrix}$
Quarks	$\begin{matrix} u \\ d \end{matrix}$	$\begin{matrix} c \\ s \end{matrix}$	$\begin{matrix} t \\ b \end{matrix}$	
Left handed SU(2) doublet	$\begin{matrix} -1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{matrix}$	$\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}$	$\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}$	$\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}$
Leptons	$\begin{matrix} \nu_e \\ e \end{matrix}$	$\begin{matrix} \nu_\mu \\ \mu \end{matrix}$	$\begin{matrix} \nu_\tau \\ \tau \end{matrix}$	

	1st	2nd	3rd
0.0023	1.275	173.07	
$\begin{matrix} u \\ d \end{matrix}$	$\begin{matrix} c \\ s \end{matrix}$	$\begin{matrix} t \\ b \end{matrix}$	
$\begin{matrix} \nu_e \\ e \end{matrix}$	$\begin{matrix} \nu_\mu \\ \mu \end{matrix}$	$\begin{matrix} \nu_\tau \\ \tau \end{matrix}$	
0.000511	0.105658	1.77682	



THE STANDARD MODEL LAGRANGIAN

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{gauge}/\psi} . \quad (1)$$

Here,

$$\mathcal{L}_{\text{Dirac}} = i\bar{e}_L^i \not{\partial} e_L^i + i\bar{\nu}_L^i \not{\partial} \nu_L^i + i\bar{e}_R^i \not{\partial} e_R^i + i\bar{u}_L^i \not{\partial} u_L^i + i\bar{d}_L^i \not{\partial} d_L^i + i\bar{u}_R^i \not{\partial} u_R^i + i\bar{d}_R^i \not{\partial} d_R^i ; \quad (2)$$

$$\mathcal{L}_{\text{mass}} = -v \left(\lambda_e^i \bar{e}_L^i e_R^i + \lambda_u^i \bar{u}_L^i u_R^i + \lambda_d^i \bar{d}_L^i d_R^i + \text{h.c.} \right) - M_W^2 W_\mu^+ W^{-\mu} - \frac{M_W^2}{2 \cos^2 \theta_W} Z_\mu Z^\mu ; \quad (3)$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} (G_{\mu\nu}^a)^2 - \frac{1}{2} W_{\mu\nu}^+ W^{-\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{WZA} , \quad (4)$$

where

$$\begin{aligned} G_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_3 f^{abc} A_\mu^b A_\nu^c \\ W_{\mu\nu}^\pm &= \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm \\ Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu , \end{aligned} \quad (5)$$

and

$$\begin{aligned} \mathcal{L}_{WZA} &= ig_2 \cos \theta_W \left[(W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+) \partial^\mu Z^\nu + W_{\mu\nu}^+ W^{-\mu} Z^\nu - W_{\mu\nu}^- W^{+\mu} Z^\nu \right] \\ &+ ie \left[(W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+) \partial^\mu A^\nu + W_{\mu\nu}^+ W^{-\mu} A^\nu - W_{\mu\nu}^- W^{+\mu} A^\nu \right] \\ &+ g_2^2 \cos^2 \theta_W (W_\mu^+ W_\nu^- Z^\mu Z^\nu - W_\mu^+ W_\nu^- Z_\nu Z^\mu) \\ &+ g_2^2 (W_\mu^+ W_\nu^- A^\mu A^\nu - W_\mu^+ W_\nu^- A_\nu A^\mu) \\ &+ g_2 e \cos \theta_W [W_\mu^+ W_\nu^- (Z^\mu A^\nu + Z^\nu A^\mu) - 2W_\mu^+ W_\nu^- Z^\mu A^\nu] \\ &+ \frac{1}{2} g_2^2 (W_\mu^+ W_\nu^-) (W^{+\mu} W^{-\nu} - W^{+\nu} W^{-\mu}) ; \end{aligned} \quad (6)$$

and

$$\mathcal{L}_{\text{gauge}/\psi} = -g_3 A_\mu^a J_{(3)}^{\mu a} - g_2 (W_\mu^+ J_{W^+}^\mu + W_\mu^- J_{W^-}^\mu + Z_\mu J_Z^\mu) - e A_\mu J_A^\mu , \quad (7)$$

where

$$\begin{aligned} J_{(3)}^{\mu a} &= \bar{u}^i \gamma^\mu T_{(3)}^a u^i + \bar{d}^i \gamma^\mu T_{(3)}^a d^i \\ J_{W^+}^\mu &= \frac{1}{\sqrt{2}} (\bar{\nu}_L^i \gamma^\mu e_L^i + V^{ij} \bar{u}_L^i \gamma^\mu d_L^j) \\ J_{W^-}^\mu &= (J_{W^+}^\mu)^* \\ J_Z^\mu &= \frac{1}{\cos \theta_W} \left[\frac{1}{2} \bar{\nu}_L^i \gamma^\mu \nu_L^i + \left(-\frac{1}{2} + \sin^2 \theta_W \right) \bar{e}_L^i \gamma^\mu e_L^i + (\sin^2 \theta_W) \bar{e}_R^i \gamma^\mu e_R^i \right. \\ &\quad + \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \bar{u}_L^i \gamma^\mu u_L^i + \left(-\frac{2}{3} \sin^2 \theta_W \right) \bar{u}_R^i \gamma^\mu u_R^i \\ &\quad \left. + \left(-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right) \bar{d}_L^i \gamma^\mu d_L^i + \left(\frac{1}{3} \sin^2 \theta_W \right) \bar{d}_R^i \gamma^\mu d_R^i \right] \\ J_A^\mu &= (-1) \bar{e}^i \gamma^\mu e^i + \left(\frac{2}{3} \right) \bar{u}^i \gamma^\mu u^i + \left(-\frac{1}{3} \right) \bar{d}^i \gamma^\mu d^i . \end{aligned}$$

THE STANDARD MODEL LAGRANGIAN



SUMMARY

- The **Standard Model** is a framework that describes the **elementary particles** and their **electromagnetic, weak and strong** interactions.
 - Furthermore, it provides an explanation for the electromagnetic and weak interactions in terms of a **spontaneously broken electroweak symmetry**.
- The Standard Model predicts **cross-sections**, couplings.
- Latest success: the **discovery** of the Higgs boson!
- However, the Standard Model is **not** completely satisfactory:
 - 26 free parameters...
 - We know it is **incomplete**: it does not fully describe nature.

STANDARD MODEL LIMITATIONS

STANDARD MODEL LIMITATIONS

OBSERVED DISCREPANCIES

- How can we describe **gravity** and the three Standard Model forces consistently?
 - We know gravity exists, but **incompatible** with Gauge theories...
- What is the **mass** of **neutrinos**, and what is its origin?
 - We **know** neutrino mass is finite.
 - Simple extensions to the Standard Model **may** be able to accommodate neutrino mass.
- What are **dark matter** and **dark energy**?
 - We know they exist but we **don't know** what they are. Theories beyond the Standard Model might offer solutions...
- How did we end up in a **matter** dominated universe?
 - The Standard Model might explain this...

STANDARD MODEL LIMITATIONS

PHILOSOPHICAL NUISANCES (“WHY” RATHER THAN “WHAT/HOW”)

- Why are there **three** generations of matter?
 - “Because” this is required for CP violation? And CP violation required for matter-dominated universe: us?
 - Anthropic principle – perhaps not great Science...
- Why are some of the Standard Model parameters “**unnaturally**” large / small?
 - Parameters that look like “1” give theorists a warm and fuzzy feeling
- Why are the **masses** what they are?
 - And why are they so **different**?! $m_t/m_\nu \sim 10^{14}$

FACING THE STANDARD MODEL LIMITATIONS

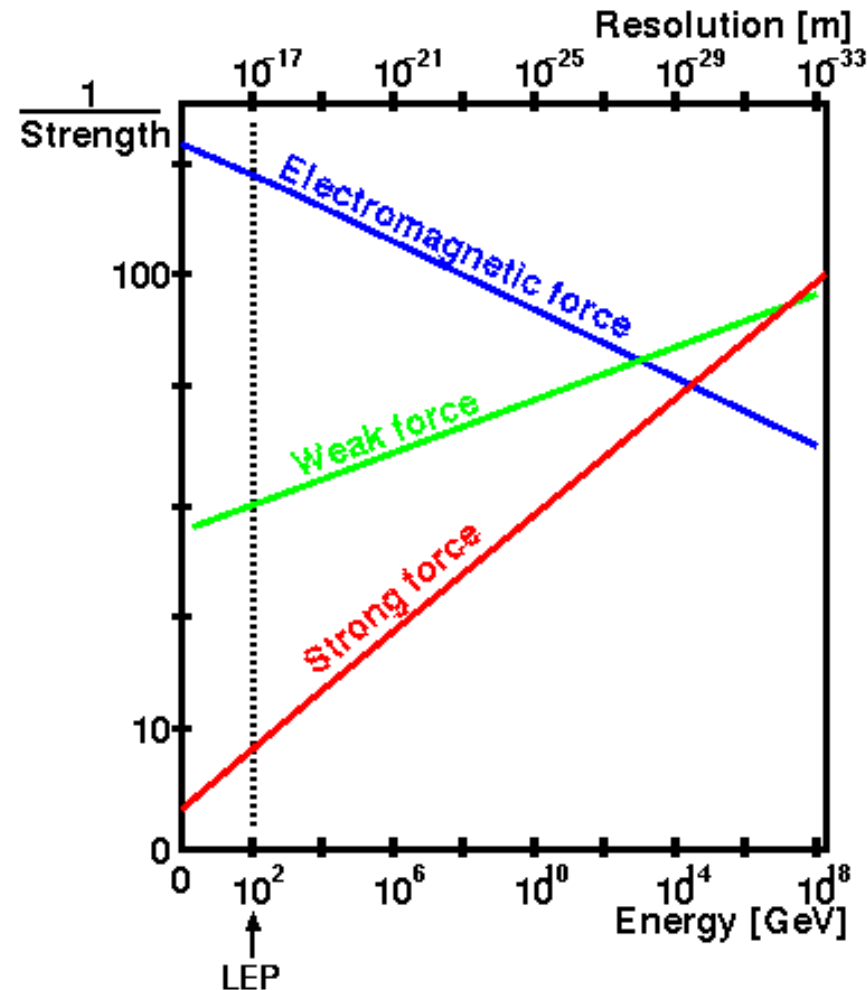
- Over the history of particle physics, a great deal of time has been spent by both **theorists** and **experimentalists** to try to **resolve** the limitations of the Standard Model.
- Generally, the strategy has been to **extend** the Standard Model in a way or another:
 - **Grand Unified Theories.**
 - Predicts **proton decay**.
 - **Supersymmetry.**
 - Might explain dark matter.
- A longer term, and more **ambitious** project has been to formulate a **Theory of Everything**, that includes gravity.
 - So far these theories have **lacked predictive power...**

GUT SCALE

- Grand unification scale:

$$10^{16} \text{ GeV}$$

- The Standard Model provides **no explanation** for what may happen beyond this unification scale, nor why the forces have such different strengths at low energies.



GRAND UNIFIED THEORIES

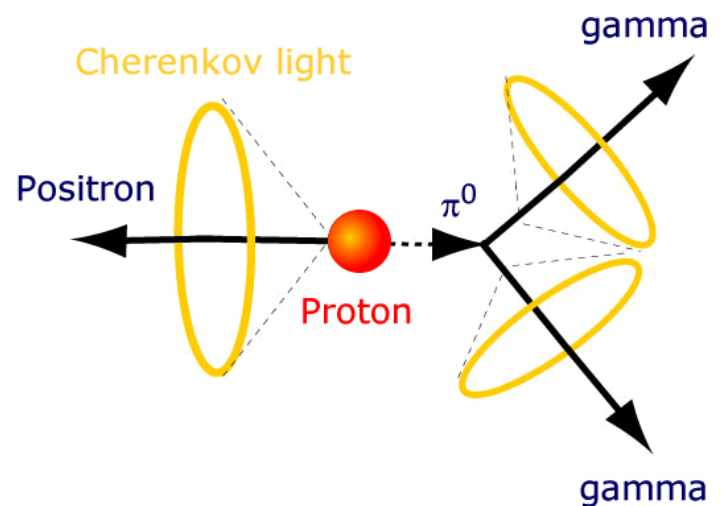
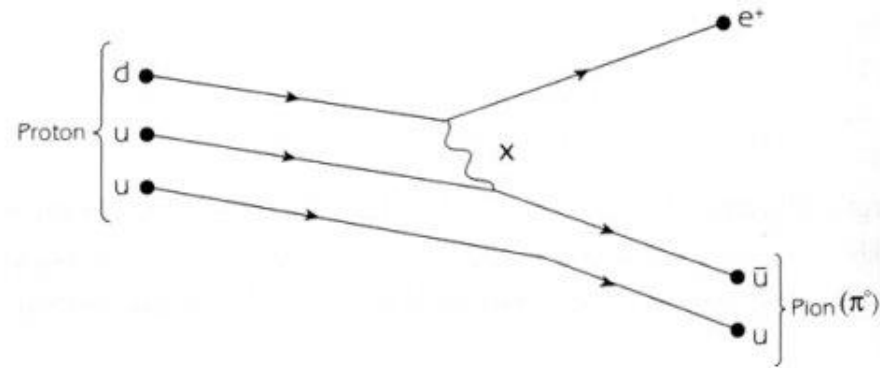
- In the 1970's, people started to think a lot about how to combine the $SU(3)$, $SU(2)$, and $U(1)$ gauge symmetries of the Standard Model into a more fundamental, global symmetry.
- The first such Grand Unified Theory is a 1974 model based on $SU(5)$ symmetry.
- This model groups all of the known fermions – i.e., the leptons and quarks – into multiplets.
- Inside the multiplets, quarks and leptons can couple to each other and transform into one another.
- In essence, this theory imposes a grand symmetry: all fermions, whether quarks or leptons, are fundamentally the same.

THE SU(5) GUT

- In this early model, interactions between the quarks and leptons are mediated by two new massive bosons, called the X and Y.
- To conserve electric and color charge, the X and Y have electric charges of $-4e/3$ and $-e/3$, and one of three possible colors. They are also incredibly massive, close to the grand unification scale of 10^{16} GeV.
- Hence, including both particles and antiparticles, the model predicts 12 types of X and Y.
- In addition to these 12, there are also 8 gluons, 3 weak bosons, and 1 photon, for a total of 24. This makes sense, for recall that a theory exhibiting SU(n) gauge symmetry requires the existence of n^2-1 gauge bosons.

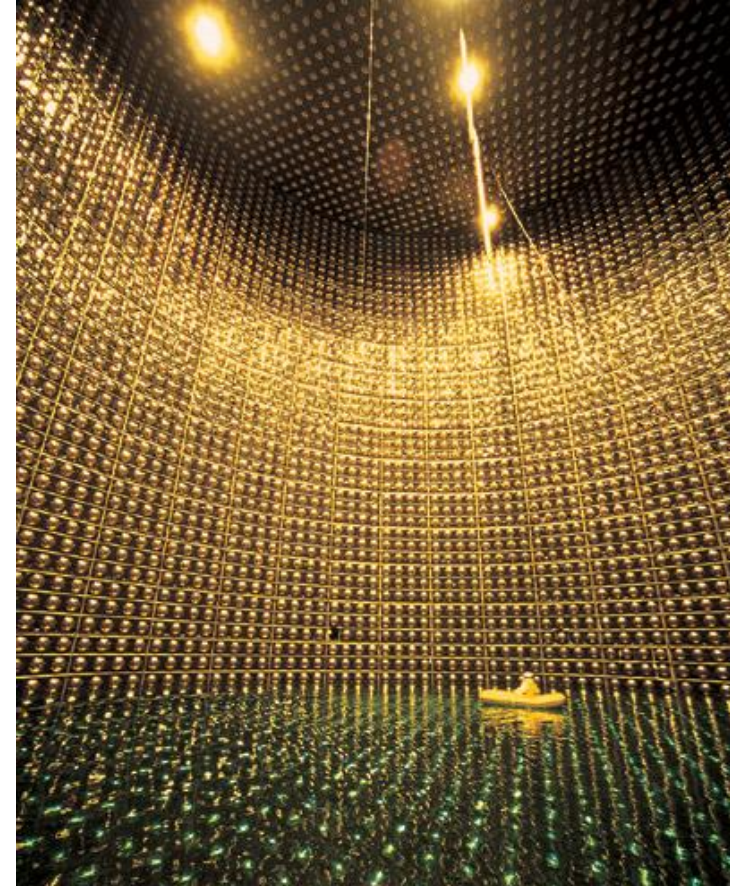
SU(5) GUT OBSERVATIONAL CONSEQUENCES

- The SU(5) GUT implies that it would take **huge** energies to even hope to **see** an X or Y particle “in the wild”.
- However, even at “**normal**” (i.e. low) energies, virtual X and Y exchanges **can** take place.
- This is major: if **quarks** can **decay into leptons** via **virtual** X and Y exchange, then “stable” particles might actually be unstable!
- Example: the **proton** could possibly decay via exchange of a virtual X.



PROTON DECAY

- The **instability** of the proton is one of the few **tests** of GUT physics that would be manifest at everyday energies.
- Computations show that relative to most elementary particles, the proton is very stable; its lifetime according to the SU(5) GUT is 10^{30} years!
- How can we detect such an effect?
- Put **many** protons together – e.g., in a huge tank of water – and **wait** for some to decay...
- The **Super-Kamiokande** water Cherenkov detector, was built to **search** for proton decay, although it has made major contributions to understanding neutrino oscillations!



PROTON DECAY



Neil did not see a proton decay, but he was very excited about neutrino oscillations!

PROTON DECAY

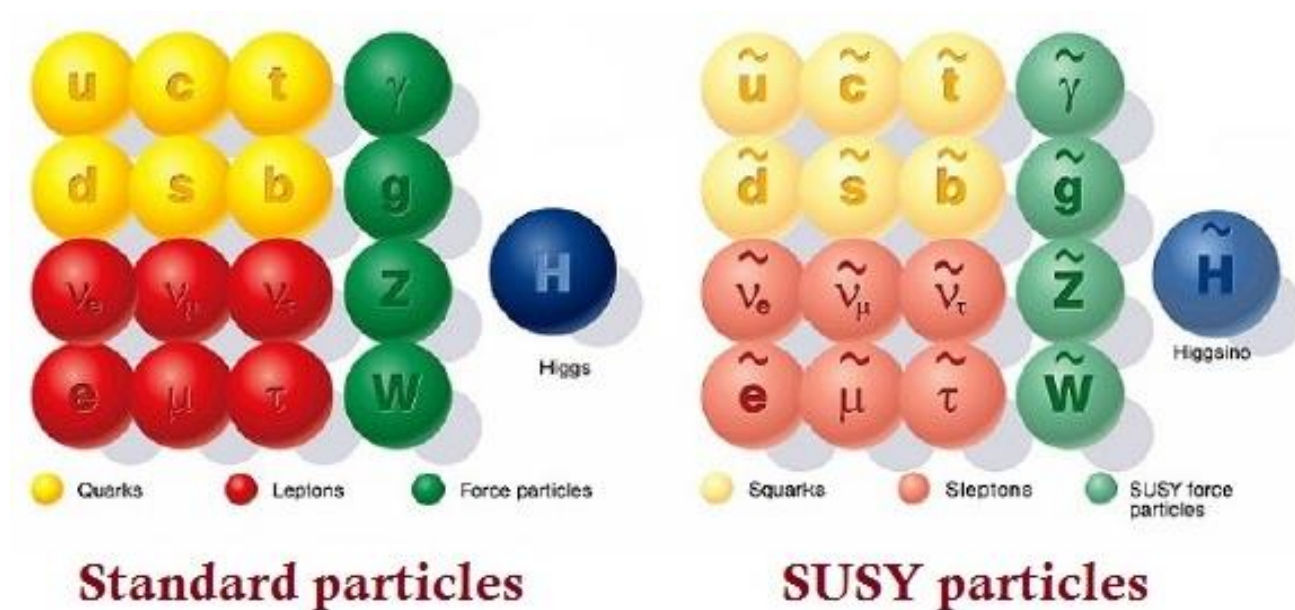
- Even though the proton lifetime is **very long**, a kiloton of material contains about 10^{32} protons, so about **one decay per day** should occur in such a sample.
- The Super-Kamiokande detector holds **50 kilotons** of water viewed by 11 000 photomultiplier tubes. It is located **underground**, shielded from background noise due to cosmic radiation at the Earth's surface.
- **No proton decays have been observed** despite more than **two decades** of searching, leading to lower limits on the proton lifetime of about 10^{34} years.
- Is this a disaster for the theory? Not necessarily. GUT physics can be saved if we introduce **supersymmetry**...

SUPERSYMMETRY

- The idea behind GUTs like $SU(5)$ symmetry is to present different particles as **transformed versions** of each other.
- The $SU(5)$ GUT treats quarks and leptons as the same underlying “something”, **unifying the fermions** in the Standard Model.
- However, these early approaches to grand unification **failed** to incorporate the gauge bosons and Higgs scalars of the Standard Model into a unified scheme.
- Hence, some eventually suggested that the **bosons** and **fermions** should be **united** somehow by invoking a new kind of symmetry: **supersymmetry** (or **SUSY** for short).
- Aside: theorists did not develop supersymmetry explicitly for the Standard Model; the field started obscurely in the early 1970’s during investigations of the spacetime symmetries in quantum field theory.

SUPERSYMMETRY

- So what, exactly, does supersymmetry **predict**?
- It says that for each known **boson**, there should be a **fermion** of **identical mass** and **charge**, and similarly for each known fermion, there exists a boson of identical mass and charge.

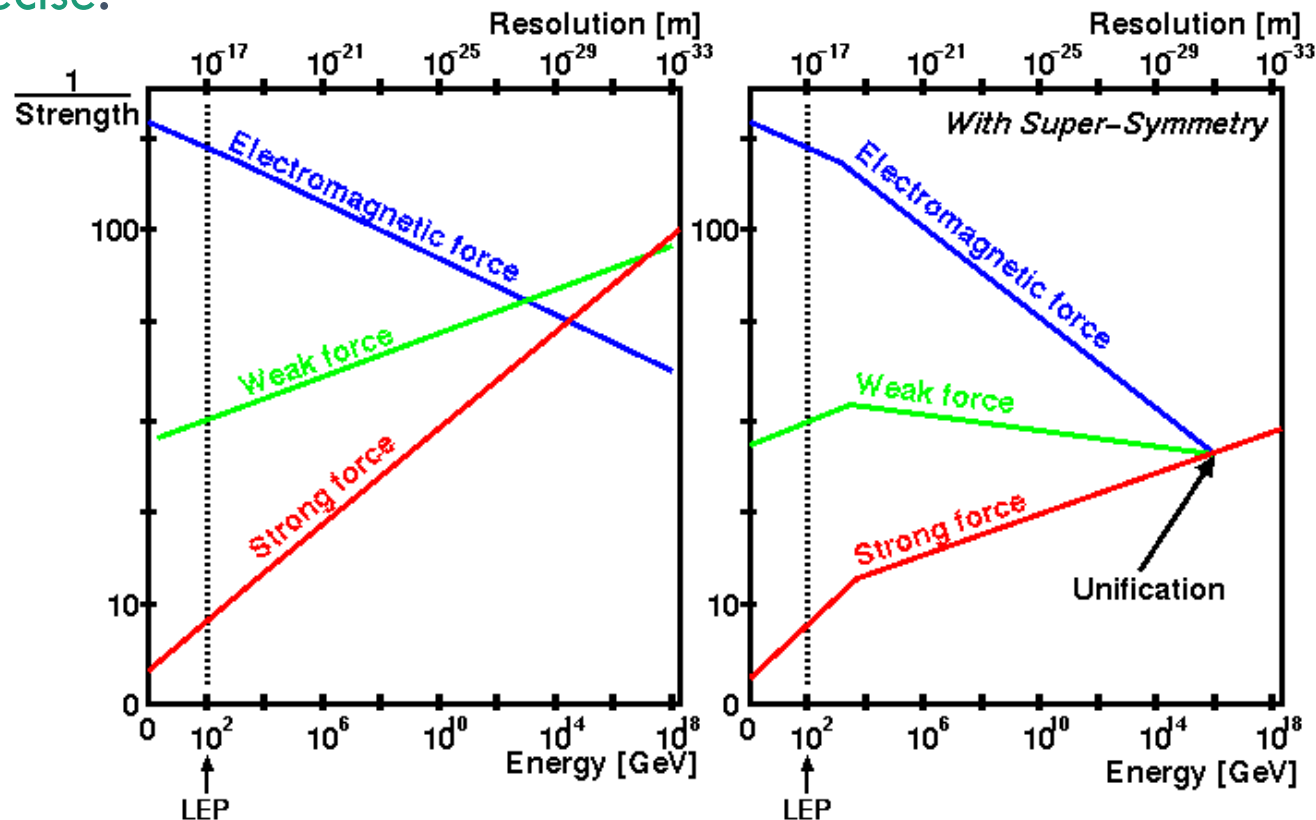


SUPERSYMMETRY

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- It says that for each known **boson**, there should be a **fermion** of **identical mass** and **charge**, and similarly for each known fermion, there exists a boson of identical mass and charge.
- If this is the case, why haven't we observed a **selectron** of mass **0.511 MeV**?
 - Supersymmetry is a **broken symmetry** at our energy scale.
 - **All sparticles** are expected to be **more massive** than their Standard Model partners.

SUPERSYMMETRY AND THE GUT SCALE

- If instead of the Standard Model, we start from the **Minimal Supersymmetric Standard Model**, the alignment of the three coupling constants at the GUT scale becomes much **more precise**.



SUPERSYMMETRY AT THE LHC

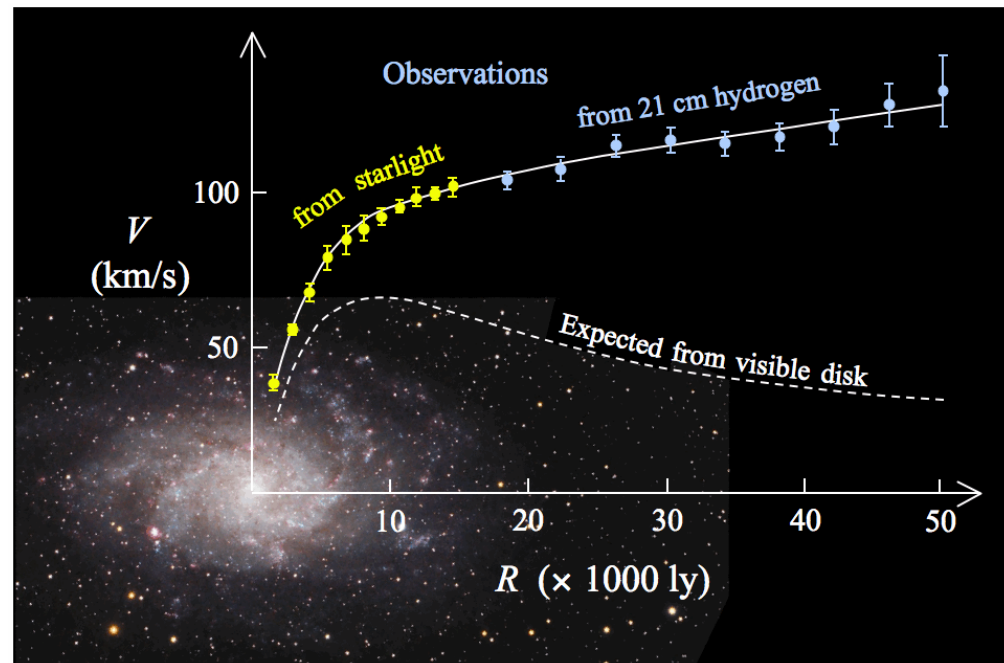
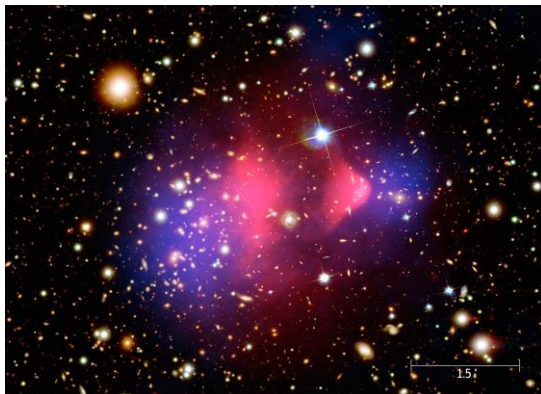
- If SUSY is a symmetry of **nature**, some of the sparticles might have a mass around **1000 GeV**.
- We hope to see **evidence** for SUSY at the **LHC**.
 - But so far **no candidates** have been found...



Excited LHC physicists still haven't found SUSY.

DARK MATTER

- Galaxies rotate **faster** than expected, implying **more mass** than what is **seen**.
- A host of **other observations** point in the same direction...
- Alternatively, **general relativity** might need **modification**.
 - Probably not the solution...
- Scientific consensus leaning towards the weakly interacting particle hypothesis.
 - We need to observe it in the lab!

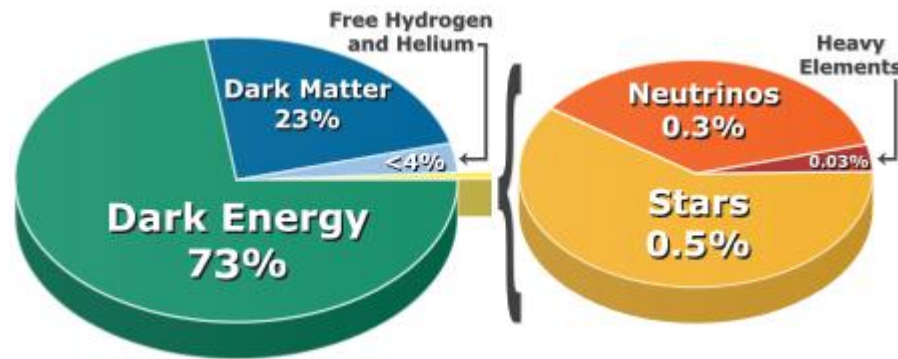


SUPERSYMMETRY AND DARK MATTER

- The **neutrinos** that we know are **not** good candidates to explain **dark matter**.
 - They are “**dark**” but also too **light** – moving too fast to **cluster** around galaxies.
- To account for dark matter, we need a particle that looks a lot like a neutrino, but **more massive**.
 - No electric charge or color.
- **Lightest Supersymmetric Particle**
 - We haven’t seen **proton decay** so, if SUSY exists, decays of **sparticles** into **particles** must be **highly suppressed**.
 - We call this R-parity.
 - Hence the **LSP** is **stable**, much **heavier** than neutrinos, and in several SUSY models, **charge neutral**.
 - A dark matter **candidate**!

...AND THERE'S ALSO DARK ENERGY

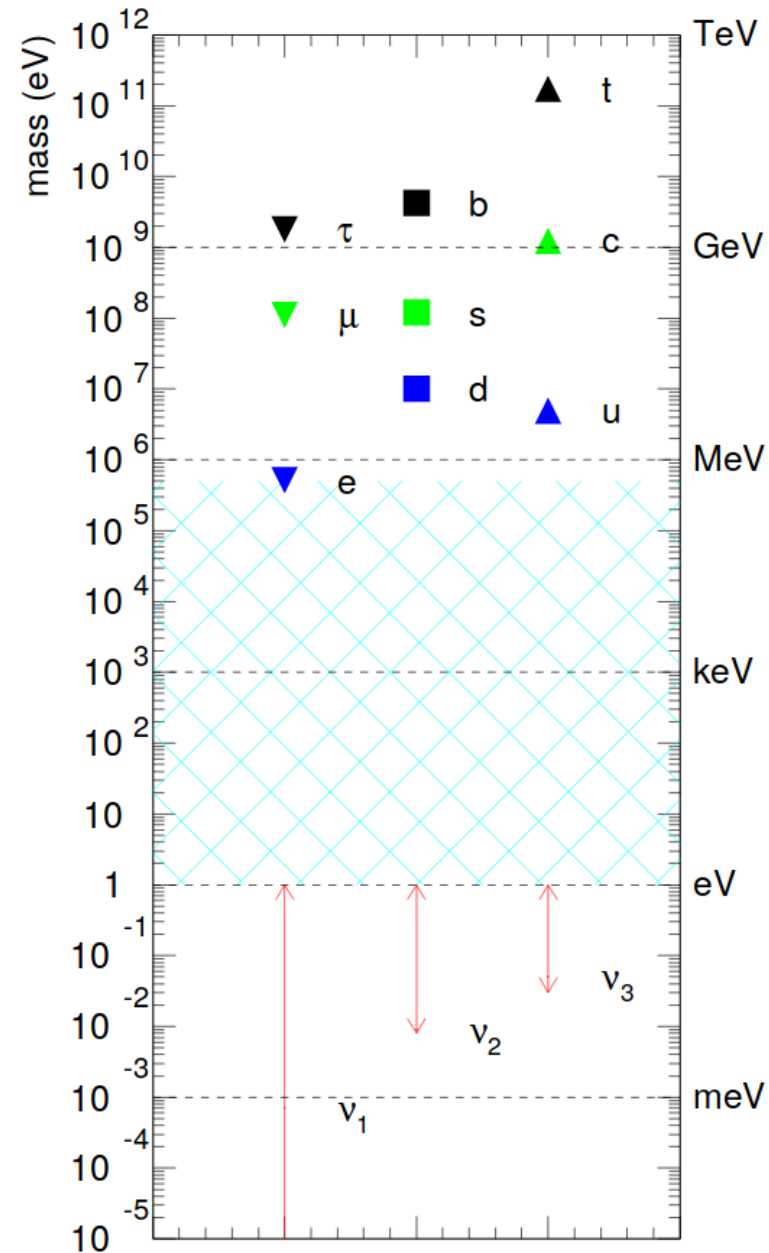
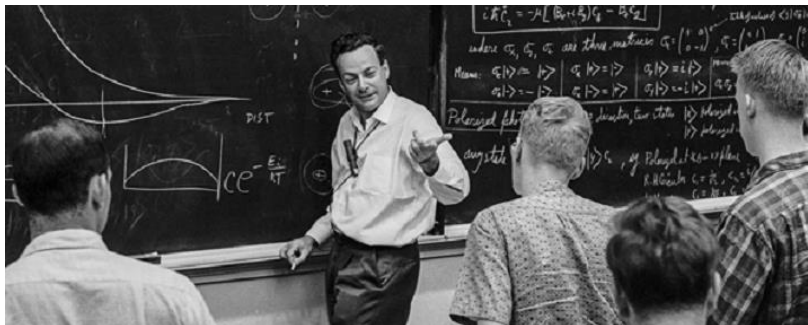
- The universe is **expanding** at an **accelerating** rate!
 - This can be accommodated in **general relativity** by having a non-zero **cosmological constant** (Λ), also called vacuum energy.
- What is the **origin** of this vacuum energy? Quantum field theories tend to predict vacuum energies that are **far too large...**



PARTICLE MASSES

- “There remains **one especially unsatisfactory** feature [of the Standard Model of particle physics]: the **observed masses** of the particles, m . There is **no theory** that adequately **explains** these numbers. We **use** the numbers in all our theories, but we **do not understand** them — **what** they are, or where they **come from**. I believe that from a **fundamental** point of view, this is a very **interesting** and **serious problem**.”

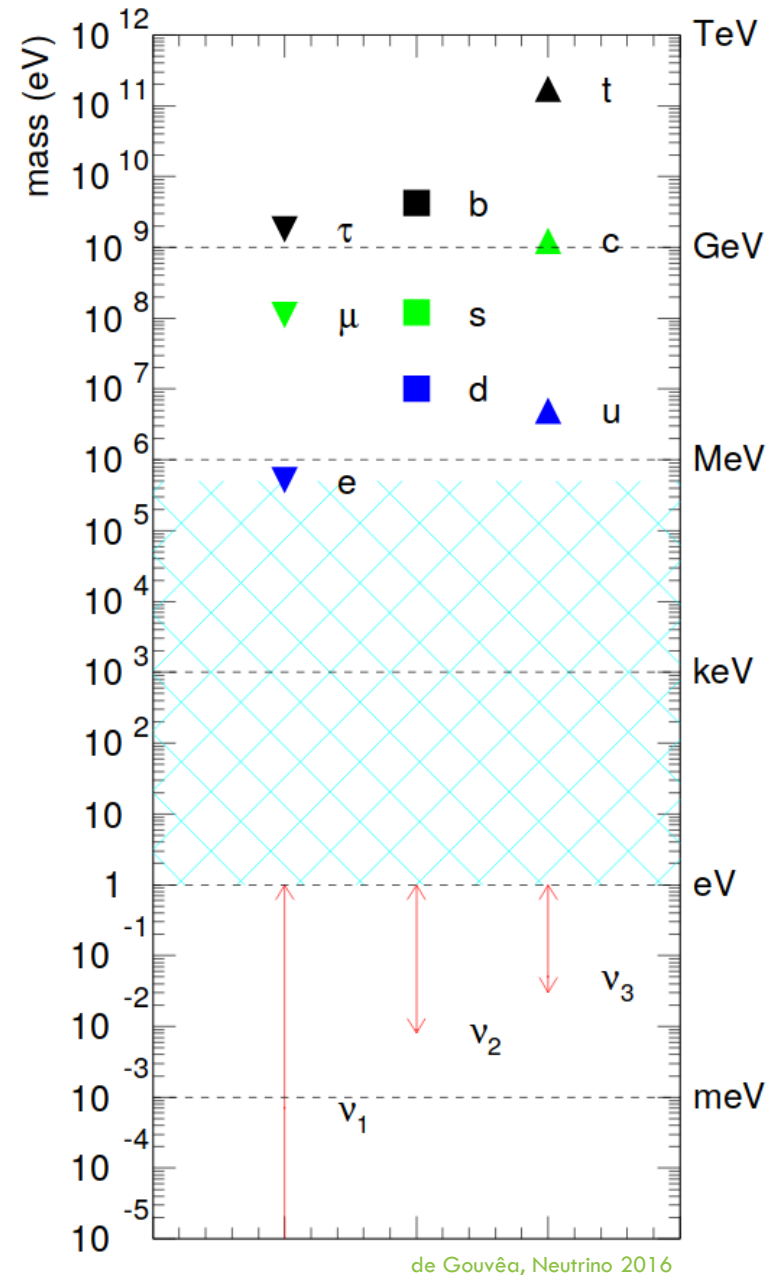
- R. P. Feynman



de Gouvêa, Neutrino 2016

PARTICLE MASSES

- Is there a theory that can accurately **predict** the masses of the fermions?
 - None so far...
 - Would it **elucidate** the nature of the **three** generations?
- Why are **neutrino masses** so much smaller than the other fermion masses?
 - Is the **mechanism** that generates neutrino masses **qualitatively different** from the mechanism that generates other fermion masses?

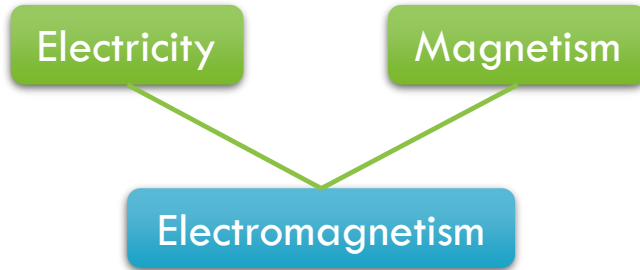


A THEORY OF EVERYTHING?

Electricity

Magnetism

A THEORY OF EVERYTHING?



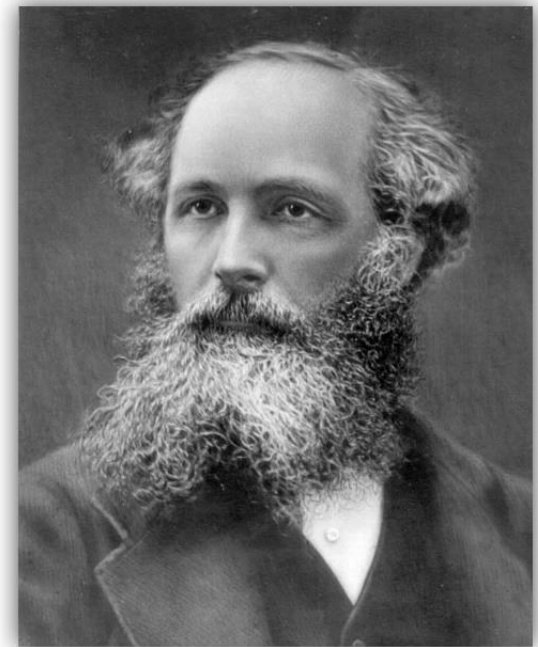
$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

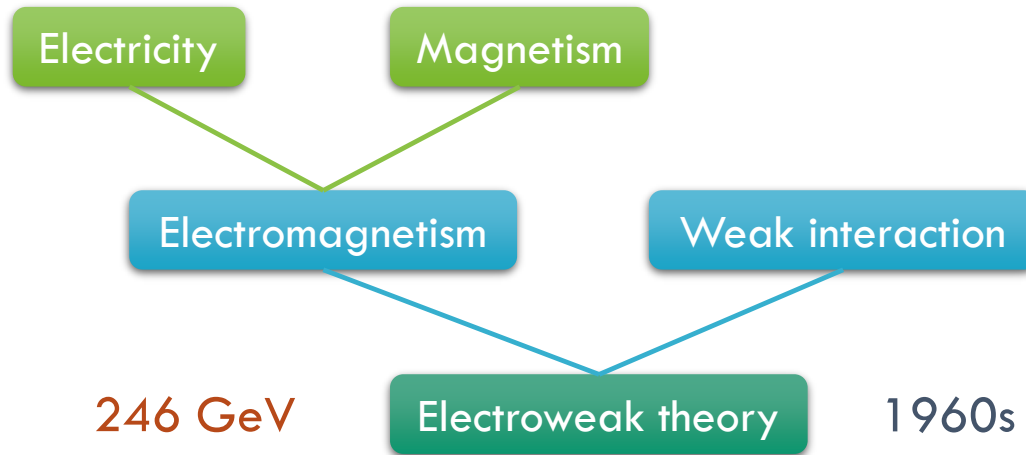
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

James Clerk Maxwell
1800s



A THEORY OF EVERYTHING?



Sheldon Lee
Glashow

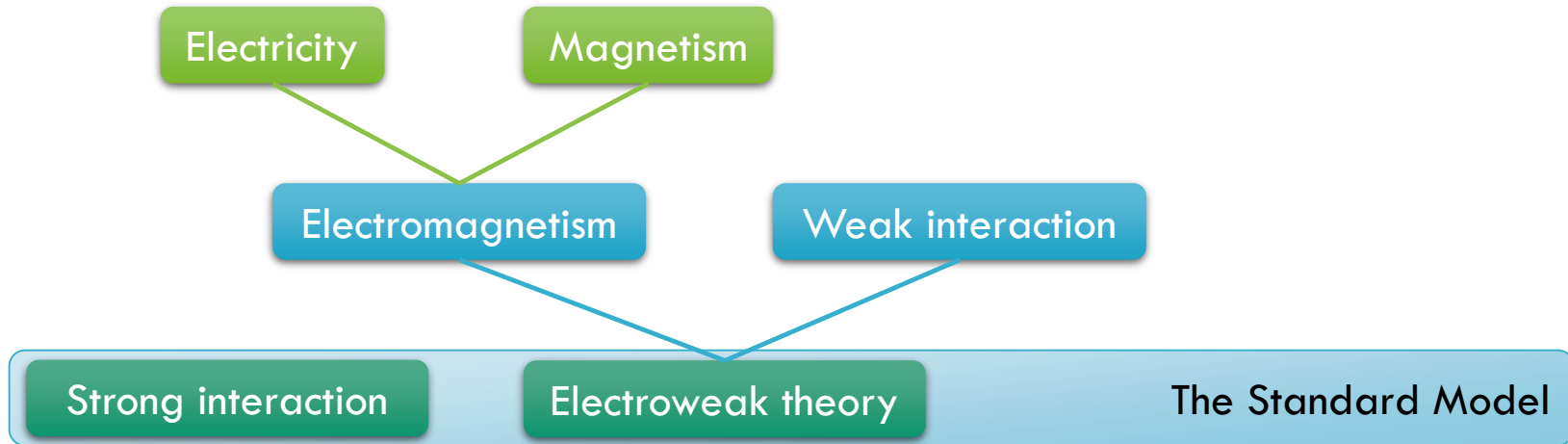


Abdus Salam



Steven Weinberg

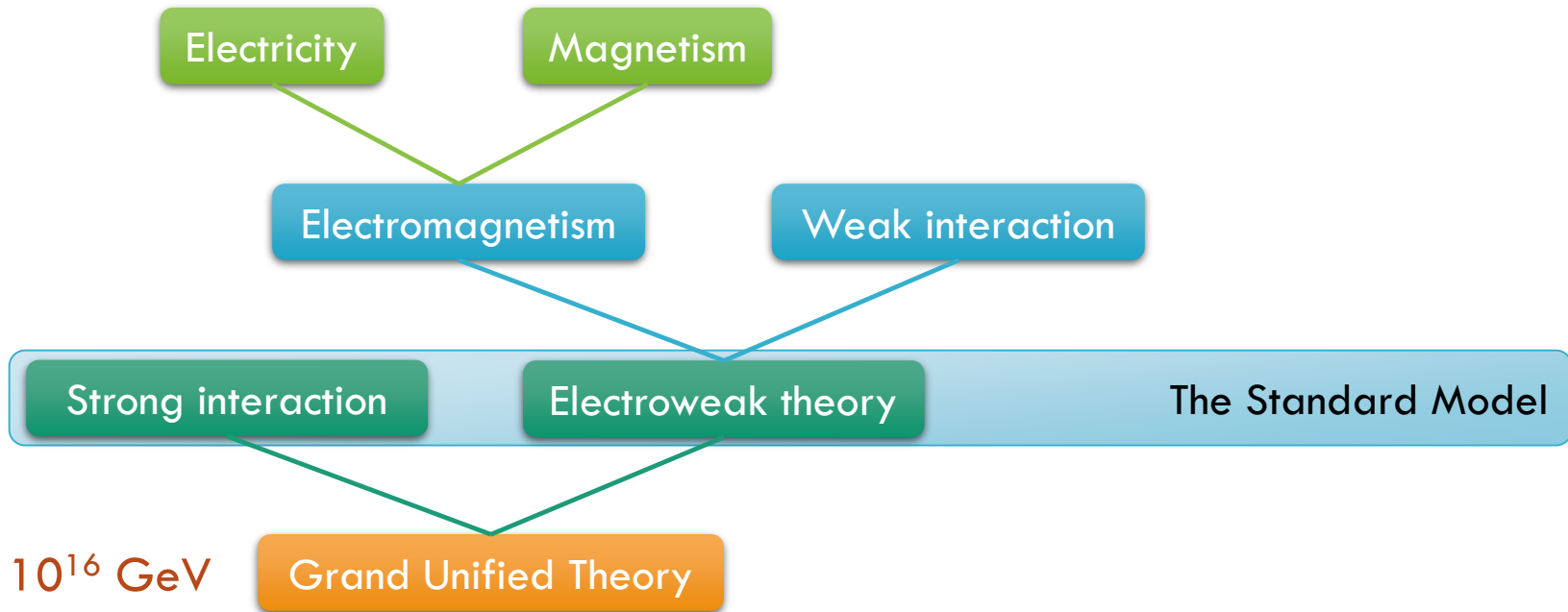
A THEORY OF EVERYTHING?



mass →	≈2.3 MeV/c ²	≈1.275 GeV/c ²	≈173.07 GeV/c ²	0	≈126 GeV/c ²
charge →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	u up	c charm	t top	g gluon	H Higgs boson
	d down	s strange	b bottom	γ photon	
	e electron	μ muon	τ tau	Z Z boson	
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	

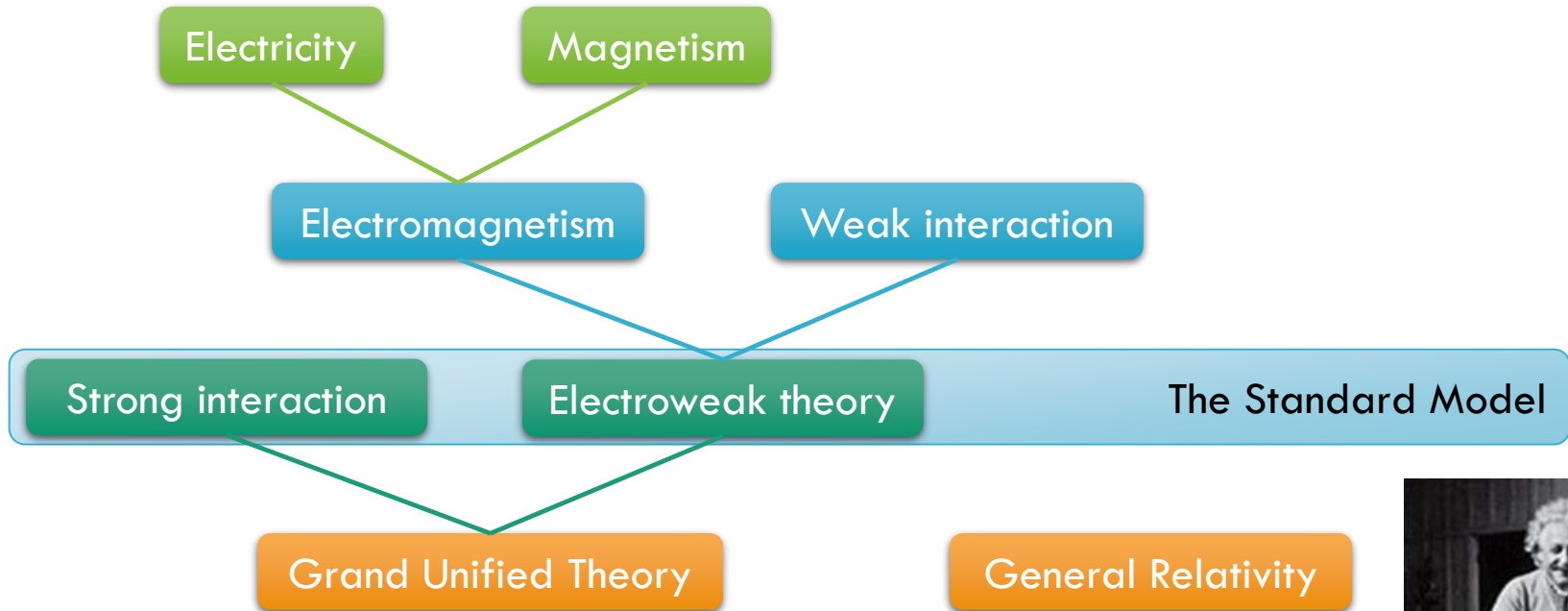
QUARKS (purple text on the left, rows 1-3)
LEPTONS (green text on the left, rows 4-5)
GAUGE BOSONS (red text on the right, rows 1-5)

A THEORY OF EVERYTHING?



$SU(5)?$ $SO(10)?$ $SU(8)?$ $O(16)?$
1970s – today...

A THEORY OF EVERYTHING?



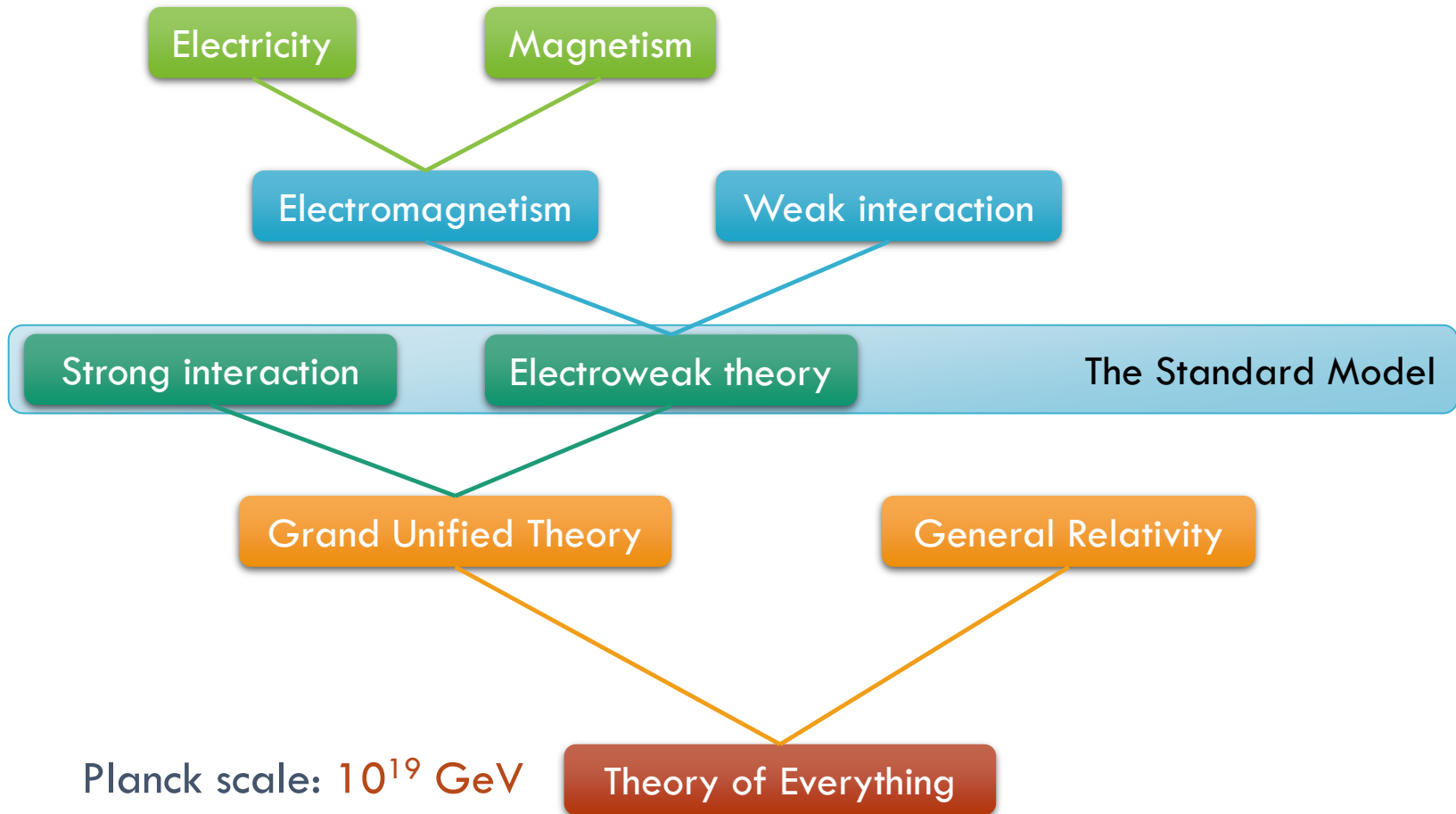
$SO(5)? SO(10)? SU(8)? O(16)?$
1970s – today...

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

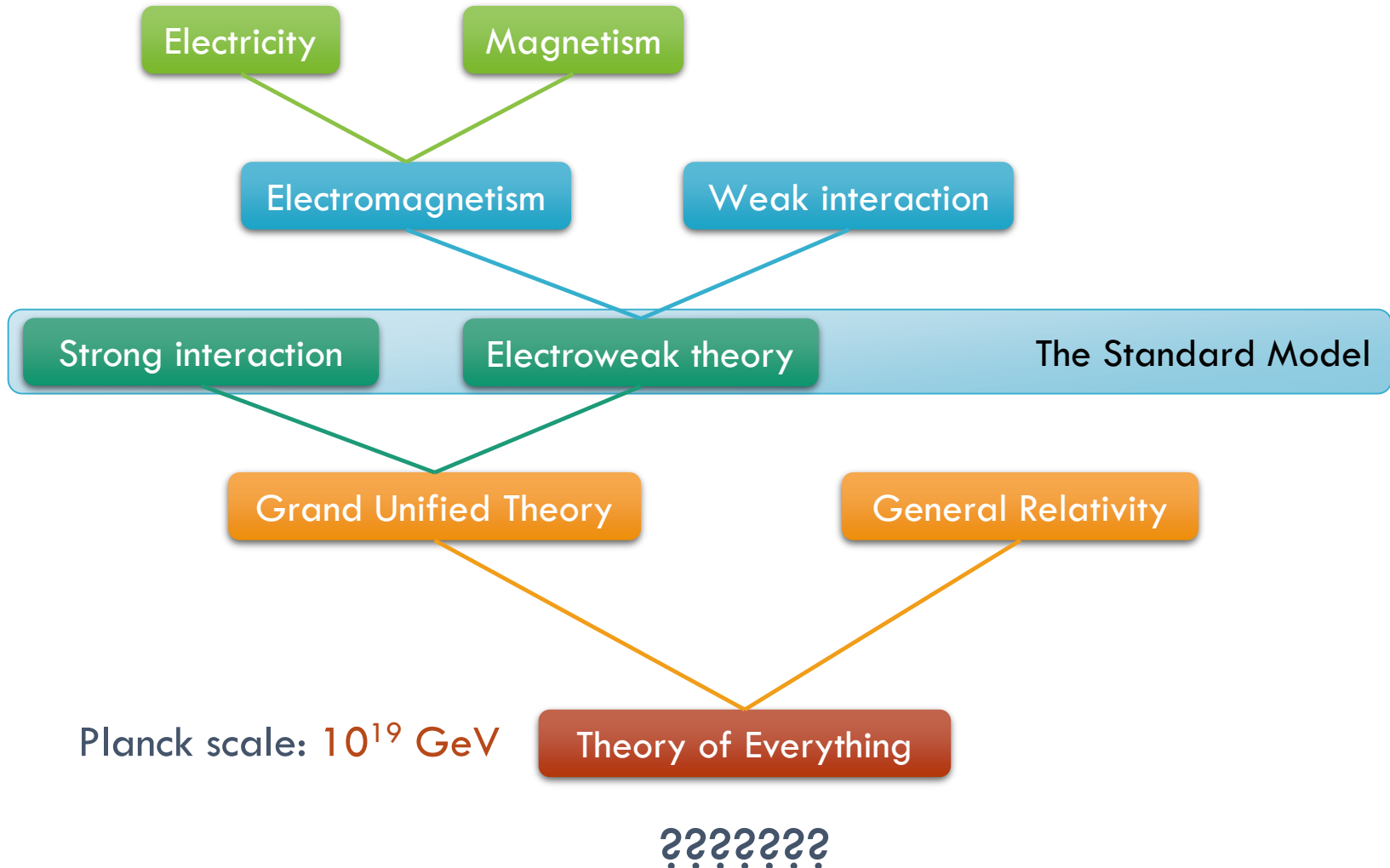
Albert Einstein
1925



A THEORY OF EVERYTHING?



A THEORY OF EVERYTHING?



A THEORY OF EVERYTHING?

- No one knows how to build a theory of everything.
 - General relativity and quantum mechanics are fundamentally different:
 - GR predicts smooth spacetime; QM predicts vacuum fluctuations.
 - A Grand Unified Theory is probably an intermediate step.
- String theory / M-theory
 - Particles are replaced by one-dimensional “strings”.
 - Different vibration modes give rise to different particle properties.
 - Needs extra dimensions to work! (9+1; 10+1)
 - Looks like supersymmetry at low energies.
- Loop quantum gravity
 - Granular description of space – quantization!
 - Needs only three space dimensions...

STANDARD MODEL LIMITATIONS SUMMARY

- We know that the Standard Model is **incomplete**: it does not **fully** account for experimental **observations**.
 - It doesn't describe **gravity** – and it seems **very difficult** to extend it in order to do so.
 - There isn't a unique way of describing **neutrino mass** in the SM context.
 - It doesn't provide a candidate for **dark matter**.
- Offers **unsatisfactory** answers to some **fundamental** questions:
 - Doesn't **predict** particle **masses**.
 - Or why there are **three** generations of matter.
 - Very large differences in the **scale** of some parameters remain unexplained.
 - Including some “lucky” **cancelations**.

SCHEDULE

- ~~1. Introduction~~
- ~~2. History of Particle Physics~~
- ~~3. Special Relativity~~
- ~~4. Quantum Mechanics~~
- ~~5. Experimental Methods~~
- ~~6. The Standard Model - Overview~~
- 7. The Standard Model - Limitations**

8. Neutrino Theory
 9. Neutrino Experiment
- } José

10. LHC and Experiments
 11. The Higgs Boson and Beyond
- } Inês

12. Particle Cosmology
- } Cris

NO CLASS NEXT WEEK
NEXT CLASS: MARCH 25th

BONUS

INTEGRAL OVER “ALL POSSIBLE PATHS”

- Where does the integral notion come from?
- Recall, QM picture of free particle motion: there is some amplitude for a free electron to travel along any path from the source to some point p . Not just the straight, classical trajectory!
 - The word “path” here doesn’t only refer to a $x(y)$ path in space, but also the time at which it passes each point in space.
 - In 3D, a path (sometimes called worldline) is defined by three functions $x(t)$, $y(t)$ and $z(t)$. An electron has an amplitude associated with a given path.
- The total amplitude for the electron to arrive at some final point is the sum of the amplitudes of all possible paths. Since there are an infinite number of paths, the sum turns into an integral.

THE QUANTUM MECHANICAL AMPLITUDE

- *Feynman: Each path has a corresponding probability amplitude. The amplitude ψ for a system to travel along a given path $x(t)$ is:*

$$\psi[x(t)] = \text{const.} \cdot e^{iS[x(t)]/\hbar}$$

where the object $S[x(t)]$ is called the **action** corresponding to $x(t)$.

- The total amplitude is the sum of contributions from each path:

$$\sum_{\text{over all paths}} \psi[x(t)]$$

THE QUANTUM MECHANICAL AMPLITUDE

$$\psi[x(t)] = \text{const.} \cdot e^{iS[x(t)]/\hbar}$$

1. What is $e^{iS[x(t)]/\hbar}$?
2. What is the action $S[x(t)]$?

UNDERSTANDING THE PHASE

- You may not have seen numbers like $e^{i\theta}$, so let's review.
- Basically, $e^{i\theta}$ is just a fancy way of writing sinusoidal functions; from Euler's famous formula:

$$e^{i\theta} = \cos \theta + i \sin \theta$$

- Note: those of you familiar with complex numbers (of the form $z=x+iy$) know that $e^{i\theta}$ is the phase of the so-called polar form of z , in which $z=re^{i\theta}$, with:

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

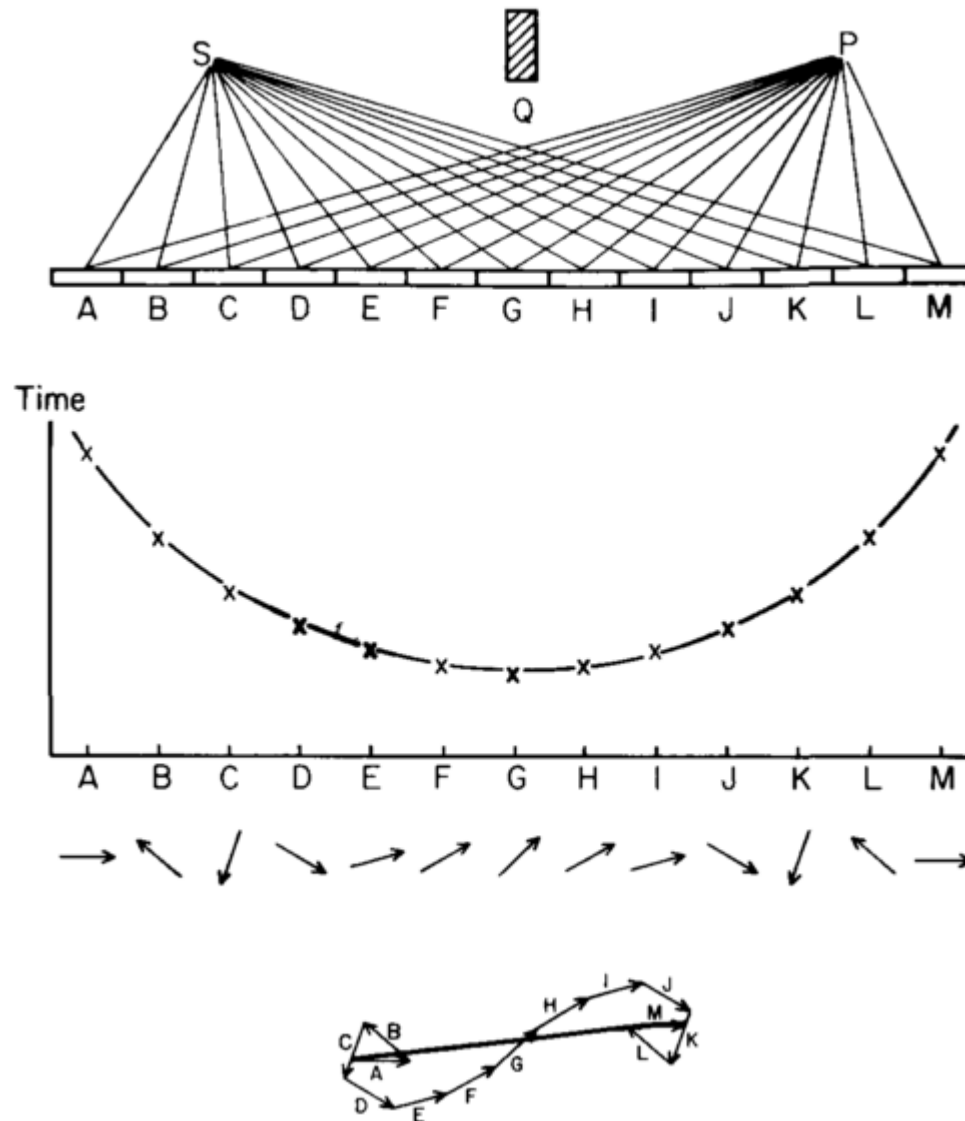
COMMENTS ON THE AMPLITUDE

- Now we can understand the probability amplitude $\psi[x(t)] \sim e^{iS[x(t)]/\hbar}$ a little better.
- The amplitude is a sinusoidal function – a wave – that oscillates along the worldline $x(t)$. The frequency of oscillation is determined by how rapidly the action S changes along the path.
- The probability that a particle will take a given path (up to some overall multiplication constant) is:

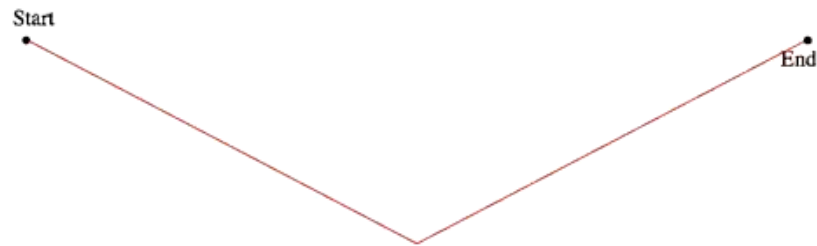
$$\begin{aligned} P &\propto |\psi|^2 = \psi^* \psi \\ &\propto e^{-iS[x(t)]/\hbar} e^{iS[x(t)]/\hbar} \\ &= e^{iS[x(t)]/\hbar - iS[x(t)]/\hbar} = e^0 \\ &= 1 \end{aligned}$$

- This is the same for every worldline. According to Feynman, the particle is equally likely to take any path through space and time!
 - Contributions from “crazy” paths will likely be suppressed by interference!

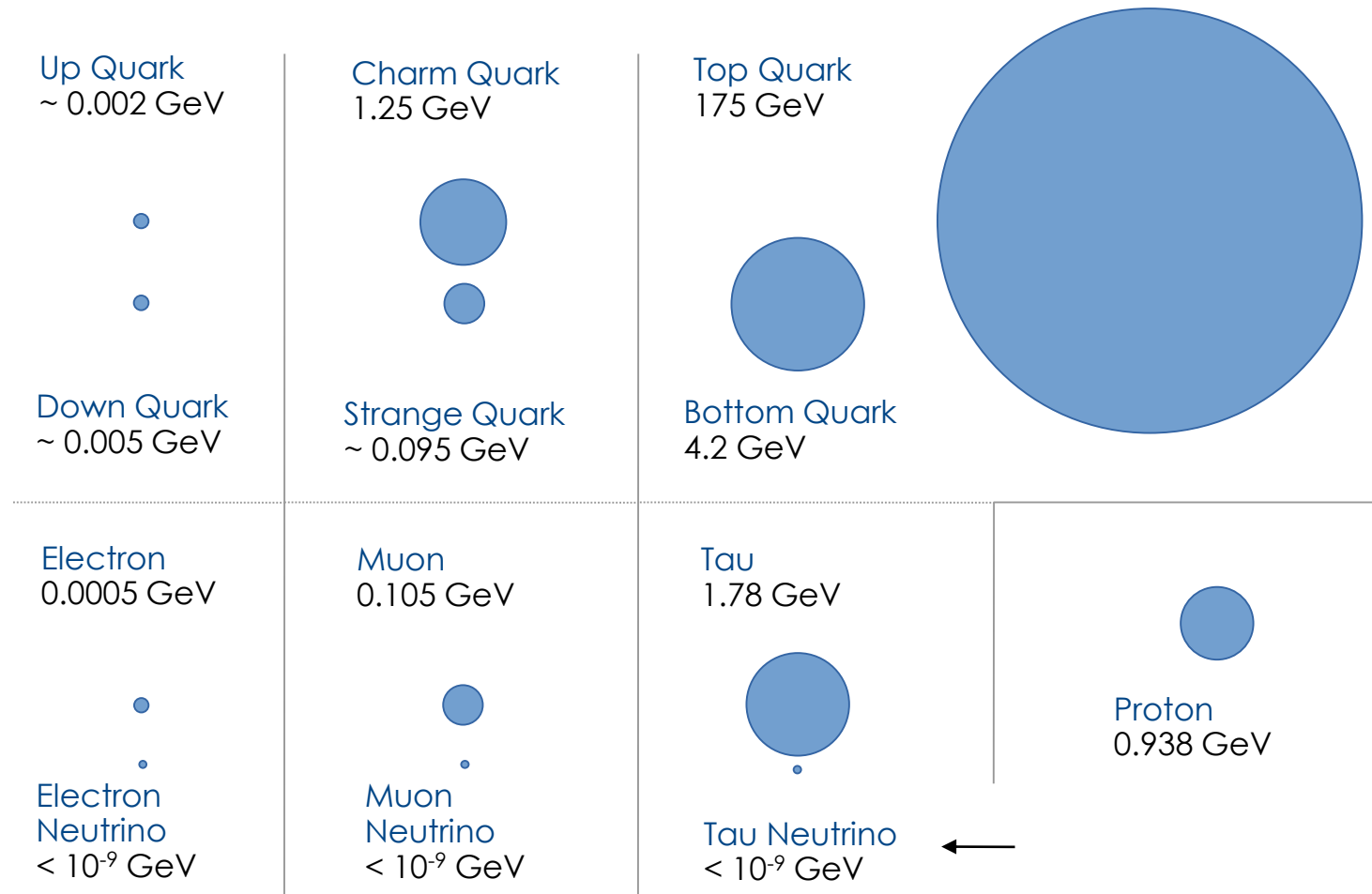
COMMENTS ON THE AMPLITUDE



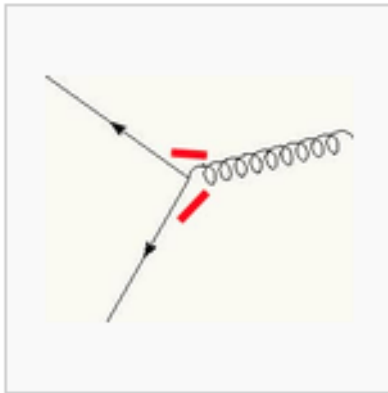
COMMENTS ON THE AMPLITUDE



MASSES OF SM FERMIONS



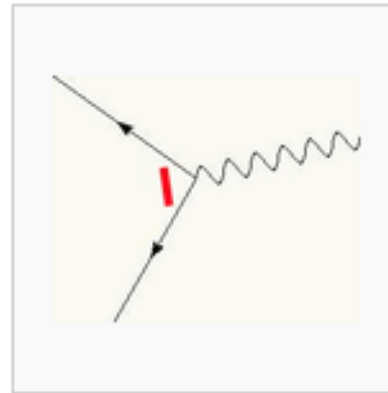
GLUON DISCOVERY AT TASSO, PETRA



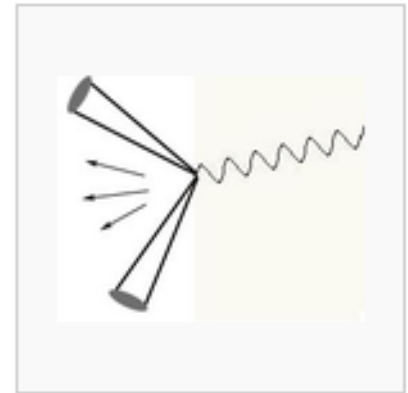
Two **quarks** (solid lines) and a **gluon** (curly line) fly apart, with the **strings** (red bars) primarily between the gluon and each quark.



As a result, three **jets** (cones) form, with extra **hadrons** (arrows) found where the strings formed.



For comparison, physicists looked at events with two quarks and a **photon** (wavy line). Here the string forms only between quarks.



Therefore extra hadrons are found only between the two jets, which is inconsistent with observations.