PARTICLE PHYSICS

COLUMBIA SCIENCE HONORS PROGRAM

WEEK 6

OVERVIEW OF THE STANDARD MODEL

Cristóvão Vilela

COURSE POLICIES

- Attendance
  - Up to four excused absences
    - Two with notes from parent/guardian
    - shpattendance@columbia.edu
  - Valid excuses:
    - Illness, family emergency, tests or athletic/academic competitions, mass transit breakdowns
  - Invalid excuses:
    - Sleeping in, missing the train…
  - I will take attendance during class
- No cell phones
- Ask questions!

LECTURE MATERIALS

- https://twiki.nevis.columbia.edu/twiki/bin/view/Main/ScienceHonorsProgram
- Questions: cristovao.vilela@stonybrook.edu

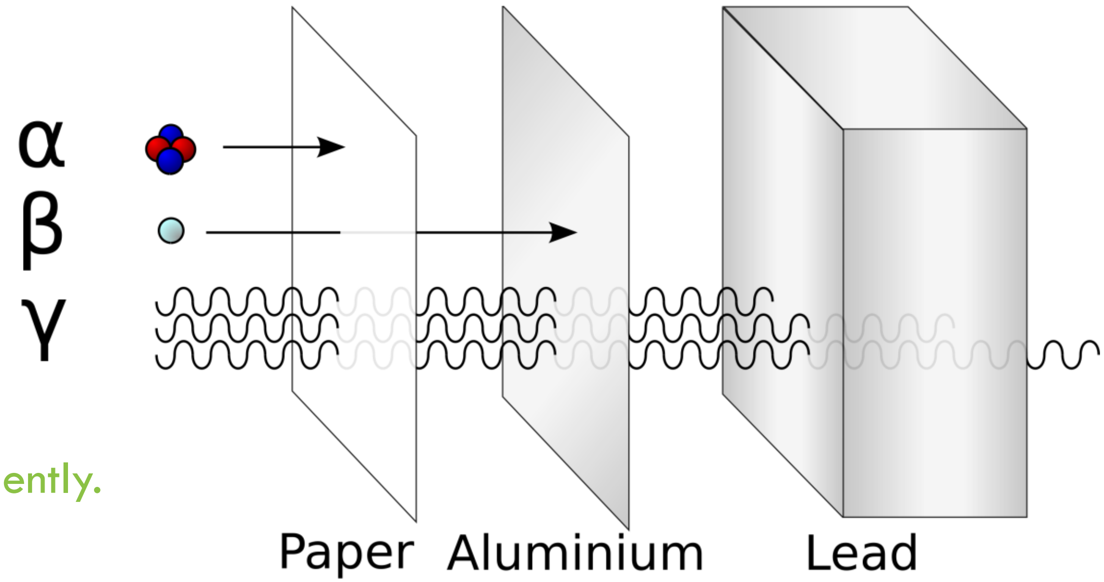
LAST WEEK…

EXPERIMENTAL METHODS

HOW WE STUDY PARTICLES IN THE LAB

PASSAGE OF PARTICLES THROUGH MATTER

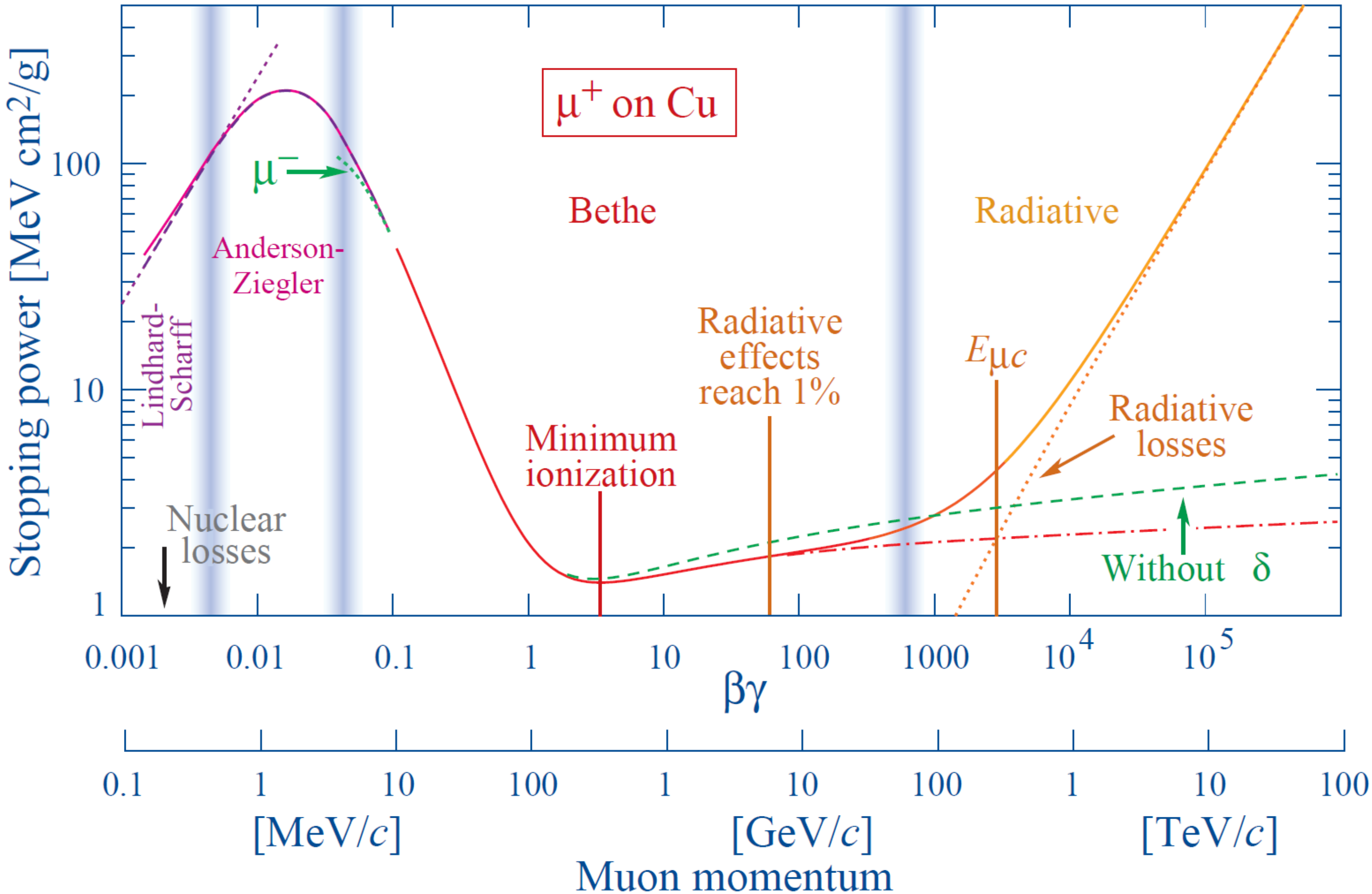
- Alpha, beta, gamma radiation:
  - Classified according to how they bend in a magnetic field.
  - Also differentiated by how easily they can be stopped.



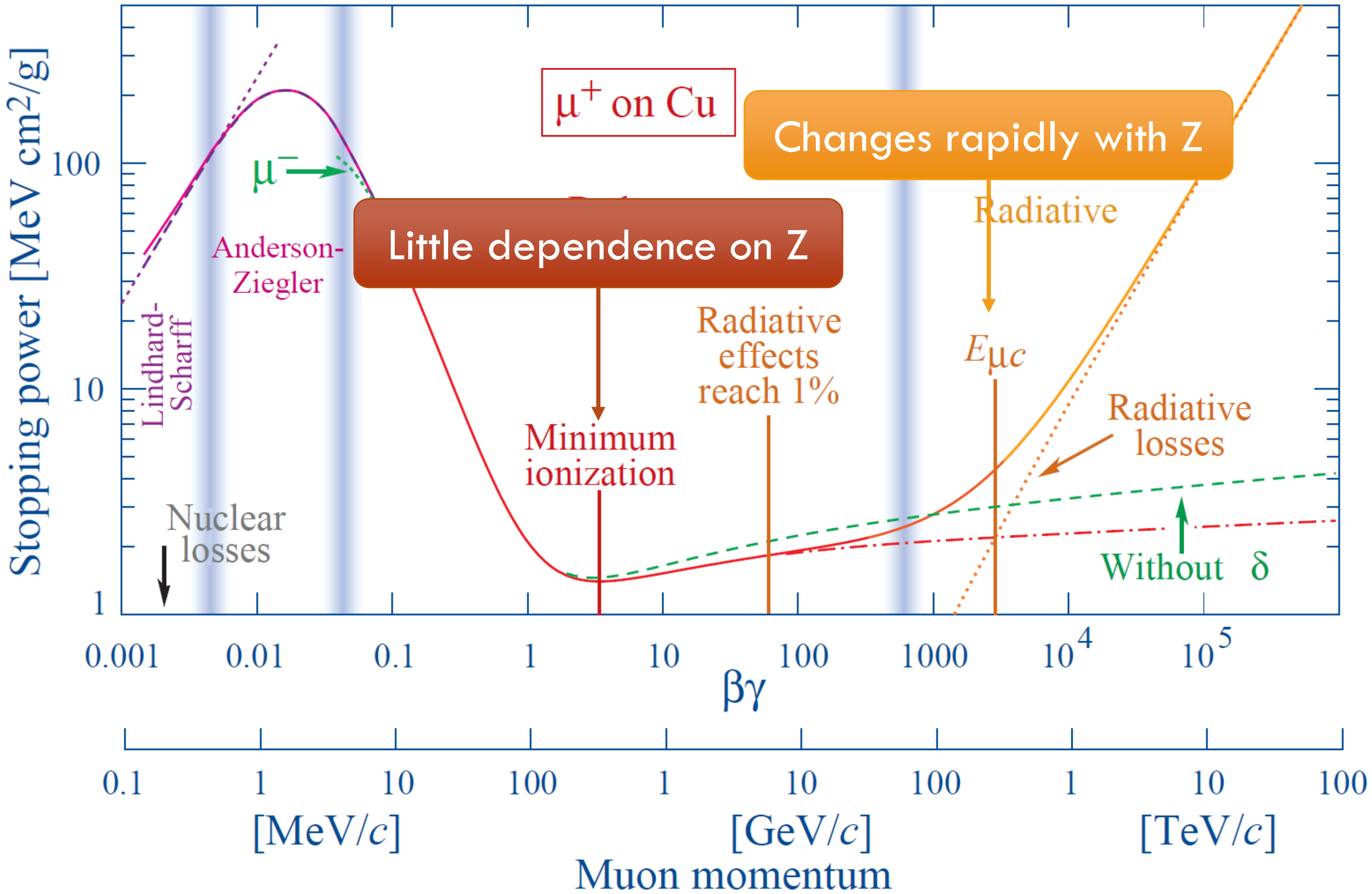
Charged particles interact more frequently. They ionize matter (directly) and lose energy in the process.

Neutral particles interact less often, losing less energy. They ionize matter indirectly.

STOPPING POWER



STOPPING POWER



PARTICLE DETECTION

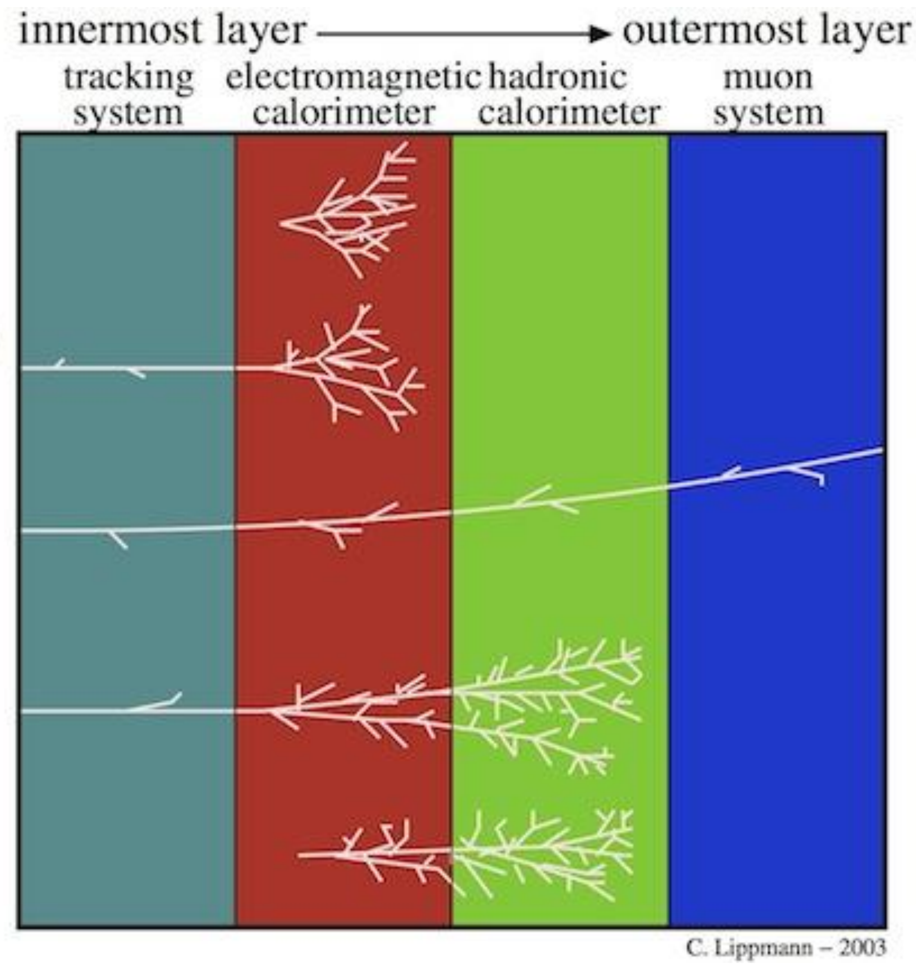
Photons
No ionization in low density tracker

Electrons
Ionize tracker, radiative losses lead to shower

Muons
Minimum ionizing particles, little energy loss

Charged hadrons
Fully absorbed in dense hadronic calorimeter

Neutral hadrons
No direct ionization

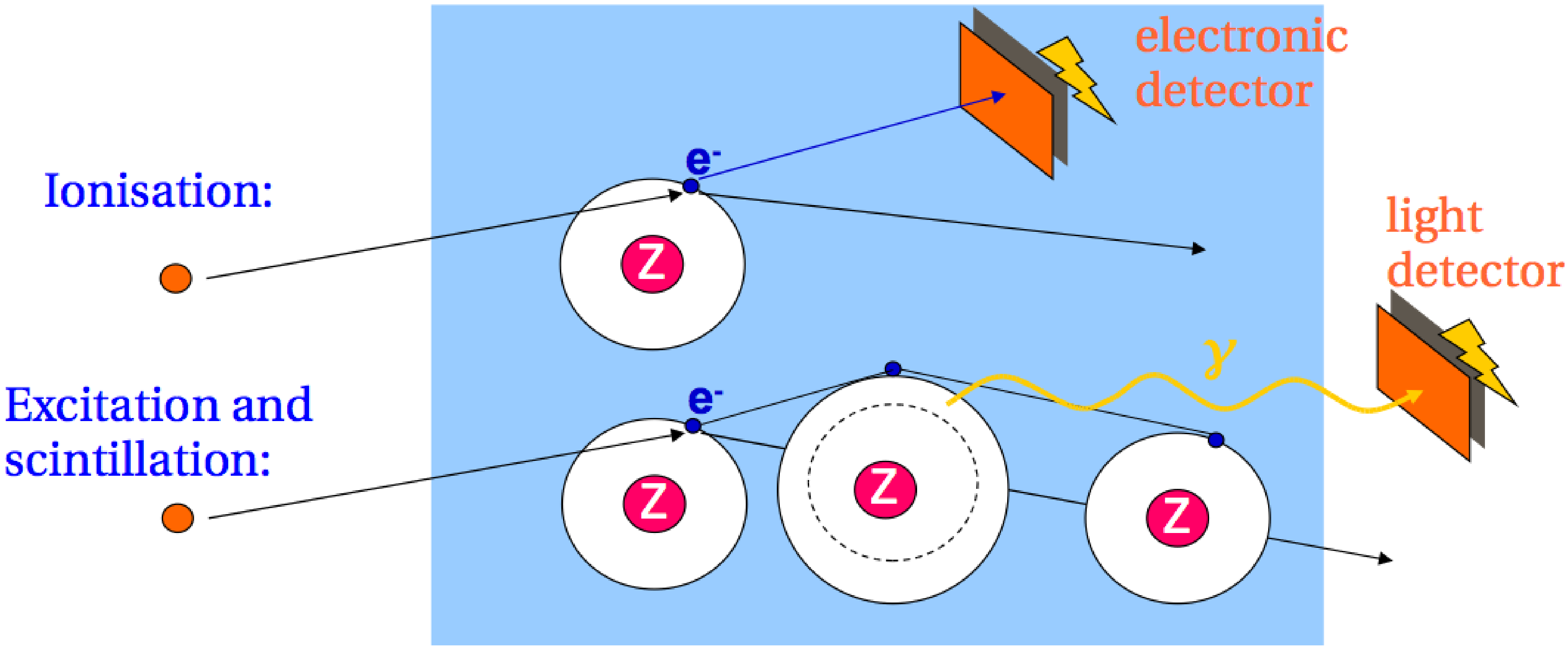


Non-destructive measurement

Destructive measurements

PARTICLE DETECTION
1. MEASUREMENT BY ELECTROMAGNETIC ENERGY LOSS

- Applies to all charged particles



IONIZATION

Bethe energy loss formula

$$-\frac{dE}{dx} = \frac{4\pi}{m_e c^2} \cdot \frac{nz^2}{\beta^2} \cdot \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \left[ln\left(\frac{2m_e c^2\beta^2}{I\cdot(1-\beta^2)}\right) - \beta^2\right]$$

$-\frac{dE}{dx}$ Energy loss per distance traveled

$\beta = \frac{v}{c}$ Particle velocity

z Particle charge (in units of electron charge)

n Density of electrons in material

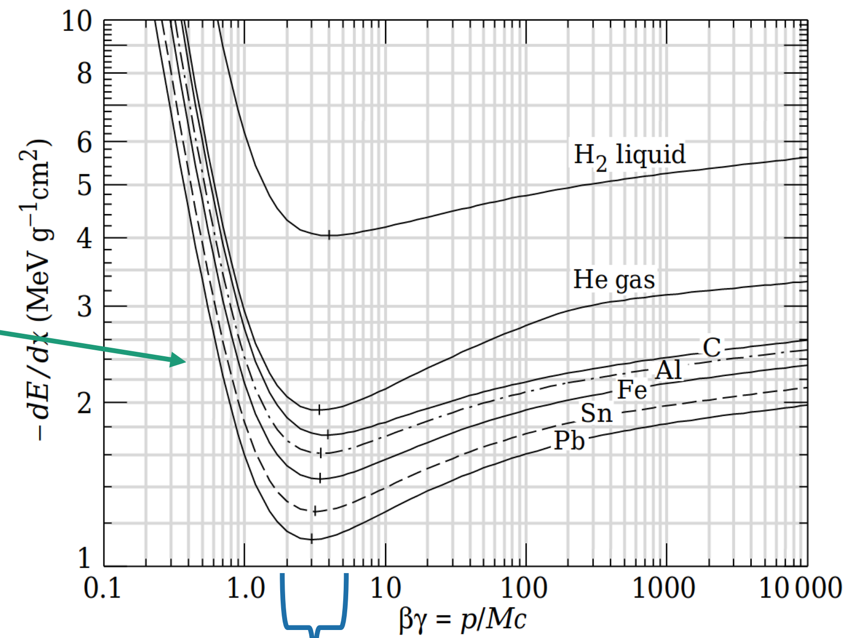
I Mean excitation potential of material

ε_0 Vacuum permittivity

e Electron charge

m_e Electron mass

c Speed of light in vacuum



Minimum Ionizing Particles

IONIZATION

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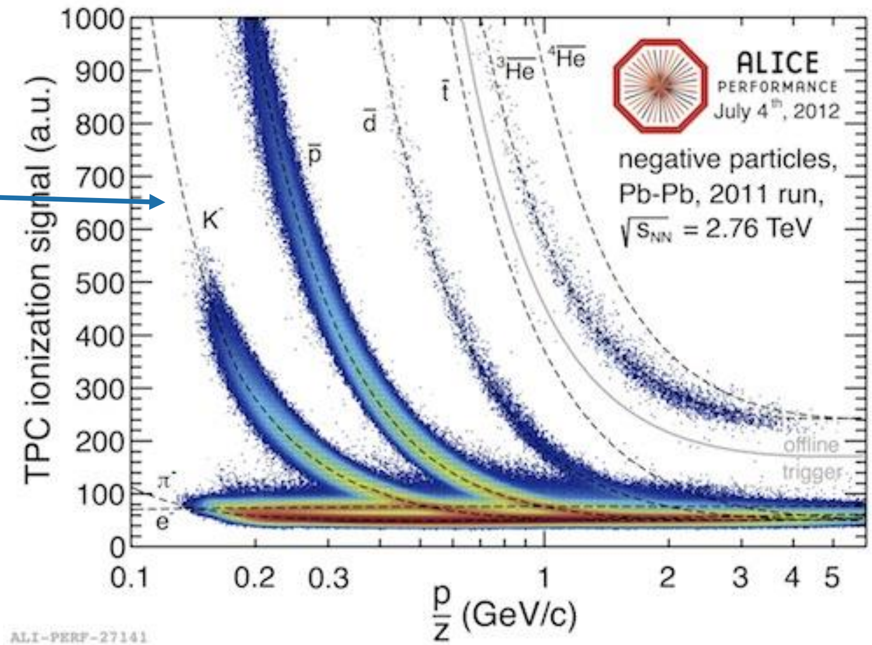


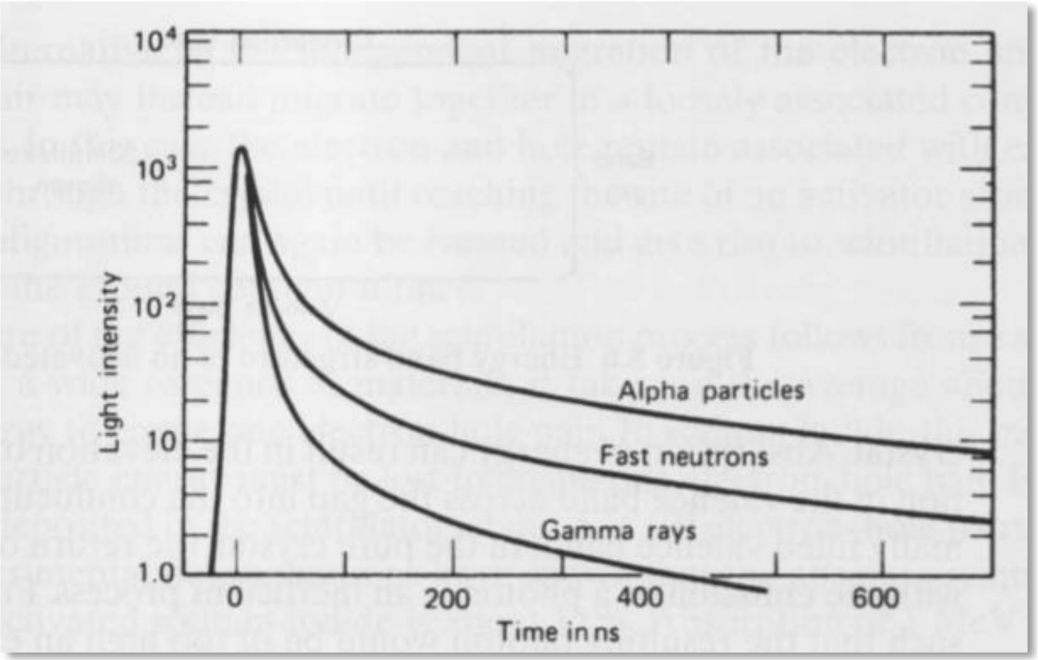
$$p = \frac{v}{\sqrt{1-\beta^2}} m$$

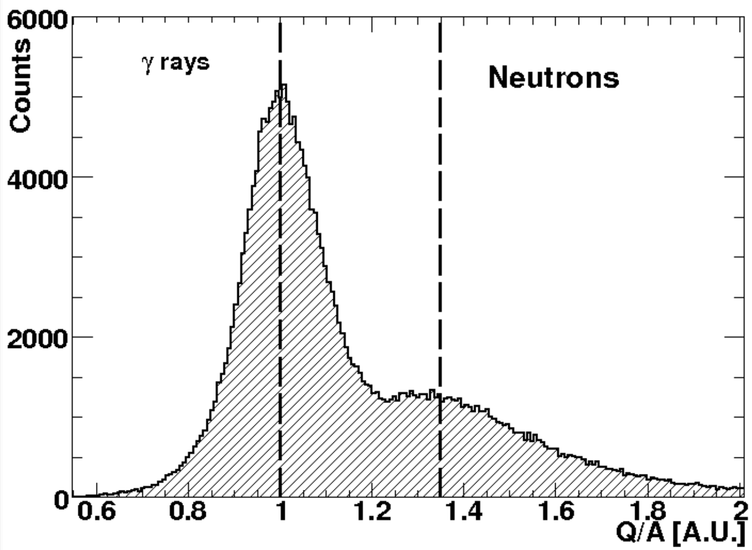
SCINTILLATION

- Scintillation is the emission of light of a characteristic wavelength spectrum, following the absorption of radiation.
  - The emitted radiation is usually less energetic than that absorbed.
- Scintillation occurs in:
  - Some types of organic molecules with complicated electronic structures
    - p-Terphenyl: $C_{18}H_{14}$
    - "PPO": $C_{15}H_{11}NO$
  - Inorganic crystals and gases / liquids
    - NaI, CaF_2
    - He, Ar, Xe
- Particles with different dE/dx populate fast and slow states differently

SCINTILLATION

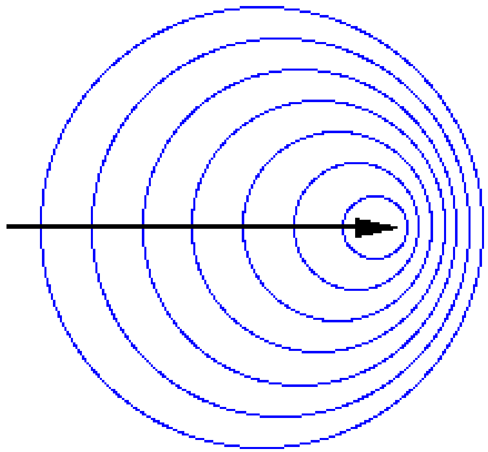
- Pulse shapes can be used to discriminate among different particle types:



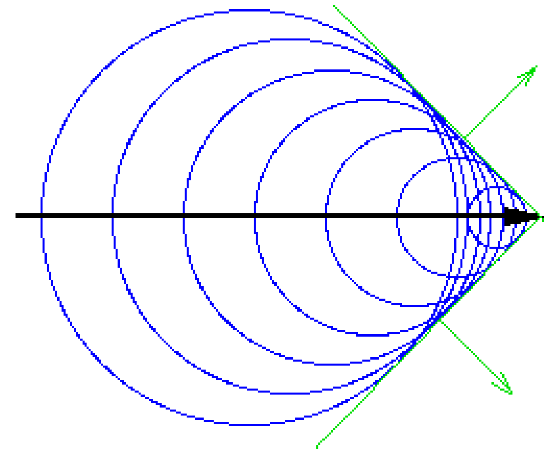


CHERENKOV RADIATION

- Cherenkov effect: a charged particle moving faster than the speed of light in a medium (v>c/n) emits Cherenkov radiation.



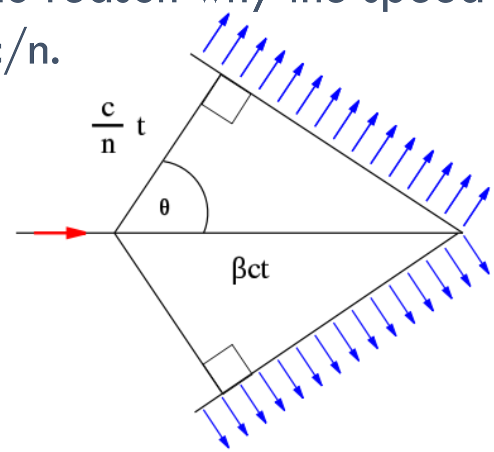
As a particle passes through matter, the surrounding atoms polarize and depolarize, and a weak electromagnetic wave spreads out from the position of the particle.
For a particle traveling more slowly than light, wave-fronts originating at different times can never meet, and no interference is possible.



For a particle traveling faster than light, the wave-fronts do overlap, and constructive interference is possible, leading to a significant, observable signal.

CHERENKOV RADIATION

- A particle can not, of course, travel faster than the speed of light in vacuum.
- In a medium of refractive index n, the speed of light is c/n, and there is no reason why the speed of the particle, βc, cannot be greater than c/n.

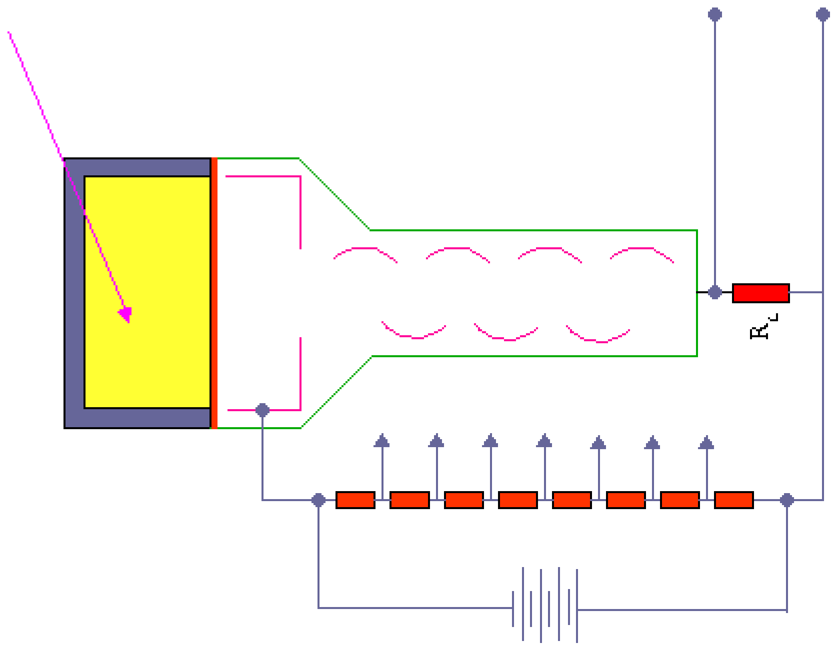


$$cos\theta_c = \frac{\frac{c}{n}t}{\beta ct} = \frac{1}{\beta n}$$

- A highly relativistic particle passing through a medium is observed to emit visible light known as Cherenkov radiation if $\beta > 1/n$. As can be seen from the above diagram, a cone of light radiates out from each point on the particle's track.
- The Cherenkov cone angle is related to the particle's β.

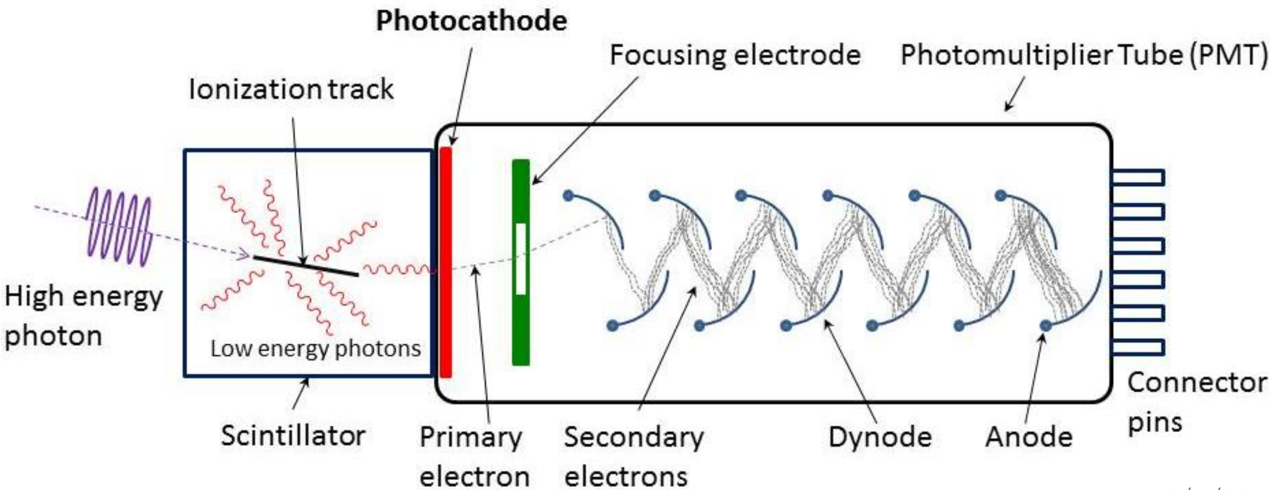
LIGHT DETECTION

- A photomultiplier tube (PMT) is a commonly used instrument for detecting visible photons.
- Basic of operation: photoelectric effect
  - Single photons converted to electrons and multiplied to a measurable electronic signal.



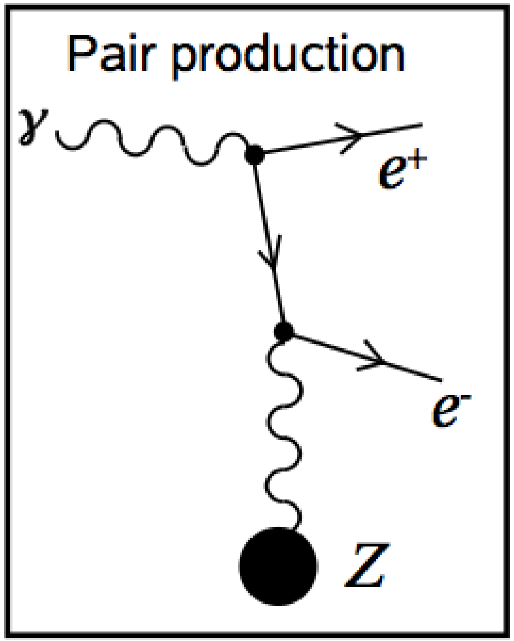
LIGHT DETECTION

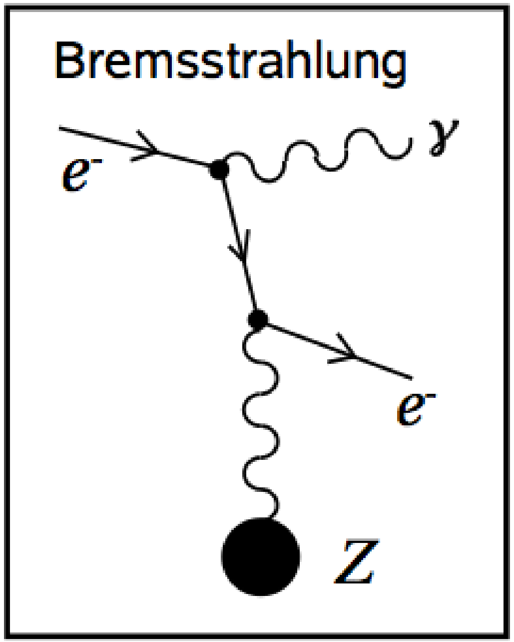
- Light falls on a photocathode and a photoelectron is emitted (photoelectric effect).
  - Quantum Efficiency depends on cathode and wavelength (QE~25%).
- Photoelectron focused and accelerated towards the first dynode by electric field.
- Photoelectron strikes dynode and several electrons are emitted (on average n~5).
- Several dynodes (~10) give high gain (10^7).
- High speed: few ns transit time!
- Gain can be much lower in magnetic fields, depending on orientation.



PAIR PRODUCTION AND BREMMSTRAHLUNG

- Pair production and Bremsstrahlung radiation are complementary processes: both lead to electromagnetic showers.

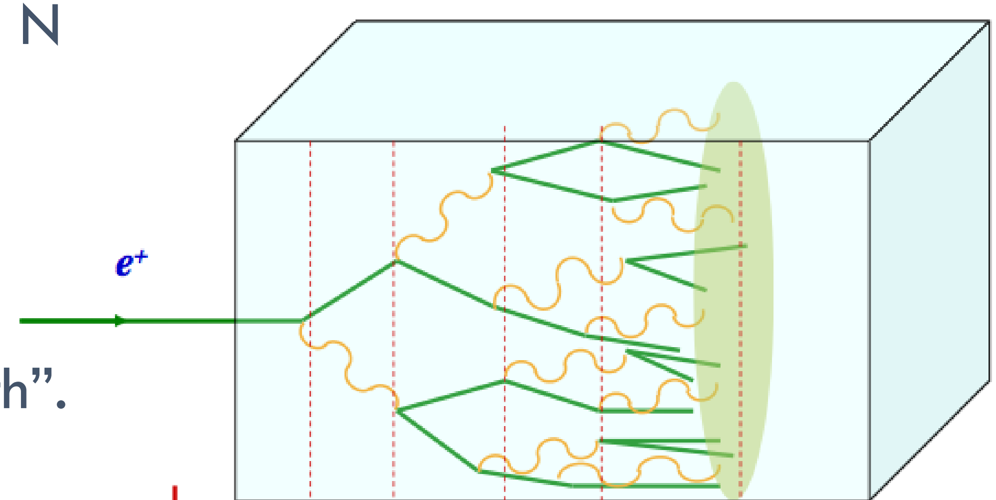




- Very similar Feynman diagrams
- Just two arms swapped
- At high energy: $\sigma_\gamma = \frac{7}{9}\sigma_e$

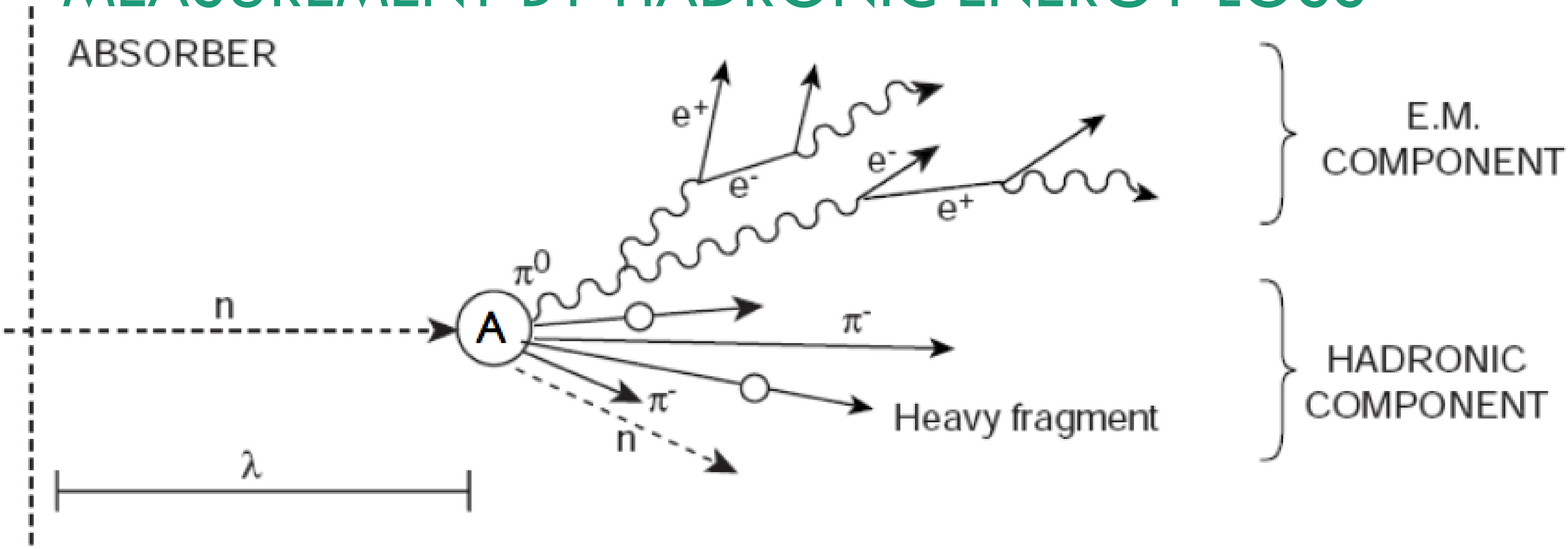
ELECTROMAGNETIC SHOWERS

- The number of particles increases as a 2^N, where N is the number of X_0 over which the shower has developed.
- X_0 is the "radiation length".
- The length of the shower depends on the primary electron energy.



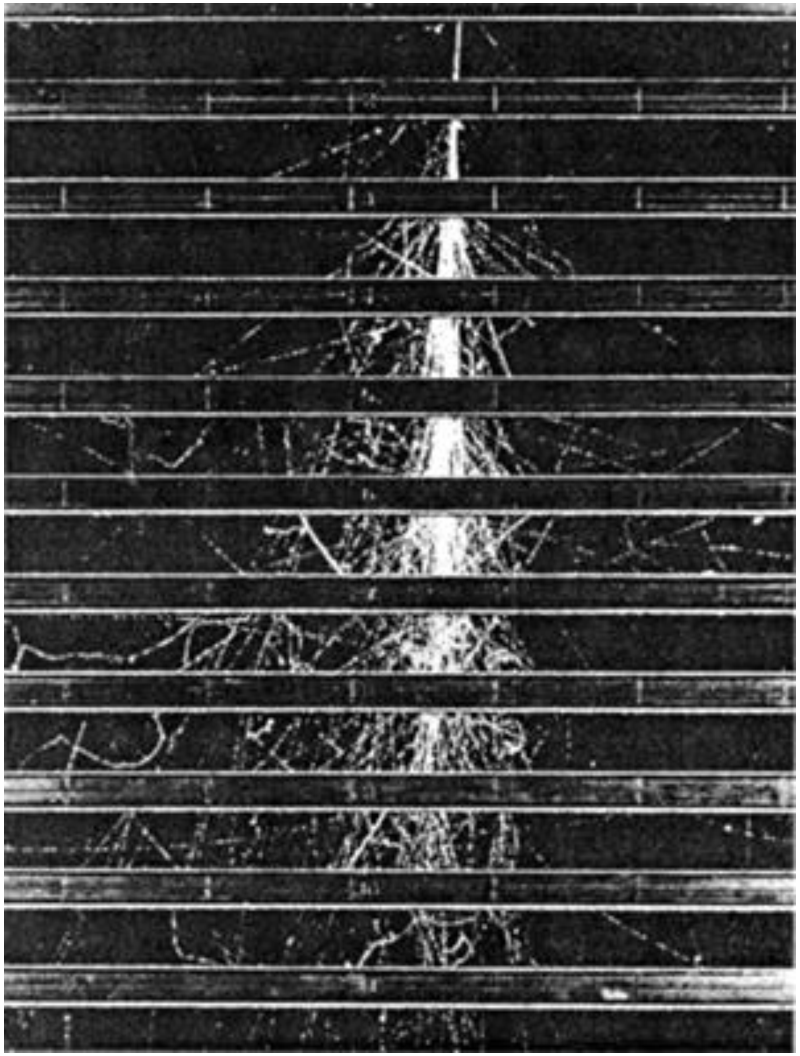
x	0	X_0	$2X_0$	$3X_0$	$4X_0$	
N	1	2	4	8	16	0
$\langle E \rangle$	E_0	$E_0/2$	$E_0/4$	$E_0/8$	$E_0/16$	$<E_c$

PARTICLE DETECTION
MEASUREMENT BY HADRONIC ENERGY LOSS



- Hadronic interactions have high multiplicity:
  - Shower is to 95% contained in ~7λ at 50 GeV (1.2 m of iron).
- Hadronic interactions produce π^0:
  - $\pi^0 \rightarrow \gamma\gamma$, leading to local EM showers.
- Some energy loss in nuclear breakup and neutrons (“invisible energy”)
- Stronger fluctuations in a hadronic shower:
  - Worse energy resolution.

HADRONIC VS EM SHOWERS

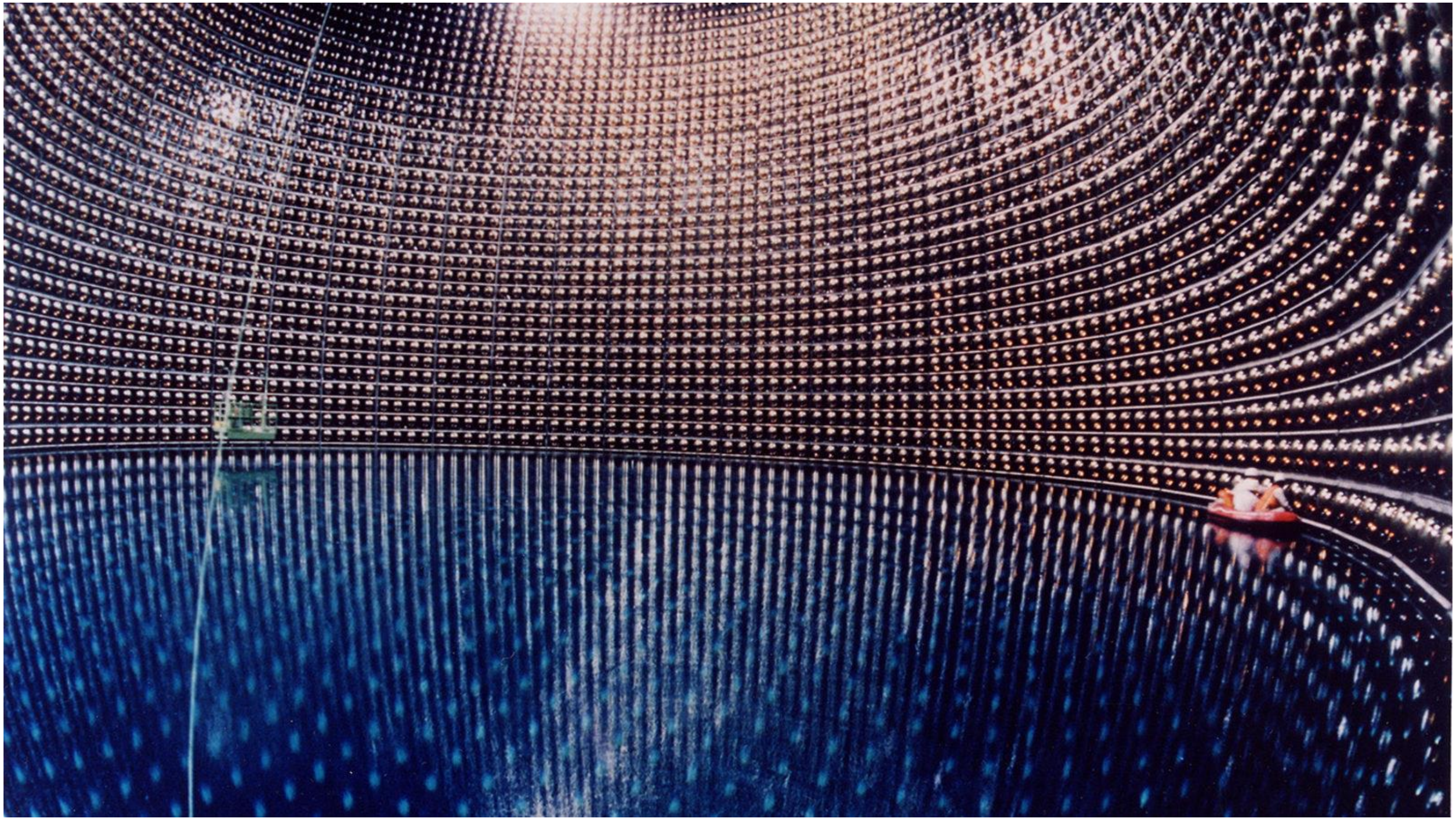


PARTICLE DETECTORS

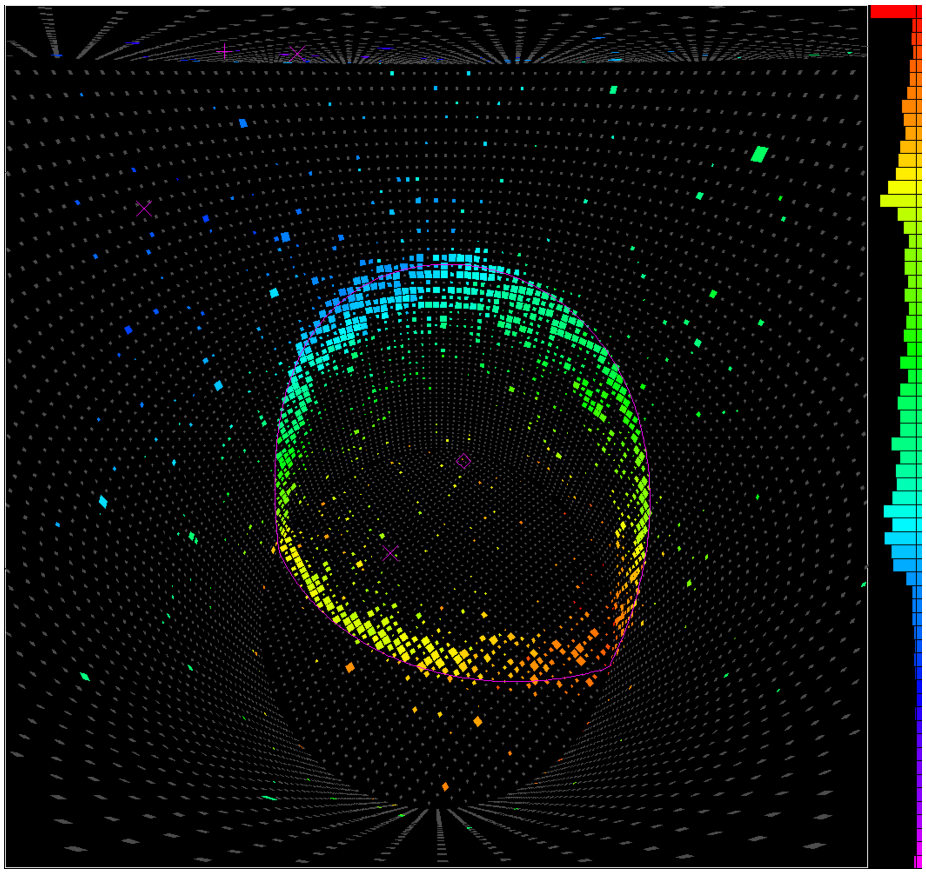
- Detectors usually specialize in:
  - Tracking: measuring positions / trajectories / momenta of charged particles, e.g.:
    - Silicon detectors
    - Drift chambers
  - Calorimetry: measuring energies of particles:
    - Electromagnetic calorimeters
    - Hadronic calorimeters
- But they can also be a combination.

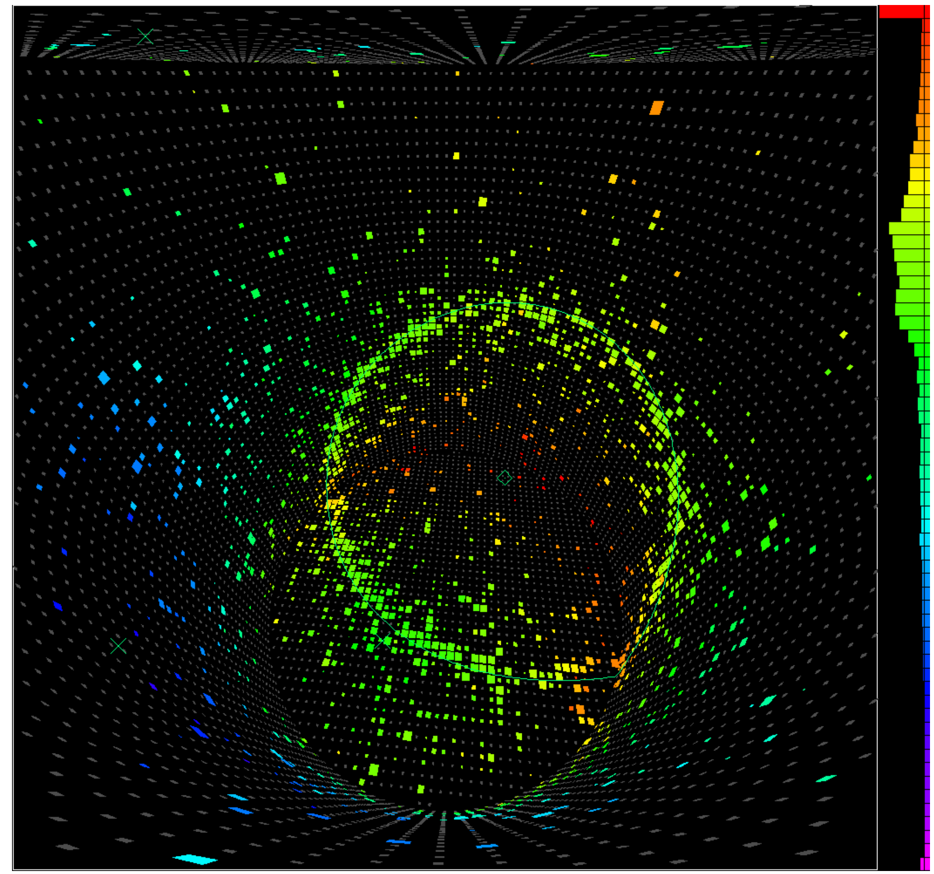
CHERENKOV DETECTORS

- Super-Kamiokande in Japan

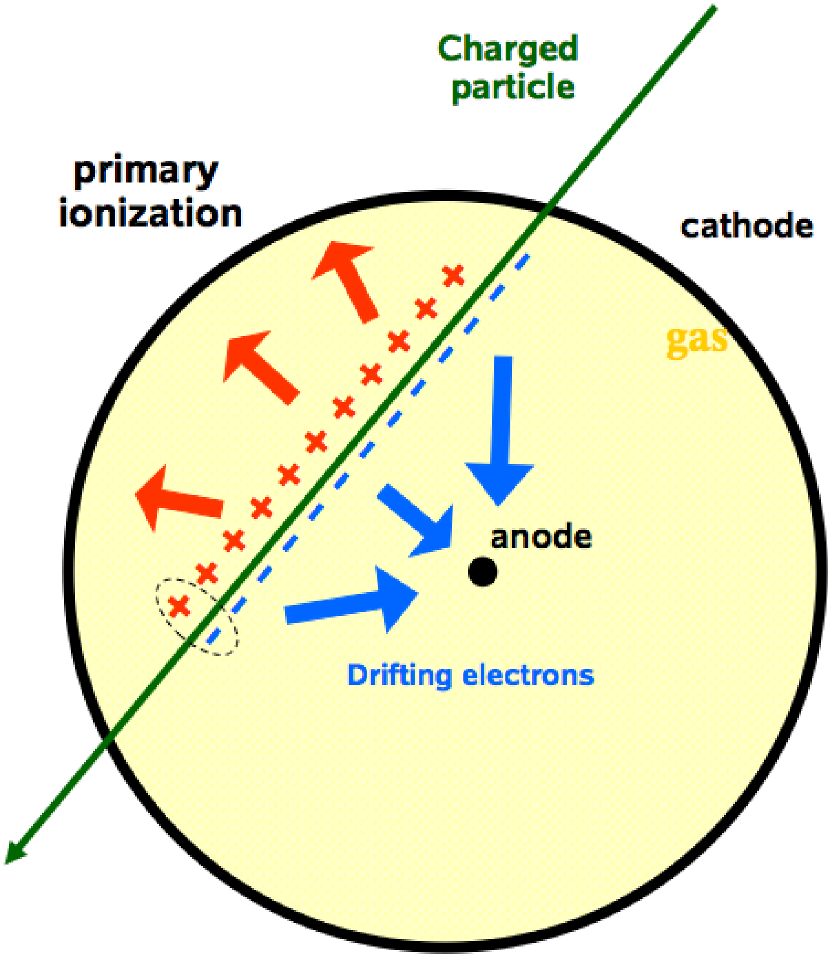


NEUTRINO DETECTION AT SUPER-K





IONIZATION DETECTOR

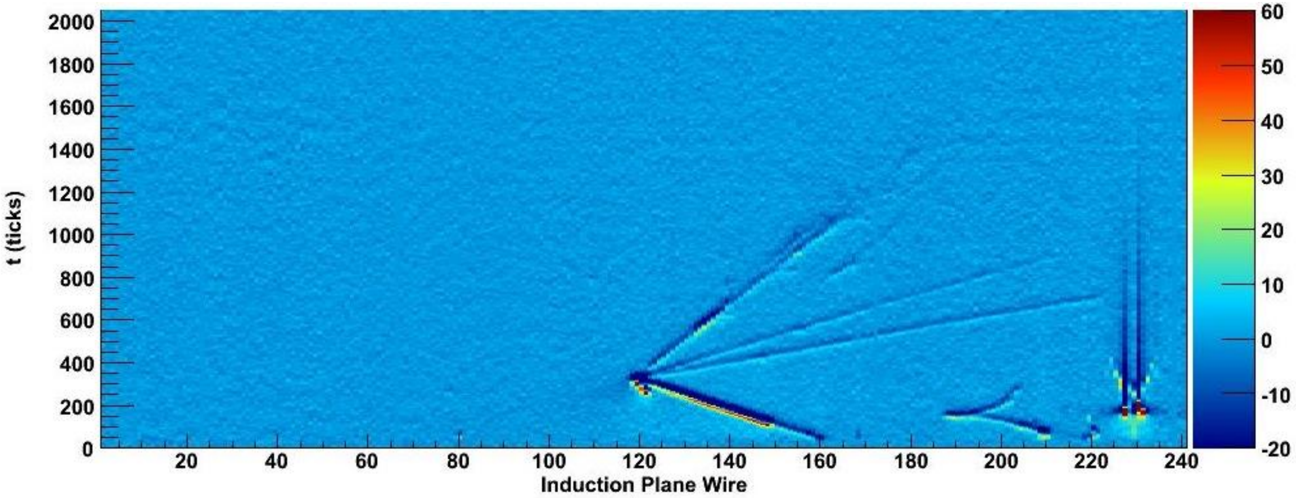


- Drifting electrons should not be trapped:
  - Use noble gas, e.g. Ar.
- Want large primary ionzation yield:
  - Ar gives 25 ions/cm at normal T, p for a minimm ionizing particle.
- The primary electrons may ionize further atoms:
  - $\times 3$ or $\times 4$ increase.
- Xe and/or higher pressure are even better (and more expensive).

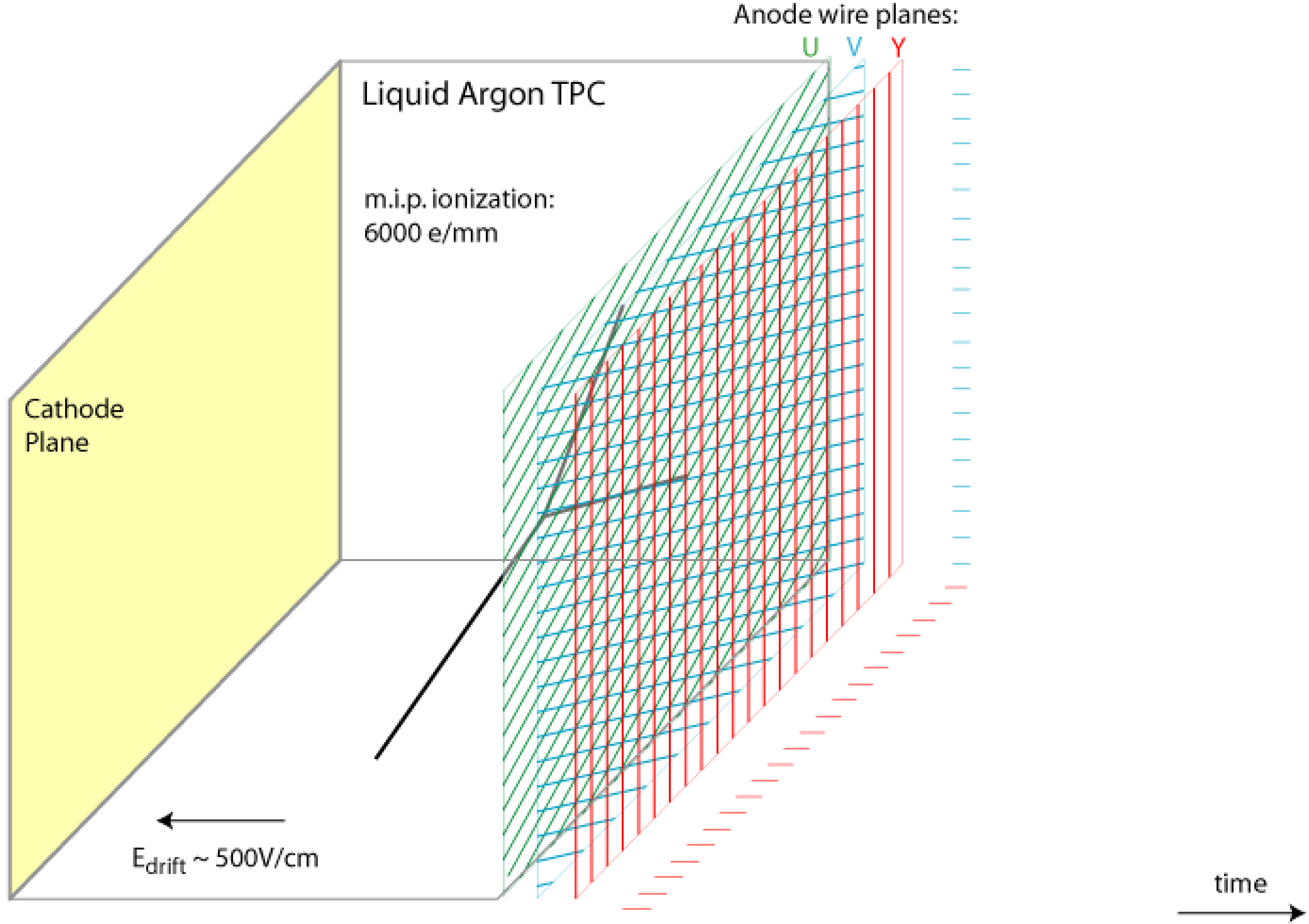
Induced electrical signal on anode can be measured to estimate number of drift electrons: E lost to ionization.

TIME PROJECTION CHAMBER

- Exploits ionization energy losses of charged particles.
- Electrons are drifted onto a fine grained plane of wires, and the particle trajectories can be mapped out, along with their ionization energy loss, dE/dx

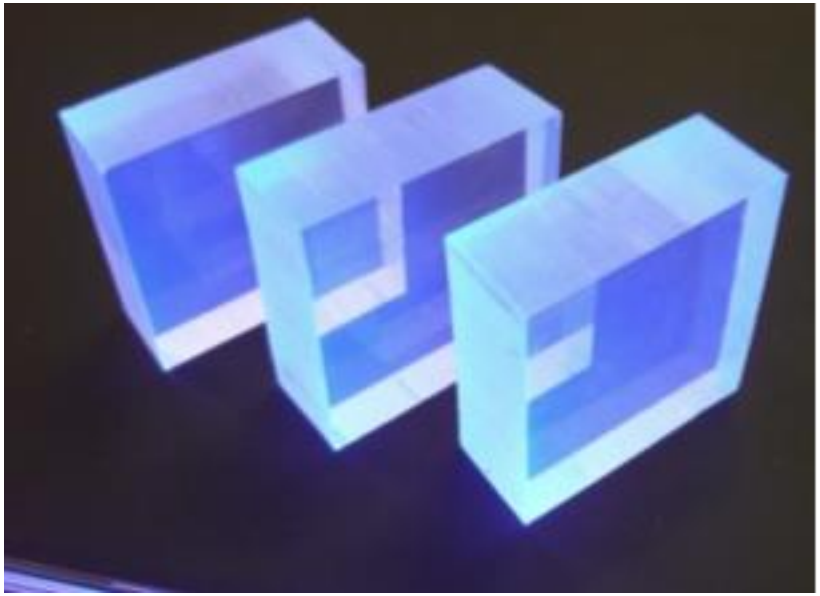


TIME PROJECTION CHAMBER



SCINTILLATION DETECTORS

- Emitted light depends on detector material.
  - Usually in the visible to UV range.
- Sometimes requires the use of wavelength-shifting materials to shift UV light to visible, so it can be efficiently measured by commonly used photomultiplier tubes.



SCINTILLATION DETECTORS

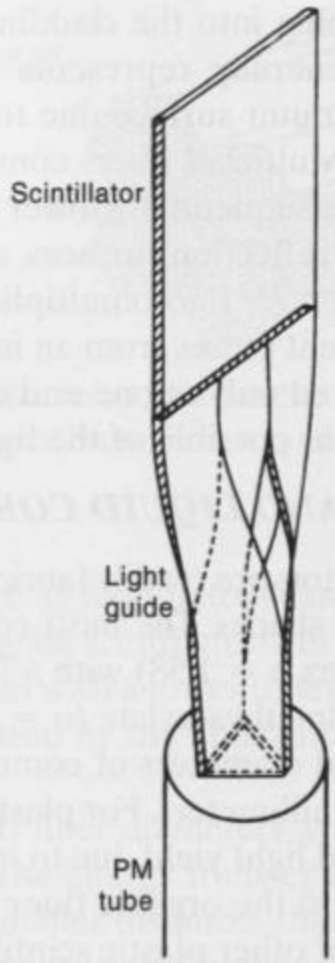
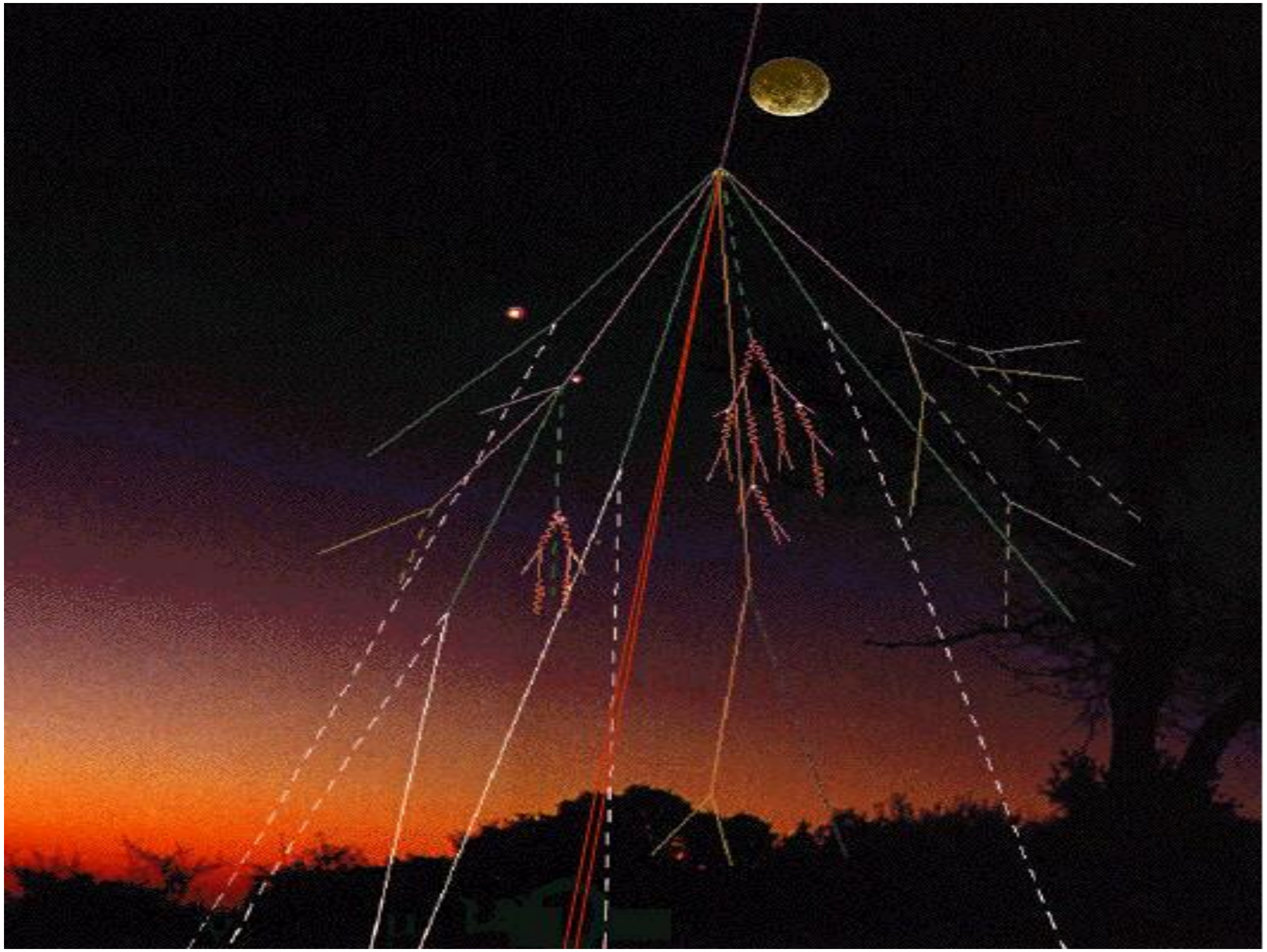


Figure 8.16 A strip light guide can be used to couple the edge of a large, flat scintillator to a PM tube.

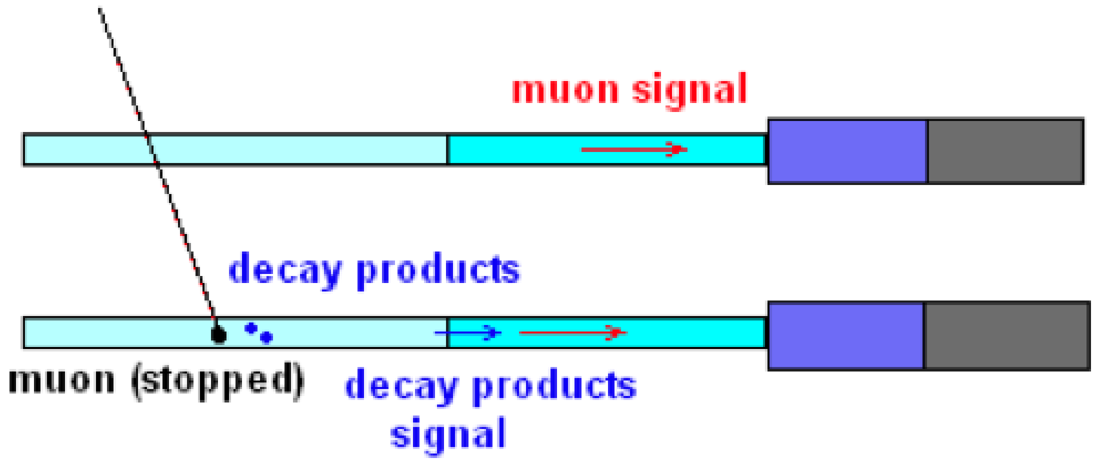
APPLICATION OF SCINTILLATION DETECTORS

- Cosmic ray muon detection



COSMIC RAY MUON DETECTION

- Measurement of the muon lifetime



1. Measure t_{decay} (difference between muon signal and decay signal in the second scintillator paddle) of a sample N_0 of low energy muons.

2. Fit the data to an exponential curve of the form:

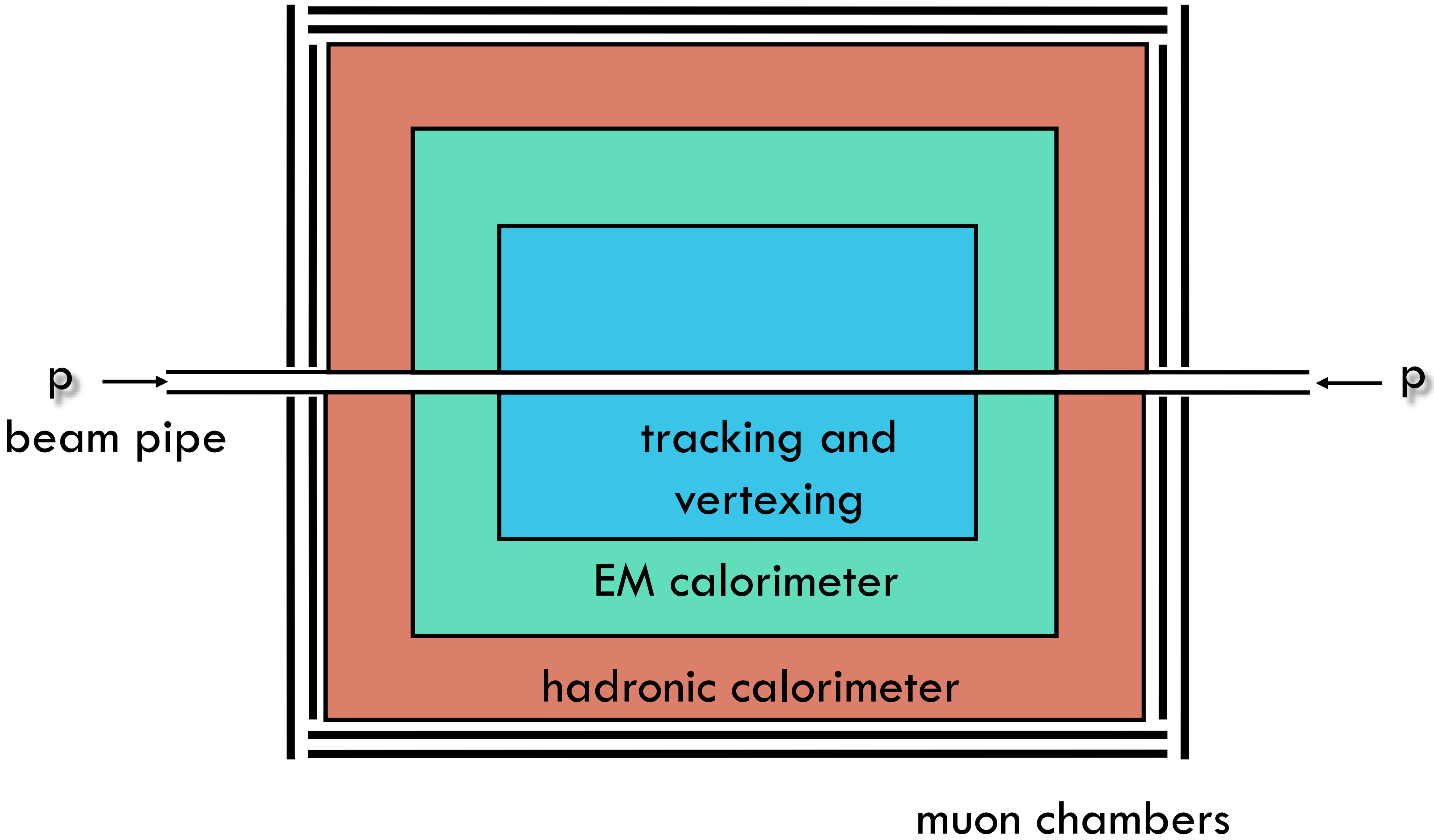
$$N(t) = N_0 e^{-t/T}$$

where T = muon lifetime

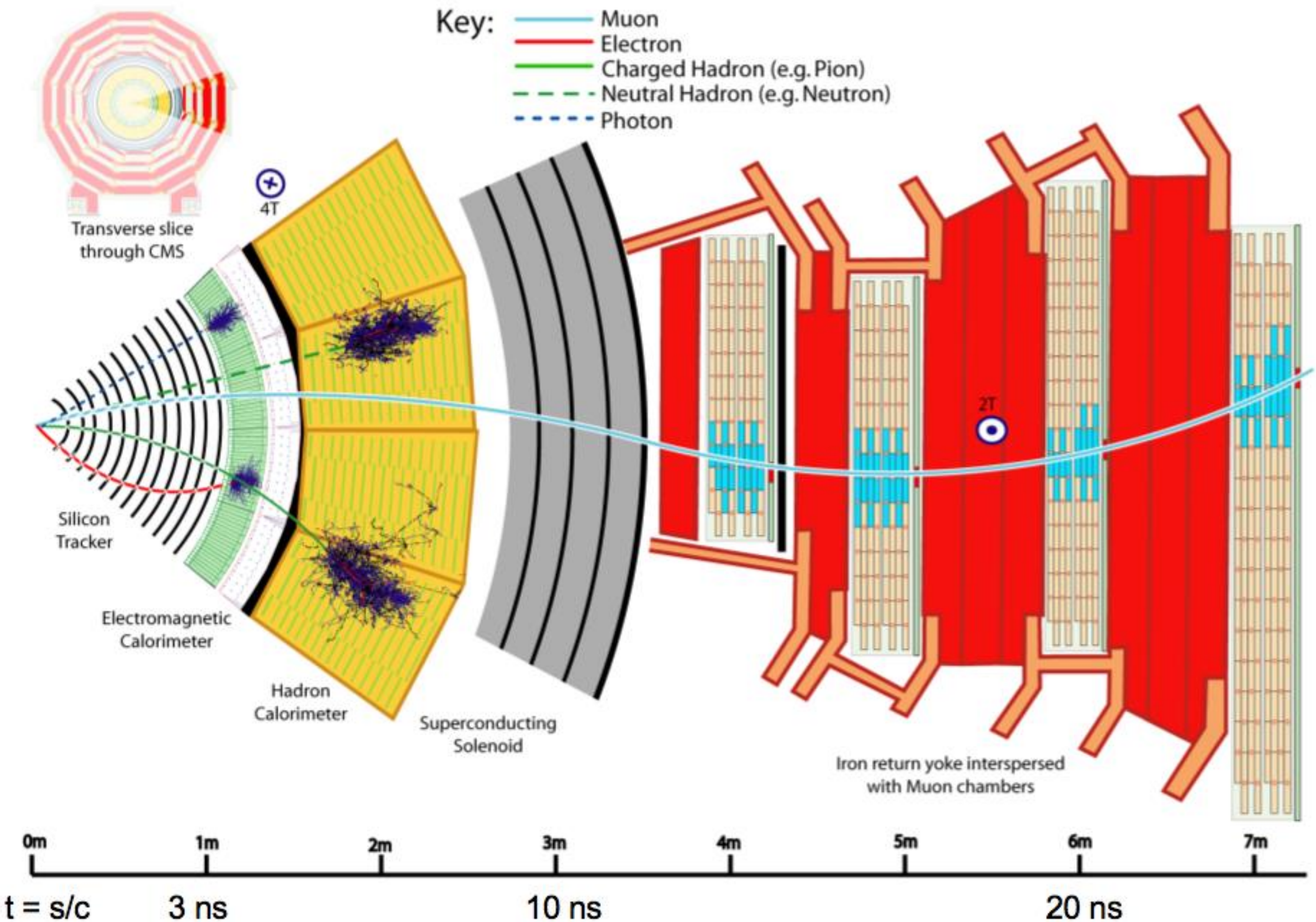
PARTICLE SOURCES

- Particle physics experiments use different sources of particles
- Artificial beams produced in accelerators
  - Colliders – beams are made to collide against each other.
    - Highest energy interactions from artificial sources
  - Beams are aimed at fixed targets / detectors
    - Lower energy, but typically more intense
- Natural sources
  - Particles resulting from cosmic ray interaction in the atmosphere
  - Radioactive sources
  - Astrophysical sources
  - Dark matter ?

A GENERAL PURPOSE DETECTOR

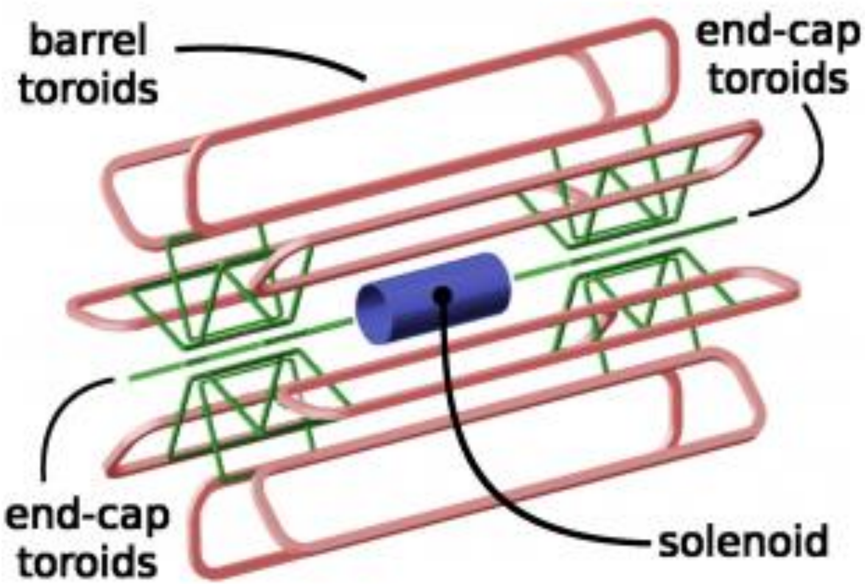


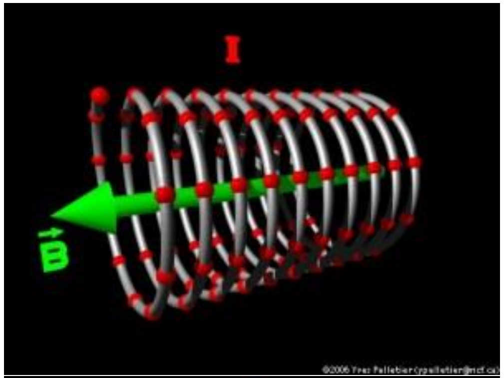
A SLICE OF CMS

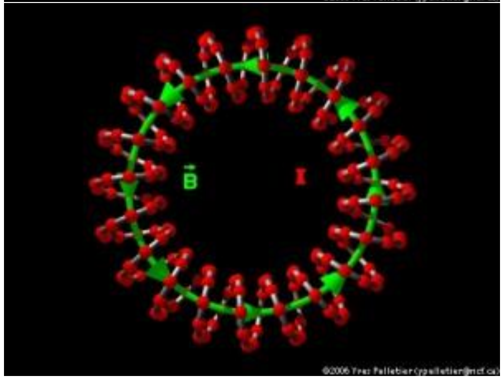


MAGNET SYSTEMS

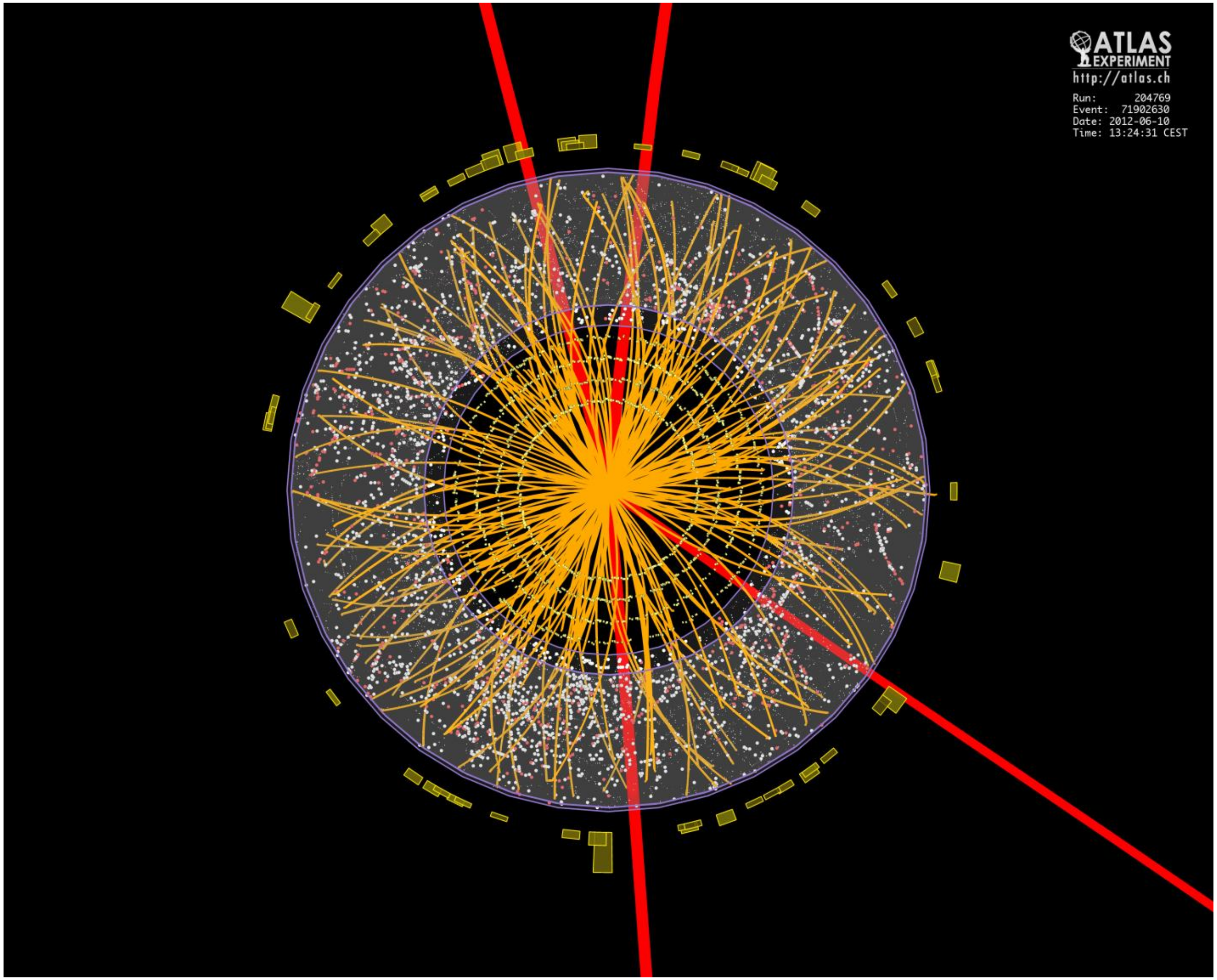
- Solenoid and toroidal magnets.
- Solenoid coils in CMS and ATLAS:
  - Field direction along beam axis.
  - Homogenous field inside the coil.
  - e.g. CMS superconducting magnet
    - I = 20 kA, B = **4 T**
    - Temperature 4K.
- For comparison, Earth's magnetic field at surface is ~50 μT.





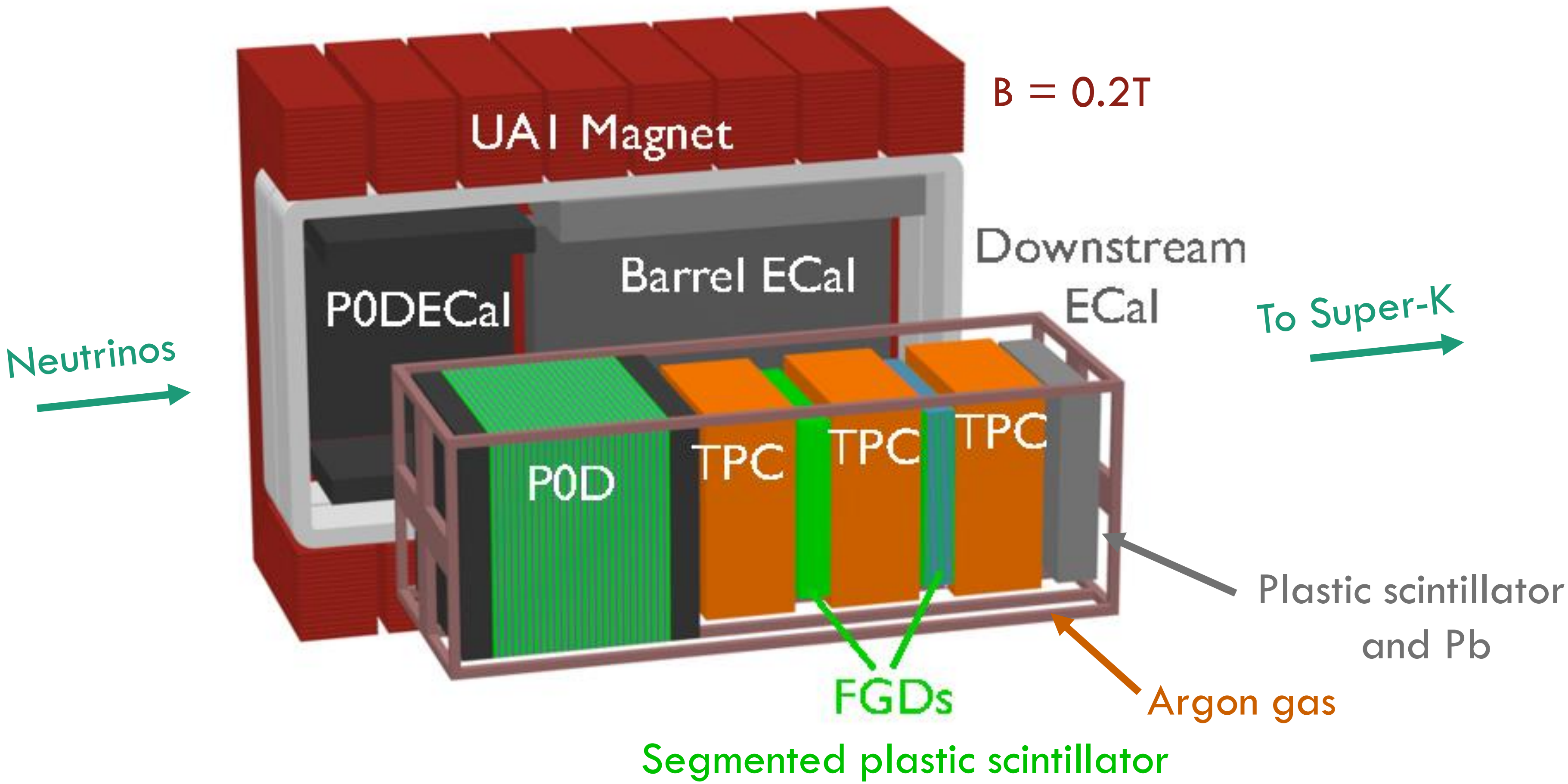


A REAL EVENT

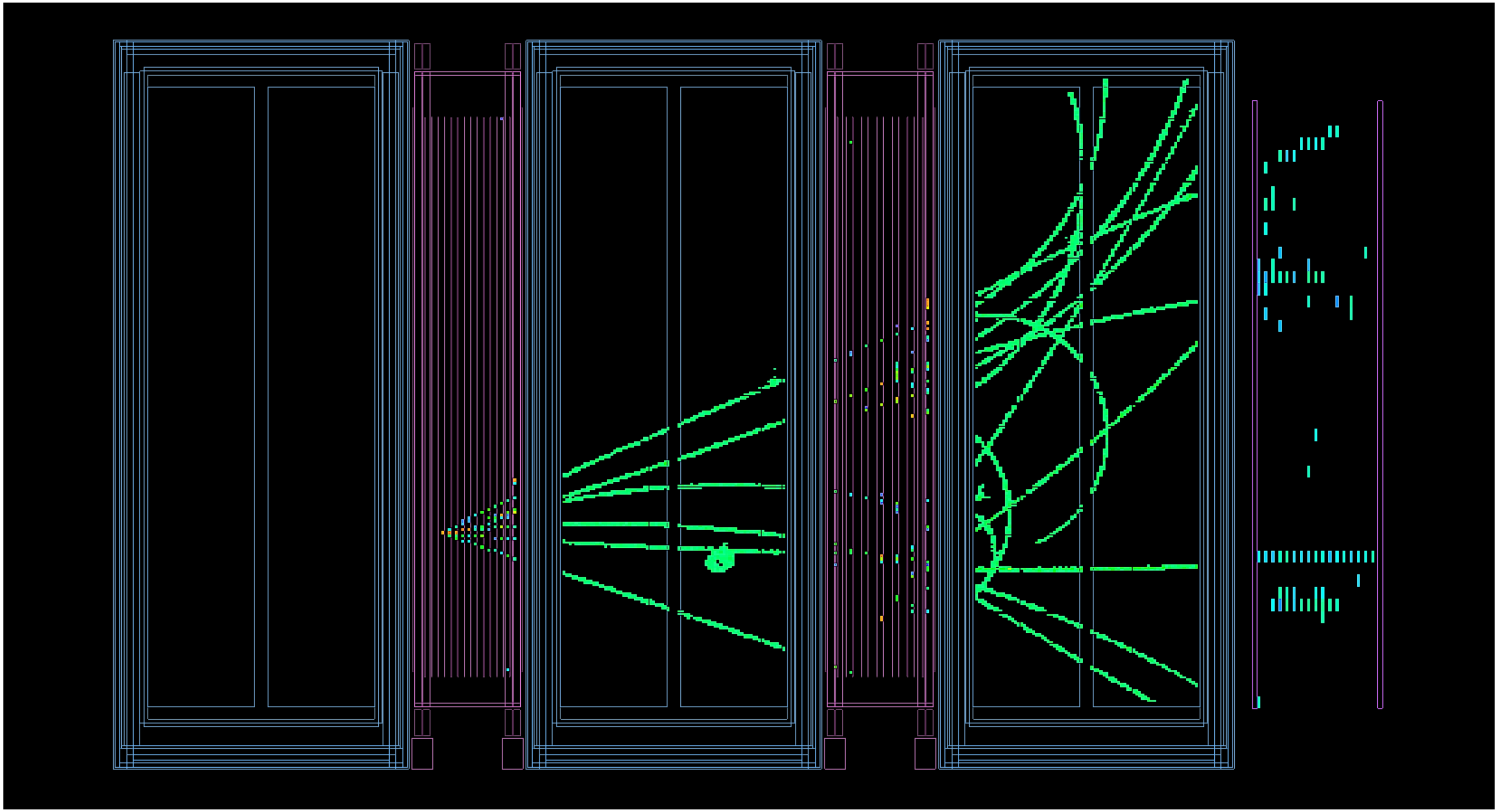

ATLAS
EXPERIMENT
http://atlas.ch
Run: 204769
Event: 71902630
Date: 2012-06-10
Time: 13:24:31 CEST

T2K: ND280
A GENERAL PURPOSE NEUTRINO DETECTOR

- Beam of neutrinos produced at the J-PARC proton accelerator in Japan

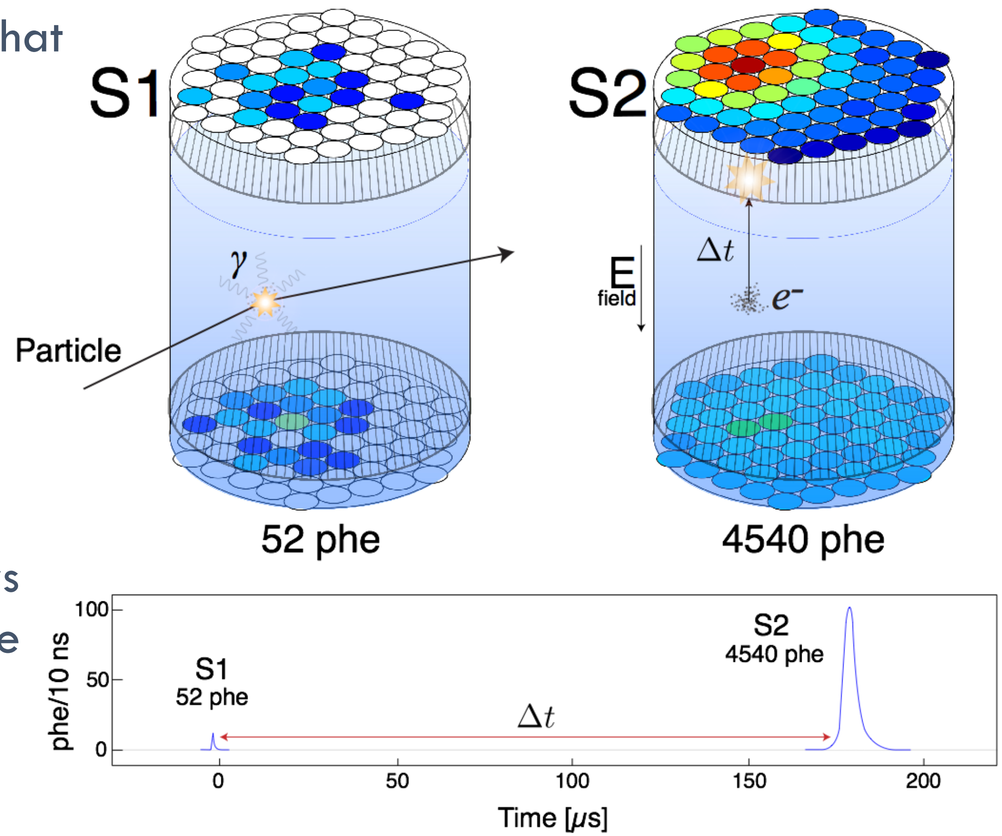


A NEUTRINO INTERACTION IN ND280



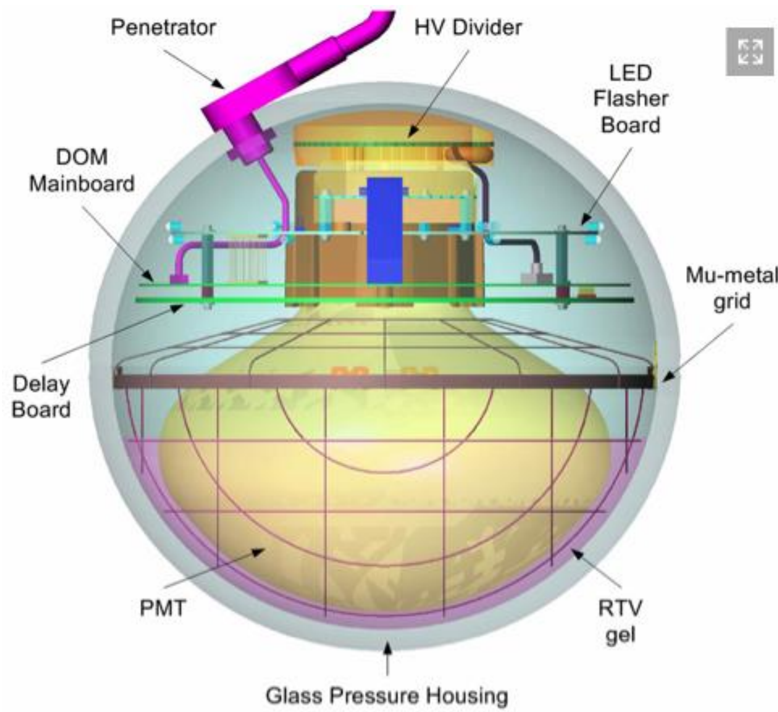
THE LUX DARK MATTER EXPERIMENT

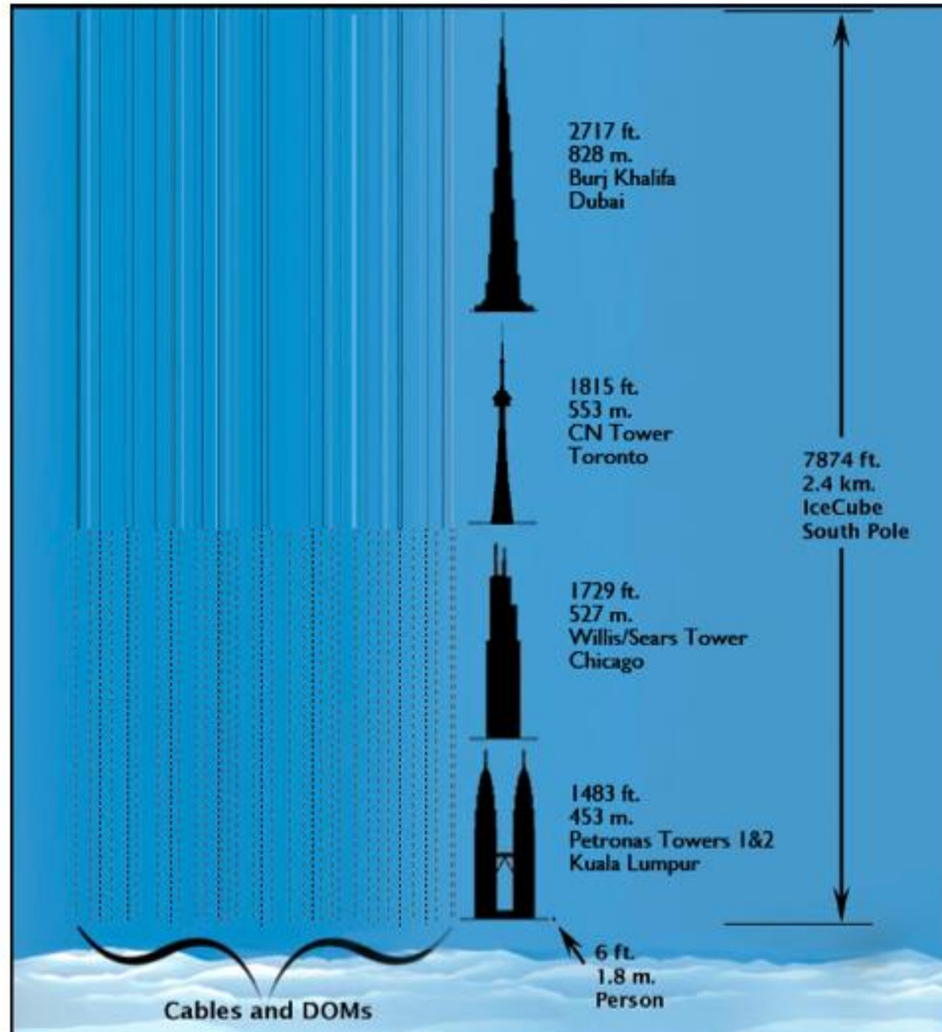
- The Large Underground Xenon experiment (LUX) physics experiment looks for evidence of weakly interacting massive particle (WIMP) dark matter interactions.
- It is a 370 kg liquid xenon TPC that aims to directly detect galactic dark matter in an underground laboratory 1 mile deep
- The detector is shielded from background particles by a surrounding water tank and the earth above.
- This shielding reduces cosmic rays and radiation interacting with the xenon.



ICE FISHING FOR SPACE NEUTRINOS!

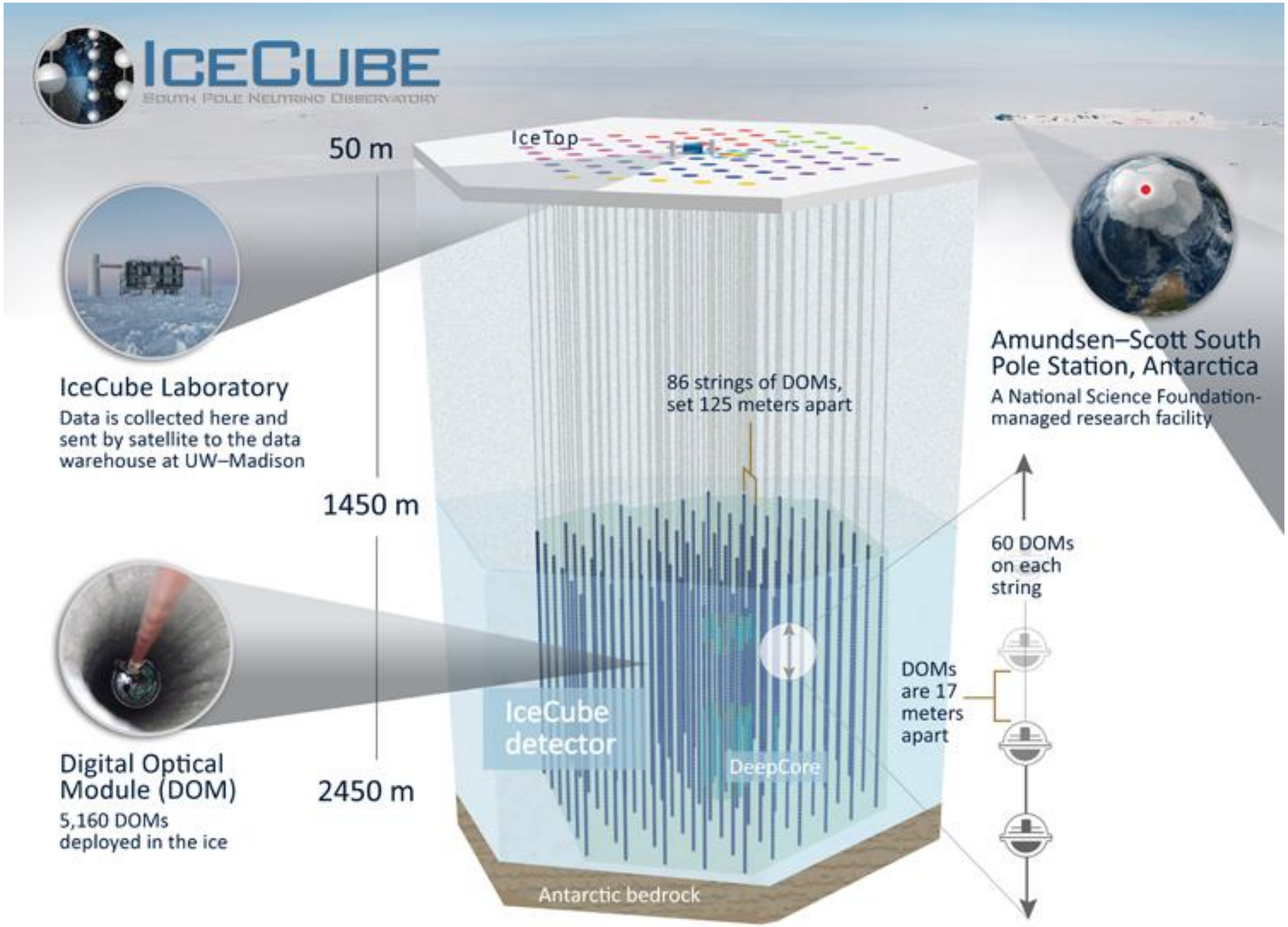
- Made up of strings of thousands of basketball-sized photon detectors
  - Digital Optical Modules





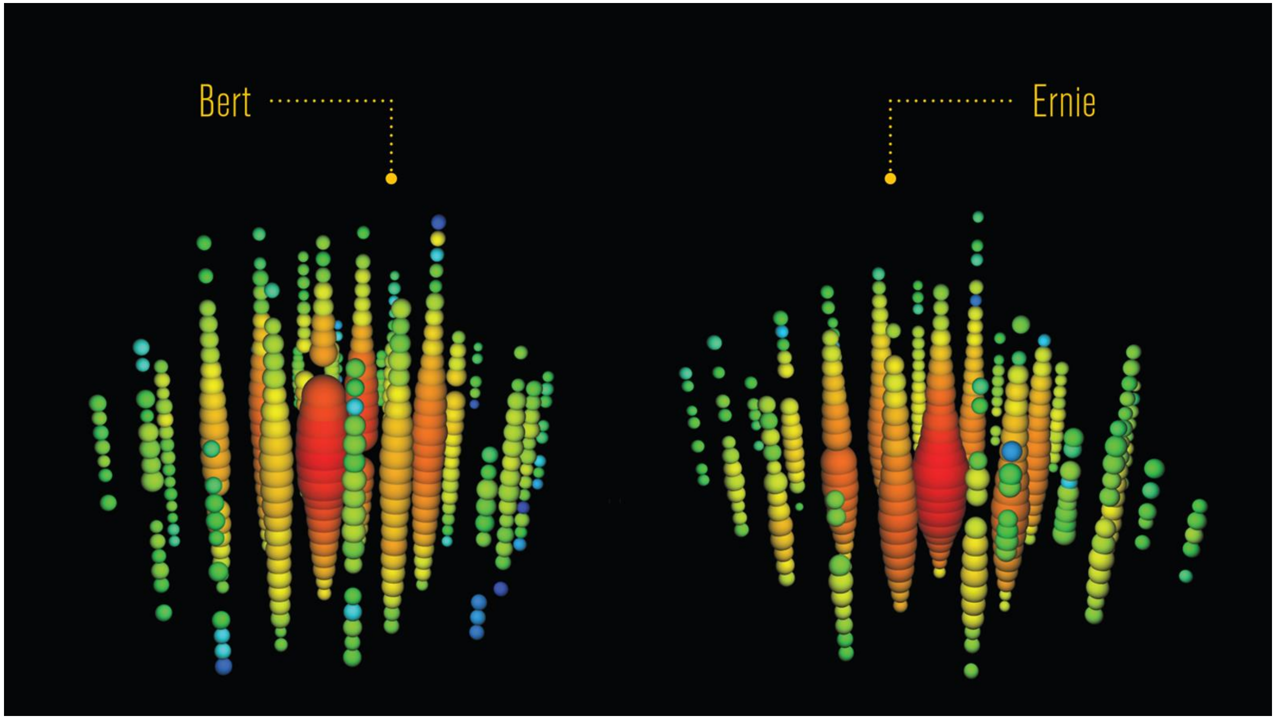
IceCube in Scale: The dashed lines above represent the portion of the cables that have DOMs attached

THE ICECUBE DETECTOR



BERT, ERNIE AND MANY OTHERS

- In 2013, IceCube announced that it had detected 28 neutrinos likely originating from outside the Solar System.
  - These are ultra-high energy (**P**eV) neutrino events.



SCHEDULE

1. ~~Introduction~~
2. ~~History of Particle Physics~~
3. ~~Special Relativity~~
4. ~~Quantum Mechanics~~
5. ~~Experimental Methods~~
6. The Standard Model - Overview
7. The Standard Model - Limitations
8. Neutrino Theory
9. Neutrino Experiment
10. LHC and Experiments
11. The Higgs Boson and Beyond
12. Particle Cosmology

AN OVERVIEW OF THE STANDARD MODEL

THE STANDARD MODEL

- The theory that attempts to fully describe the **weak**, **electromagnetic**, and **strong** interactions within a **common framework**:
  - A "common ground" that would unite all of laws and theories which describe particle dynamics into one integrated theory of everything, of which all the other known laws would be special cases, and from which the behavior of all matter and energy can be derived.
- A theory of "almost everything": does **not** accommodate gravity, dark matter, dark energy.

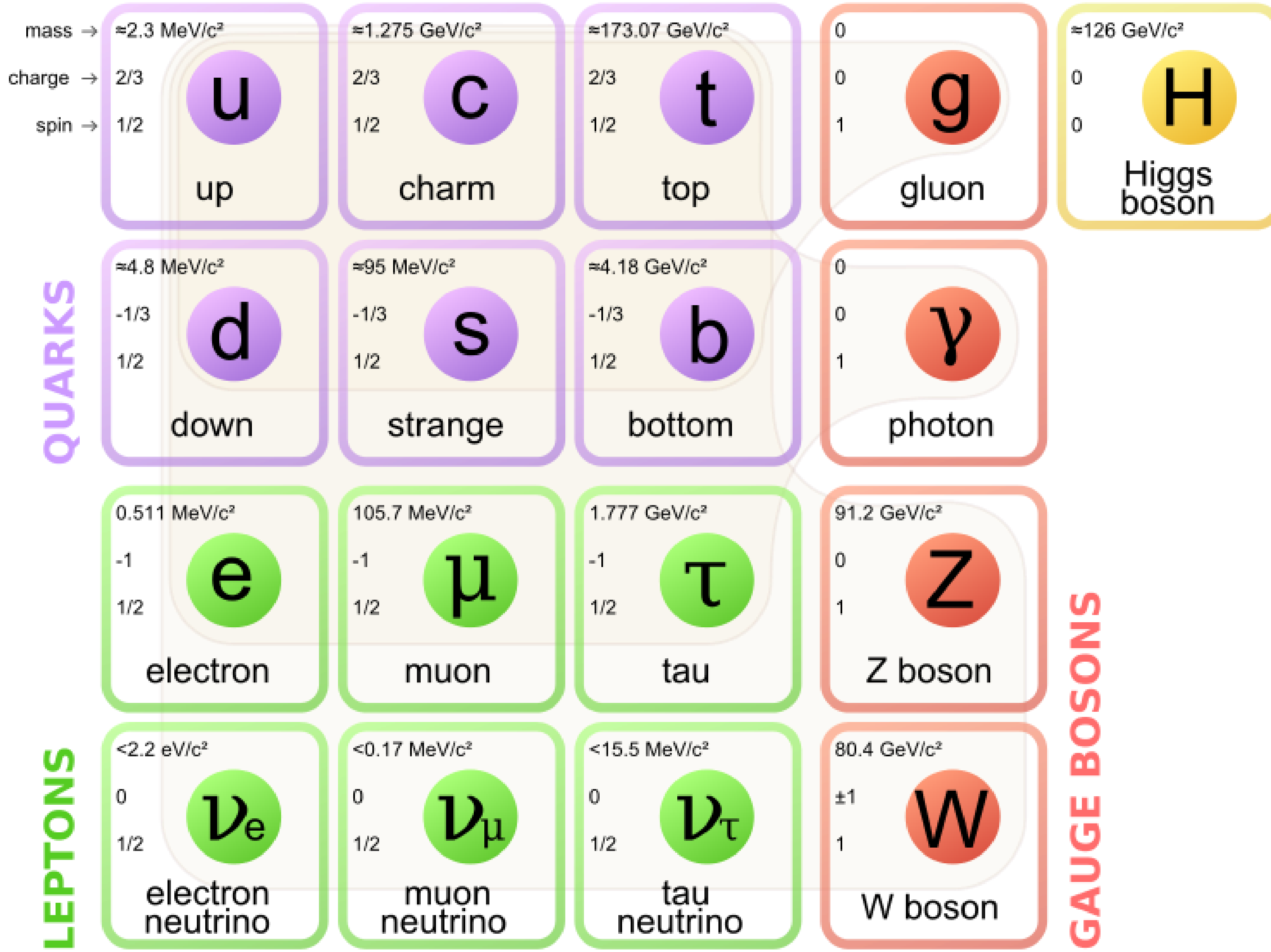
THE STANDARD MODEL

- The Standard Model was solidified in the 1970's, with the discovery of quarks
  - Confirmation of theory of strong interactions.
- Under scrutiny for the last 40 years, has managed to survive* experimental tests
  - All particles predicted by this theory have been found experimentally!
  - * If you ignore neutrino mass…
- We already know it is incomplete
  - See next lecture on this.

TODAY'S AGENDA

- Historical background (see lecture 2)
- Standard Model particle content
- Standard Model particle dynamics
  - Quantum Electrodynamics (QED)
  - Quantum Chromodynamics (QCD)
  - Weak Interactions
  - Force Unification
- Lagrangian / Field formulation
- Higgs mechanism
- Tests and predictions

THE WORLD, ACCORDING TO A PARTICLE PHYSICIST



THE WORLD, ACCORDING TO A PARTICLE PHYSICIST
COURTESY OF …

The Nobel Prize in Physics 1979
Sheldon Glashow, Abdus Salam, Steven Weinberg

Sheldon Lee Glashow

Abdus Salam

Steven Weinberg

The Nobel Prize in Physics 1979 was awarded jointly to Sheldon Lee Glashow, Abdus Salam and Steven Weinberg *"for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current"*.

THE STANDARD MODEL PARTICLE CONTENT

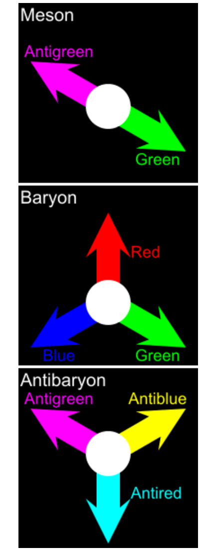
- Fermions:
  - Quarks and Leptons
  - Half-integer spin

- Bosons:
  - Force mediators and the Higgs
  - Integer spin

PARTICLE CHARGES

	Electric	Weak	Color
Quarks	✓	✓	✓

- Quarks:
  - There are **no** free quarks.
  - They form colorless composite objects, hadrons;
    - Baryons: qqq, $\overline{qqq}$
    - Mesons: $q\bar{q}$, $q\bar{q}$, $q\bar{q}$

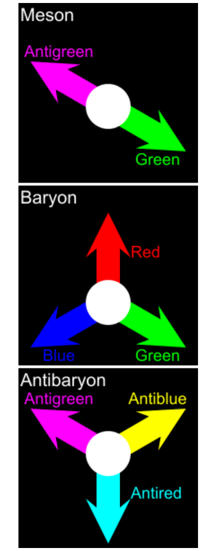


Name	Symbol	Mass (MeV/c^2)*	J	B	Q	I_3	C	S	T	B'	Antiparticle	Antiparticle symbol
						First generation						
Up	u	1.7 to 3.3	½	+⅓	+⅔	+½	0	0	0	0	Antiup	$\bar{u}$
Down	d	4.1 to 5.8	½	+⅓	−⅓	−½	0	0	0	0	Antidown	$\bar{d}$
						Second generation						
Charm	c	$1{,}270^{+70}_{-90}$	½	+⅓	+⅔	0	+1	0	0	0	Anticharm	$\bar{c}$
Strange	s	101^{+29}_{-21}	½	+⅓	−⅓	0	0	−1	0	0	Antistrange	$\bar{s}$
						Third generation						
Top	t	172,000 ±900 ±1,300	½	+⅓	+⅔	0	0	0	+1	0	Antitop	$\bar{t}$
Bottom	b	$4{,}190^{+180}_{-60}$	½	+⅓	−⅓	0	0	0	0	−1	Antibottom	$\bar{b}$

J = total angular momentum, B = baryon number, Q = electric charge, I_3 = isospin, C = charm, S = strangeness, T = topness, B' = bottomness.

PARTICLE CHARGES

	Electric	Weak	Color
Quarks	✓	✓	✓



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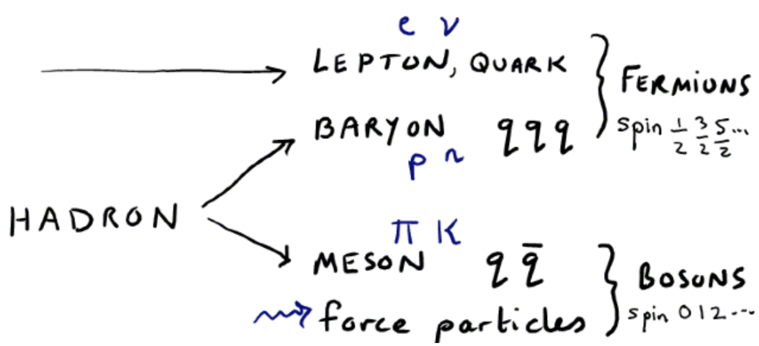


PARTICLE CHARGES

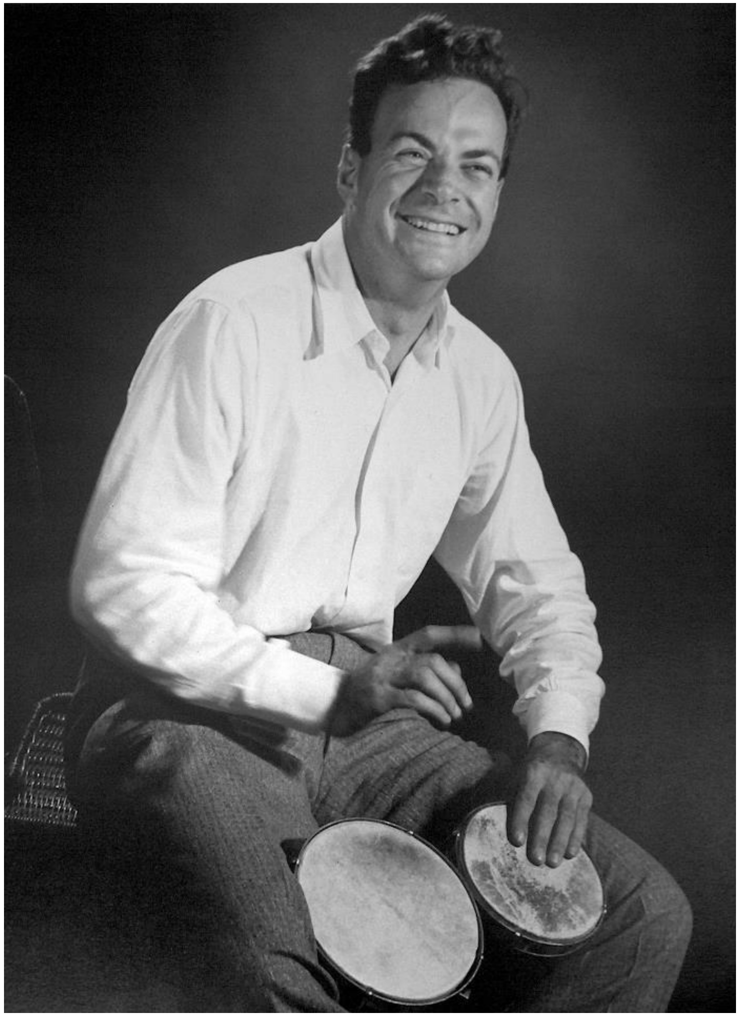
		Electric	Weak	Color
Quarks		✓	✓	✓
Leptons	Charged	✓	✓	✗
	Neutral	✗	✓	✗

- Leptons
  - Exist as free particles

Particle/Antiparticle Name	Symbol	Q (e)	S	L_e	L_μ	L_τ	Mass (MeV/c^2)	Lifetime (s)
Electron / Antielectron[17]	e^-/e^+	−1/+1	½	+1/−1	0	0	0.510 998 910(13)	Stable
Muon / Antimuon[18]	μ^-/μ^+	−1/+1	½	0	+1/−1	0	105.658 3668(38)	$2.197\,019(21) \times 10^{-6}$
Tau / Antitau[20]	τ^-/τ^+	−1/+1	½	0	0	+1/−1	1,776.84(17)	$2.906(10) \times 10^{-13}$
Electron neutrino / Electron antineutrino[33]	$\nu_e/\bar{\nu}_e$	0	½	+1/−1	0	0	< 0.000 0022[35]	Unknown
Muon neutrino / Muon antineutrino[33]	$\nu_\mu/\bar{\nu}_\mu$	0	½	0	+1/−1	0	< 0.17[35]	Unknown
Tau neutrino / Tau antineutrino[33]	$\nu_\tau/\bar{\nu}_\tau$	0	½	0	0	+1/−1	< 15.5[35]	Unknown

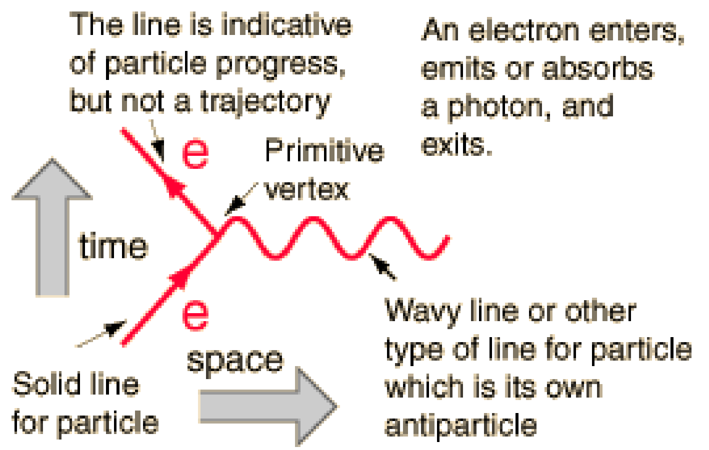
PARTICLE DYNAMICS: FEYNMAN DIAGRAMS

- Feynman Rules!
- 1948: introduced pictorial representation scheme for the mathematical expressions governing the behavior of subatomic particles.
  - Can be used to easily caclulate probability amplitudes
  - Other options: cumbersome mathematical derivations



FEYNMAN DIAGRAMS

- How to read them:



FEYNMAN DIAGRAMS

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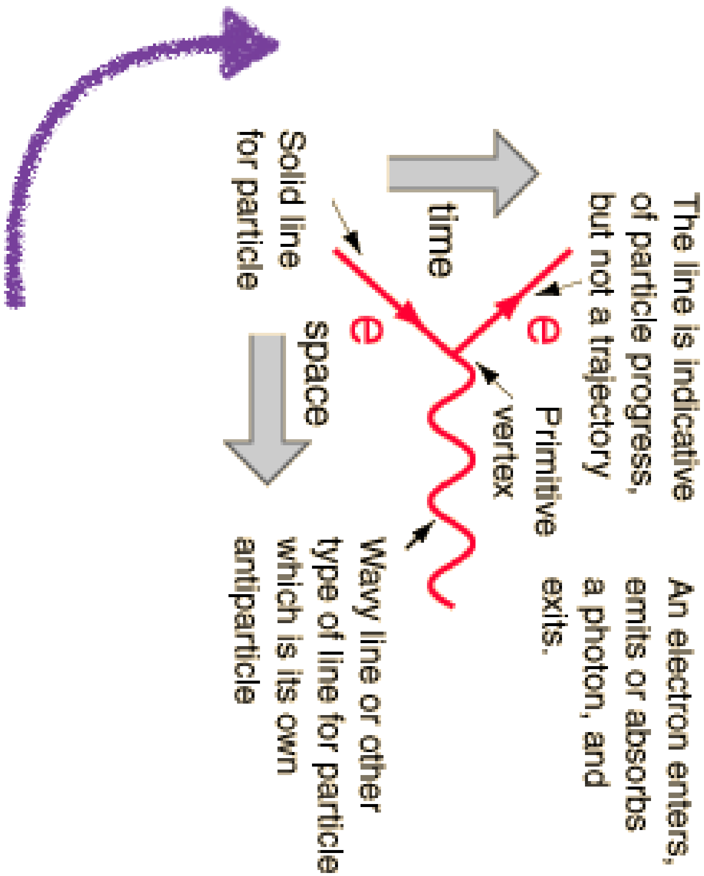
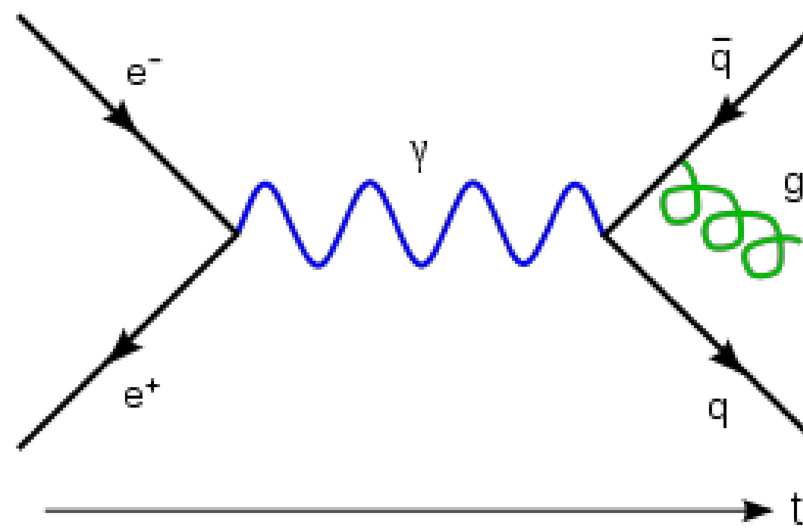


Beware of the time direction!
(You'll see it used in either way.)

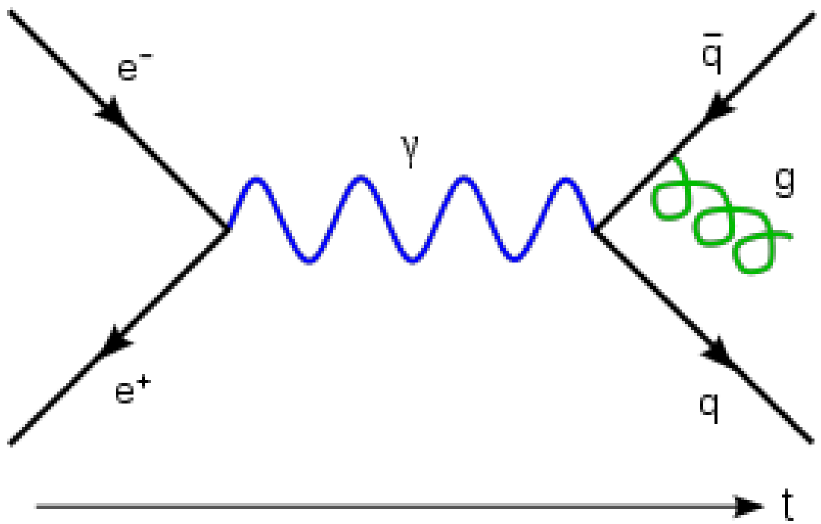
If t was on y-axis, this would be a different process.

FEYNMAN DIAGRAMS
AN EXAMPLE



1. An **electron** and a **positron** annihilate into
2. a virtual photon that produces
3. a **quark-antiquark** pair, one of which radiates
4. A gluon

FEYNMAN DIAGRAMS
AN EXAMPLE



Note, at every vertex:
Q conservation
L conservation
L_e conservation
B conservation

1. An **electron** and a **positron** annihilate into
2. a virtual photon that produces
3. a **quark-antiquark** pair, one of which radiates
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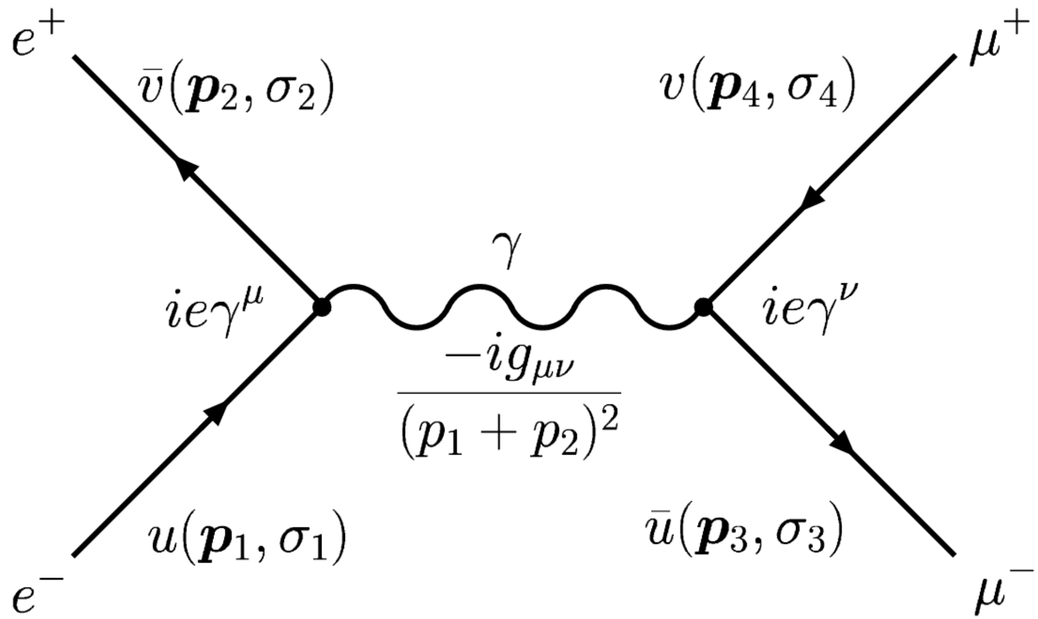
FEYNMAN DIAGRAMS
AN EXAMPLE CALCULATION



$$-i\mathcal{M} = [\bar{u}(\boldsymbol{p}_3, \sigma_3)(ie\gamma^\nu)v(\boldsymbol{p}_4, \sigma_4)] \frac{-ig_{\mu\nu}}{(p_1 + p_2)^2} [\bar{v}(\boldsymbol{p}_2, \sigma_2)(ie\gamma^\mu)u(\boldsymbol{p}_1, \sigma_1)]$$

QED, QCD AND WEAK INTERACTIONS

QUANTUM ELECTRODYNAMICS
QED

- As you already know, electromagnetism is the dominant physical force in your life. All of your daily interactions - besides your attraction to the Earth - are electromagnetic in nature.
- As a theory of electromagnetism, QED is primarily concerned with the behavior and interactions of charged particles with each other and with light.
- As a quantum theory, QED works in the submicroscopic world, where particles follow all possible paths and can blink in and out of existence (more later).

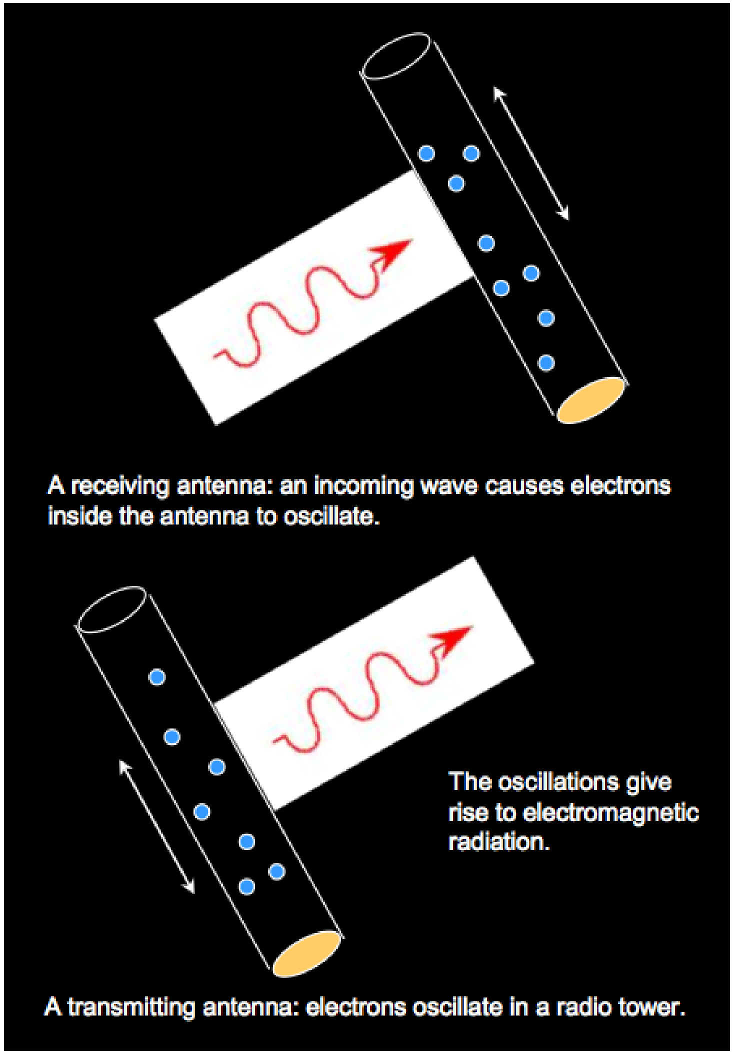
QUANTUM ELECTRODYNAMICS
QED

- In classical EM, light is a wave, and matter is made up of charged particles.
- Charge is always conserved; particles are never created or destroyed.
- EM fields interact with charges according to the Lorentz force law:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

- Accelerating charges radiate EM waves (Larmor power formula):

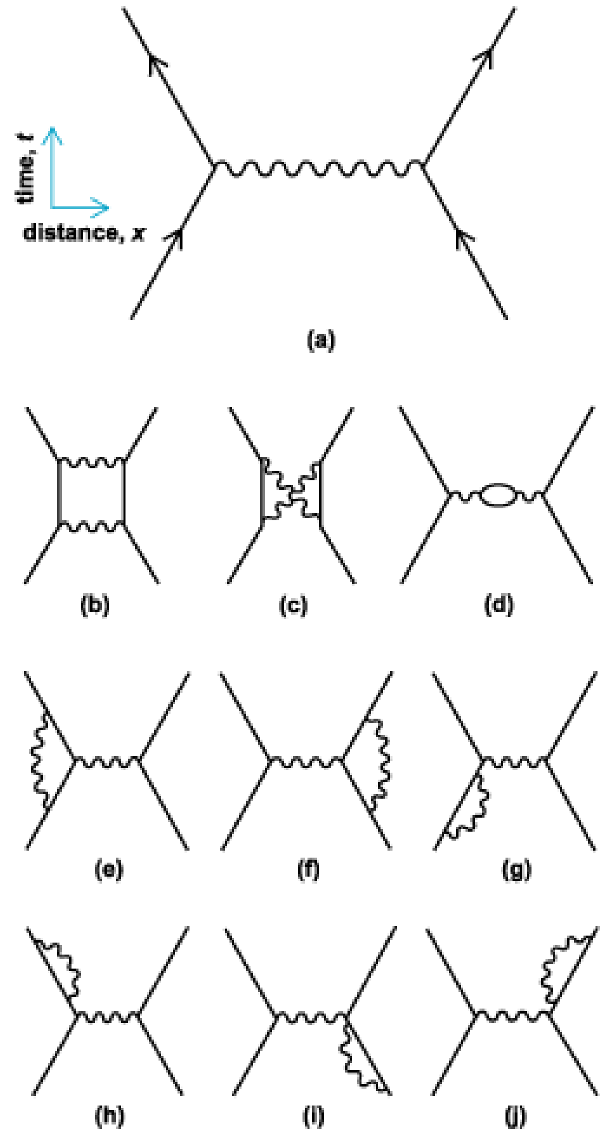
$$P = \frac{2}{3}\frac{q^2 a^2}{c^3}$$



QUANTUM ELECTRODYNAMICS

QED

- The vertices are interactions with the electromagnetic field.
- The straight lines are electrons and the wiggly ones are photons.
- Between interactions (vertices), particles propagate as free particles.
- The higher the number of vertices, the less likely for the interaction to happen.
    - Higher "order".

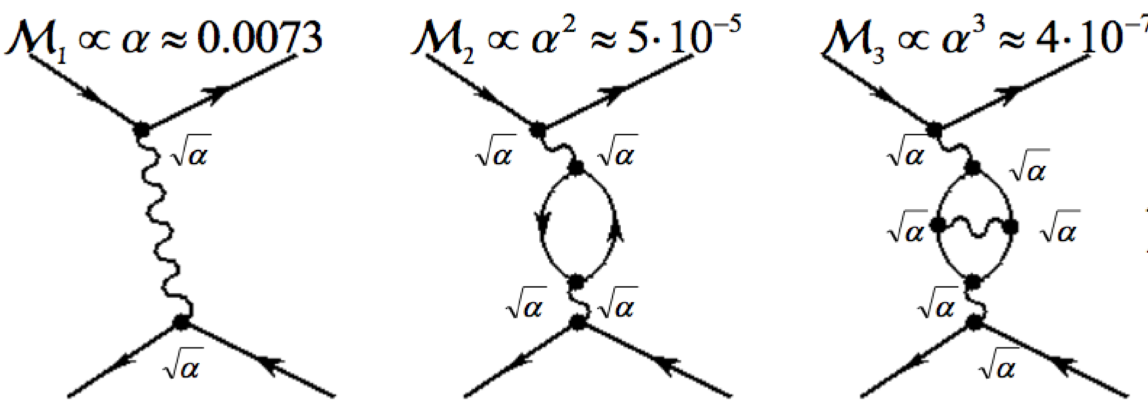


QUANTUM ELECTRODYNAMICS
QED

- Each vertex contributes a coupling constant $\sqrt{\alpha}$, where α is a small dimensionless number:

$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

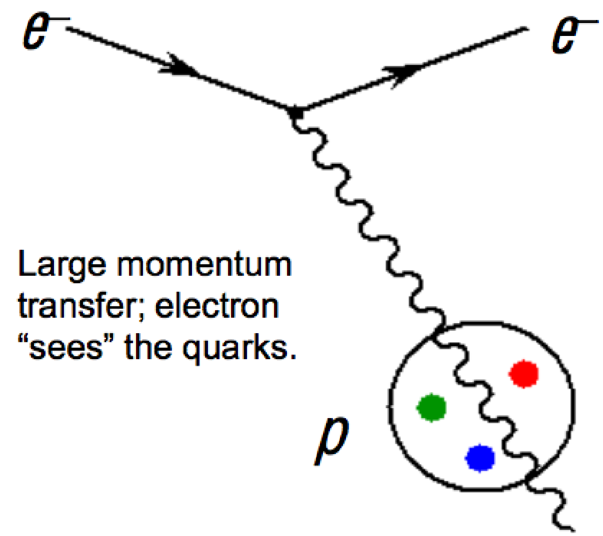
- Hence, higher-order diagrams get suppressed relative to diagrams with fewer vertices.

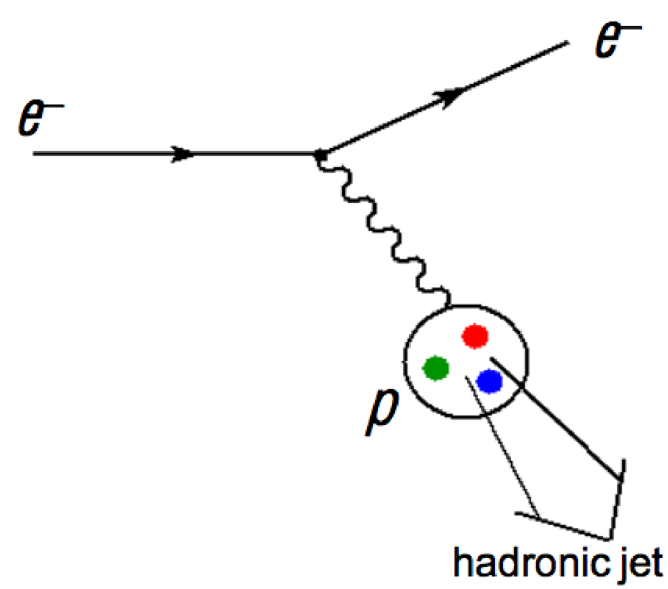


$$\begin{array}{ccccc} \mathcal{M}_1 & : & \mathcal{M}_2 & : & \mathcal{M}_3 \\ 18769 & : & 137 & : & 1 \end{array}$$

WHEN QED IS NOT ENOUGH

- Higher energy interactions involving hadrons will result in the production of new particles.
- In this type of *inelastic scattering*, in which two colliding particles can form (hundreds of) new hadrons.
- QED cannot explain phenomena like inelastic scattering.
    - We need an additional theory of particle interactions.



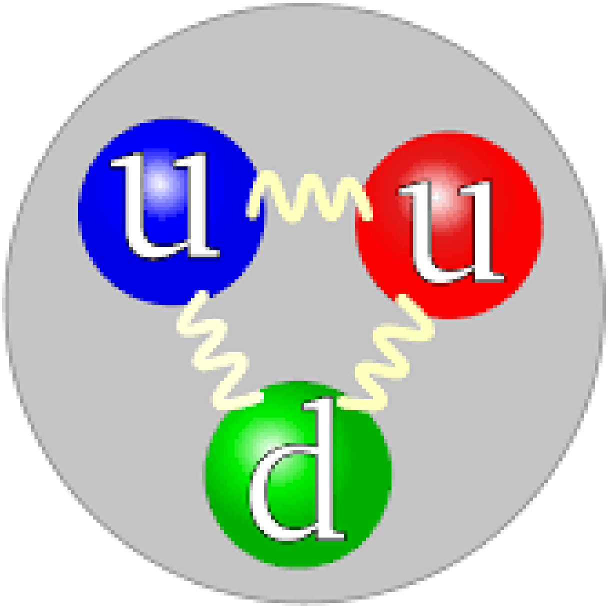


QUANTUM CHROMODYNAMICS
QCD

- QCD can explain many phenomena not covered by QED.
- The binding of nucleons in atoms and the phenomena of inelastic scattering are both explained by a single field theory of quarks and gluons: QCD.
- QCD describes the interactions between quarks via the exchange of massless gluons.
- Note: the quark-gluon interactions are also responsible for the binding of quarks into the bound states that make up the hadron zoo (ρ's, η's, Λ, Ξ, Σ's, …).
- QCD is conceptually similar to QED, but its calculations are even more complicated. We'll discuss why…

QUANTUM CHROMODYNAMICS
QCD

- Quarks and bound states:
  - Since quarks are spin-1/2 particles (fermions), they must obey the Pauli Exclusion Principle.
- **Pauli Exclusion Principle**: fermions in a bound state (e.g., the quarks inside a hadron) cannot have the same quantum numbers.
- Then, how can we squeeze three quarks into a baryon?
- Give them an additional charge, called color.
- This removes the quantum numbers degeneracy.



QUANTUM CHROMODYNAMICS
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- Quarks and bound states:
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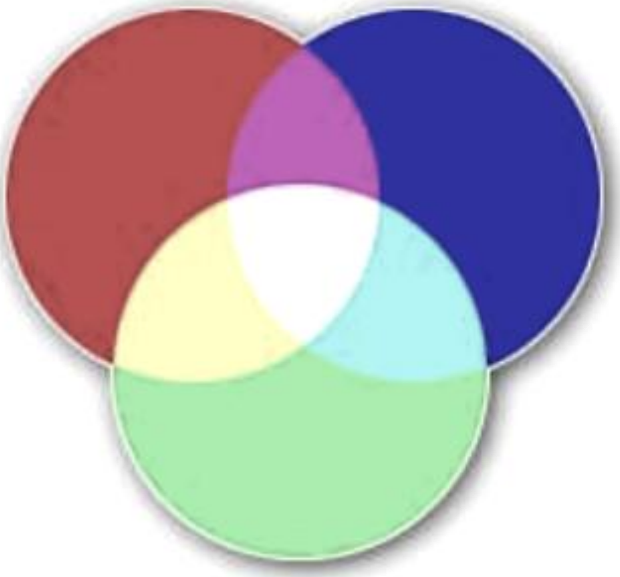
- **Pauli Exclusion Principle**: fermions in a bound state (e.g., the quarks inside a hadron) cannot have the same quantum numbers.

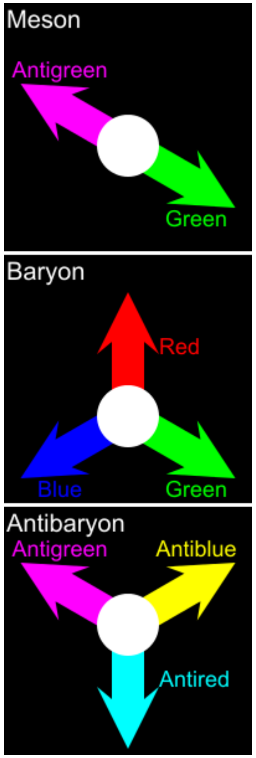
- Proposal: quark color comes in three types: red, green, and blue;
- All free, observable particles are colorless



Red, blue, and green combine to give white (color-neutral).

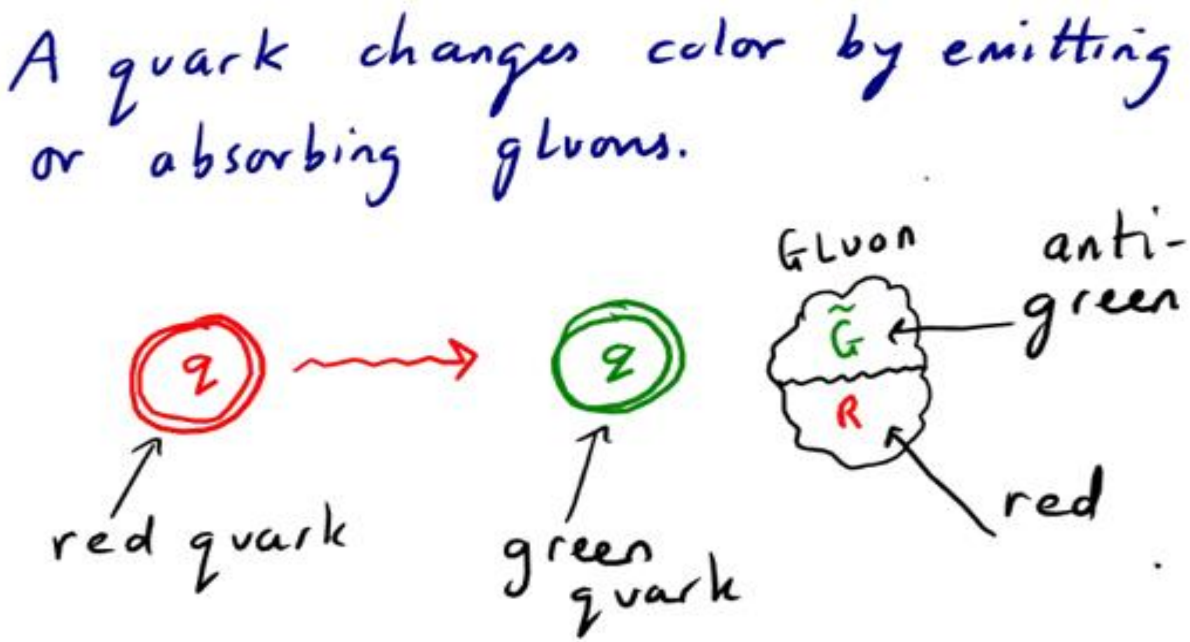
QUANTUM CHROMODYNAMICS
QCD

- Quarks and bound states:
  - Since quarks are spin-1/2 particles (fermions), they must obey the Pauli Exclusion Principle.
- **Pauli Exclusion Principle**: fermions in a bound state (e.g., the quarks inside a hadron) cannot have the same quantum numbers.
- What do the anti-colors look like?
- Red plus anti-red gives white
- Combining red with blue and green gives white.
- Hence:
  - Anti-red is blue+green
  - Anti-blue is red+green
  - Anti-green is red+blue



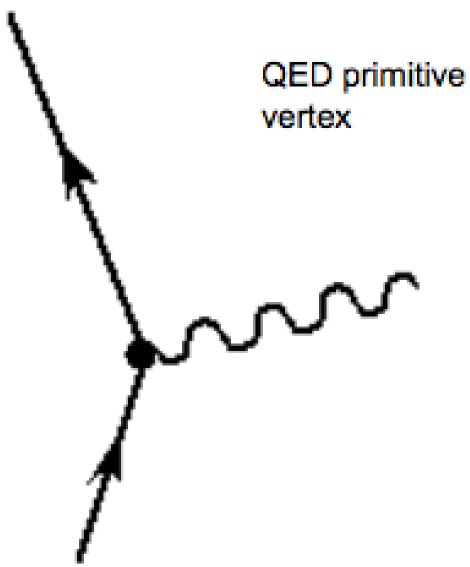
QUANTUM CHROMODYNAMICS
QCD

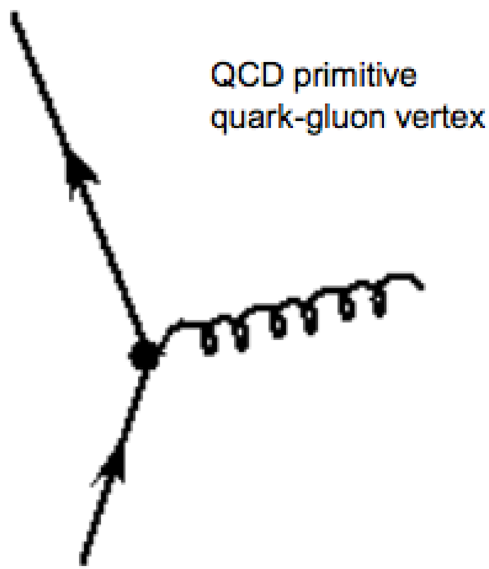
- Gluons carry a color and an anti-color.
- There are 9 possible combinations, but 1 is white, which is not allowed.
    - No evidence for colorless particles exchanging gluons.
- This leaves 8 types of gluon.



QUANTUM CHROMODYNAMICS
QCD

- Quarks are electrically charged, so they also interact via the electromagnetic force, exchanging photons.
- The strong interaction is gluon-mediated, the Feynman diagram for the *quark-gluon vertex* looks just like the primitive QED vertex.





QCD VS QED

- QCD is much harder to handle than QED.
- What makes it so difficult? Let's start with perturbation theory.

- Recall: In QED, each vertex contributes a coupling constant $\sqrt{\alpha}$, where α is a small dimensionless number:

$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}$$

- Hence, we saw that higher-order diagrams (diagrams with more vertices) get suppressed relative to diagrams with fewer vertices.

PERTURBATION THEORY
ASIDE

- Use a power series in a parameter ε (such that ε << 1) - known as perturbation series - as an approximation to the full solution.
- For example:

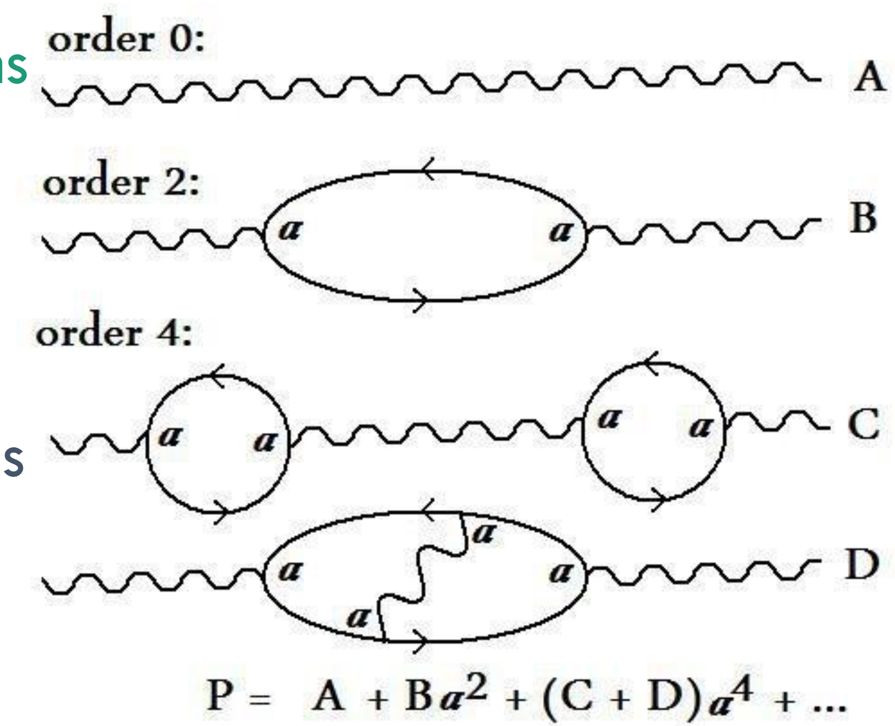
$$A = A_0 + \epsilon A_1 + \epsilon^2 A_2 + \ldots$$

- In this example, A_0 is the "leading order" solution, while A_1, A_2, … represent higher order terms.
- **Note:** if ε is small, the higher-order terms in the series become successively smaller.
- Approximation:

$$A \approx A_0 + \epsilon A_1$$

PERTURBATION THEORY IN QFT
ASIDE

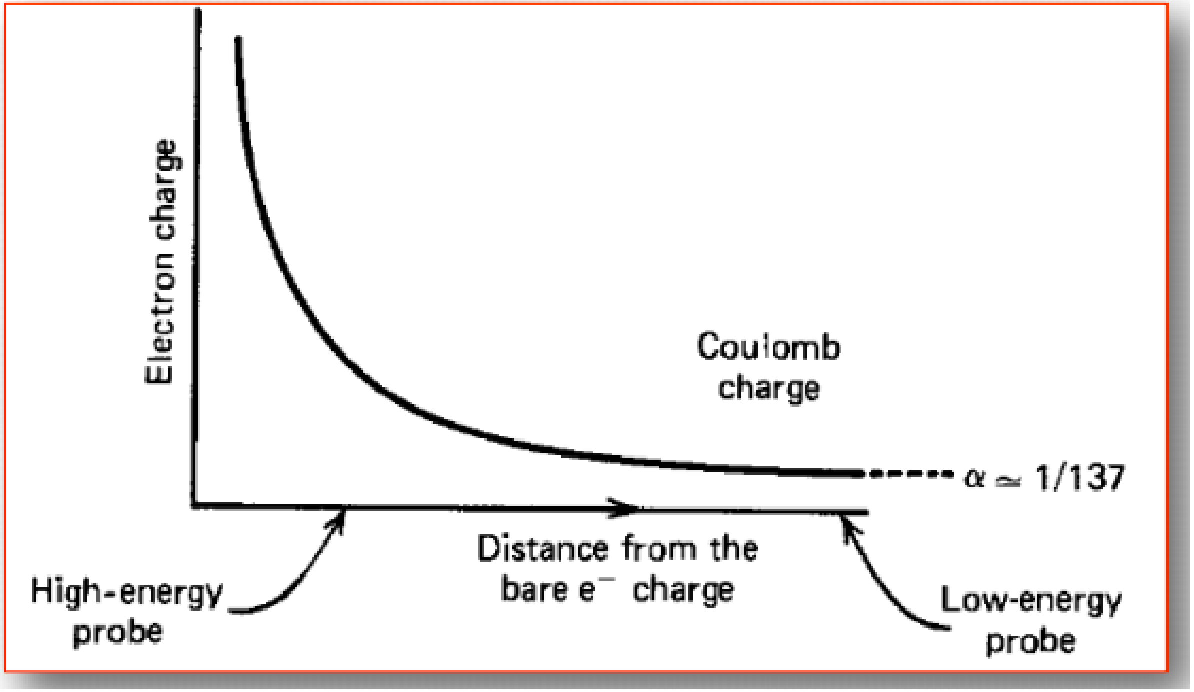
- Perturbation theory allows for well-defined predictions in quantum field theories (as long as they obey certain requirements).
- Quantum electrodynamics is one of those theories.
- Feynman diagrams correspond to the terms in the perturbation series!



Diagrams define a series in α

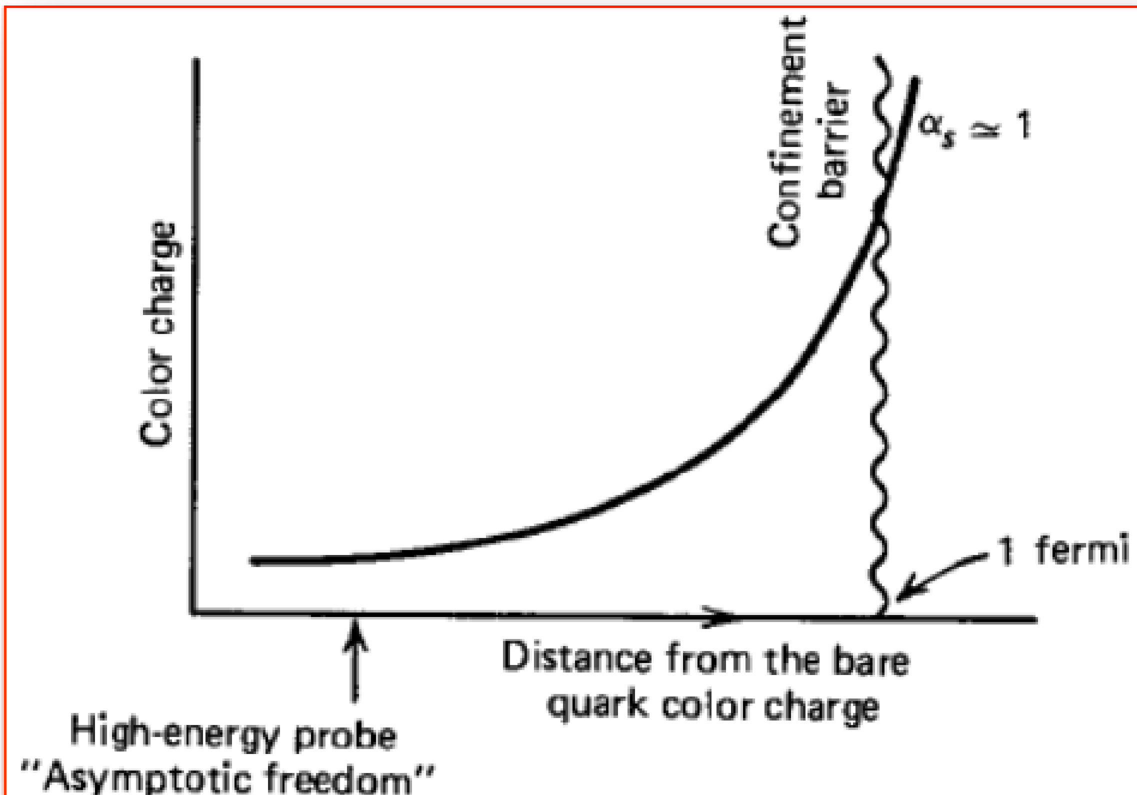
QCD VS QED

- Recall: In QED, each vertex contributes a coupling constant $\sqrt{\alpha}$.
- α is not exactly a constant though… it "runs" with the scale of the interaction.



QCD VS QED

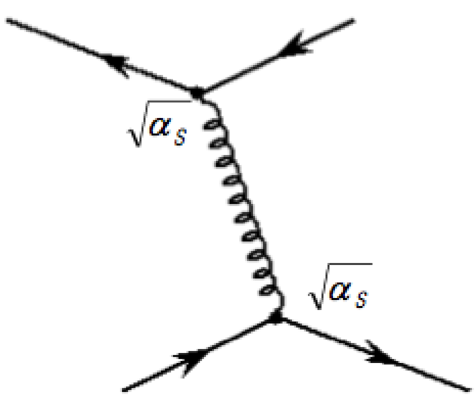
- The coupling constant for QCD, α_s, "runs" in a different way with energy.



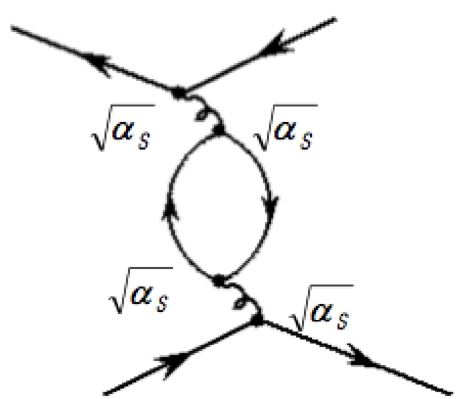
QCD VS QED

- In QCD, the coupling between quarks and gluons, given by the number α_s, is much larger than 1/137 at low energies.
- In fact, at low energies, $\alpha_s \gg 1$, making higher-order diagrams just as important as those with fewer vertices!
- This means we can't truncate the sum over diagrams.
  - Perturbation theory is not a good approximation!
- Calculations quickly become complicated!

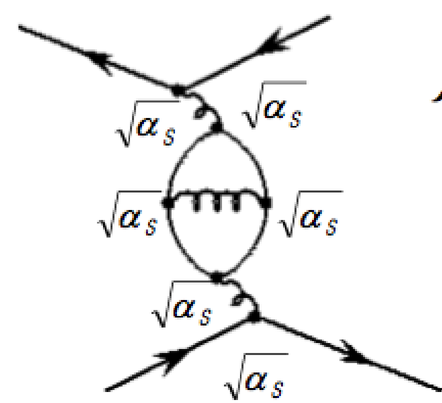
$$\mathcal{M}_1 \propto \alpha_S \approx 1$$



$$\mathcal{M}_2 \propto \alpha_S^{\,2} \approx 1$$



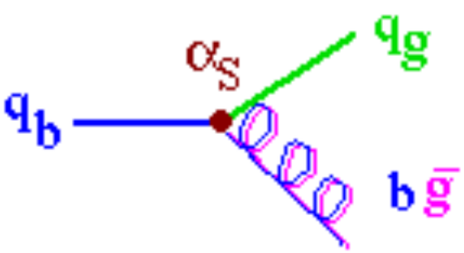
$$\mathcal{M}_3 \propto \alpha_S^{\,3} \approx 1$$

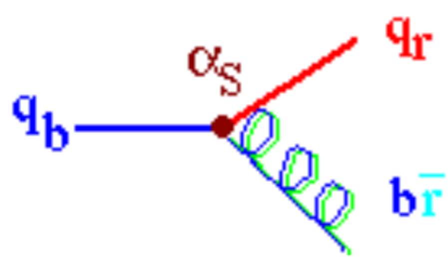


$$\mathcal{M}_1 : \mathcal{M}_2 : \mathcal{M}_3$$
$$1 : 1 : 1$$

ANOTHER COMPLICATION: GLUON COLOR

- Quark color changes at a quark-gluon vertex.
- In order to allow this, the gluons have to carry off “excess" color.
- Color is conserved at the vertex, like electric charge is conserved in QED.





Color, like electric charge, must be conserved at every vertex. This means that the gluons cannot be color-neutral, but in fact carry some color charge. It turns out that there are 8 distinct color combinations!

- Gluons themselves are not color-neutral. That’s why we don’t observe them outside the nucleus, where only colorless particles exist.
- Hence, the strong force is short-range.

GLUON CONFINEMENT

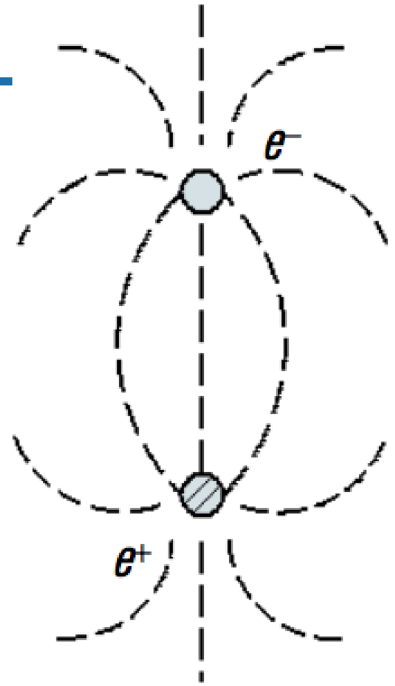
- Confinement is the formal name for what we just discussed.
- The long-range interactions between gluons are theoretically unmanageable. The math is very complicated and riddled throughout with infinities.
- If we assume the massless gluons have infinite range, we find that an infinite amount of energy would be associated with these self-interacting long-range fields.
- The solution is to assume that any physical particle must be colorless: there can be no long-range gluons.

QUARK CONFINEMENT

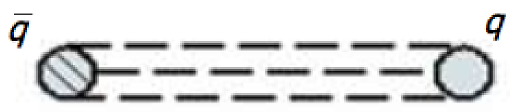
- Confinement also applies to quarks. All bound states of quarks must have a color combination such that they are white, or colorless.
- Protons, neutrons, and other baryons are bound states of three quarks of different color.
- The mesons are composed of a quark-antiquark pair with opposite colors (red and anti-red, etc…)
- As a consequence of confinement, one cannot remove just one quark from a proton, as that would create two "color-full" systems.
  - We would need an infinite amount of energy to effect such separation!
- Hence, the quarks are confined to a small region (<1 fm) near one another.

UNDERSTANDING CONFINEMENT

- The mathematics of confinement are complicated, but we can understand them in terms of a very simple picture.
- Recall, the Coulomb field between a e+e- pair looks like V(r)~1/r.
  - As we pull the pair apart, the attraction weakens.
- Imagine the color field between a quark-antiquark pair like Hooke's Law: V(r)~r.
  - As we pull the pair apart, the attraction between them increases.
- So, separating two quarks by a large r puts a large amount of energy into the color field: V(r)→∞



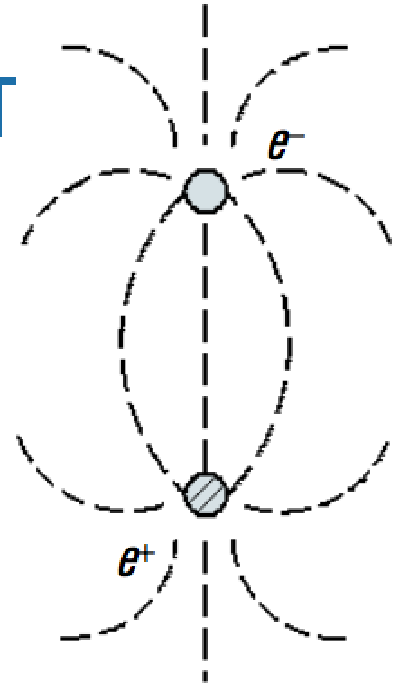
Dipole field for the Coulomb force between opposite electrical charges.



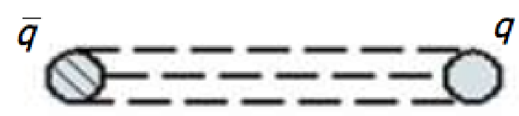
Dipole field between opposite-color quarks.

UNDERSTANDING CONFINEMENT

- How do we understand this picture?
- When a quark and anti-quark separate, their color interaction strengthens (more gluons appear in the color field).
- Through the interaction of the gluons with each other, the color lines of force are squeezed into a tube-like region.
- Contrast this with the Coulomb field: nothing prevents the lines of force from spreading out.
  - There is no self-coupling of photons to contain them.
- If the color tube has constant energy density per unit length k, the potential energy between quark and antiquark will increase with separation, V(r)~kr.



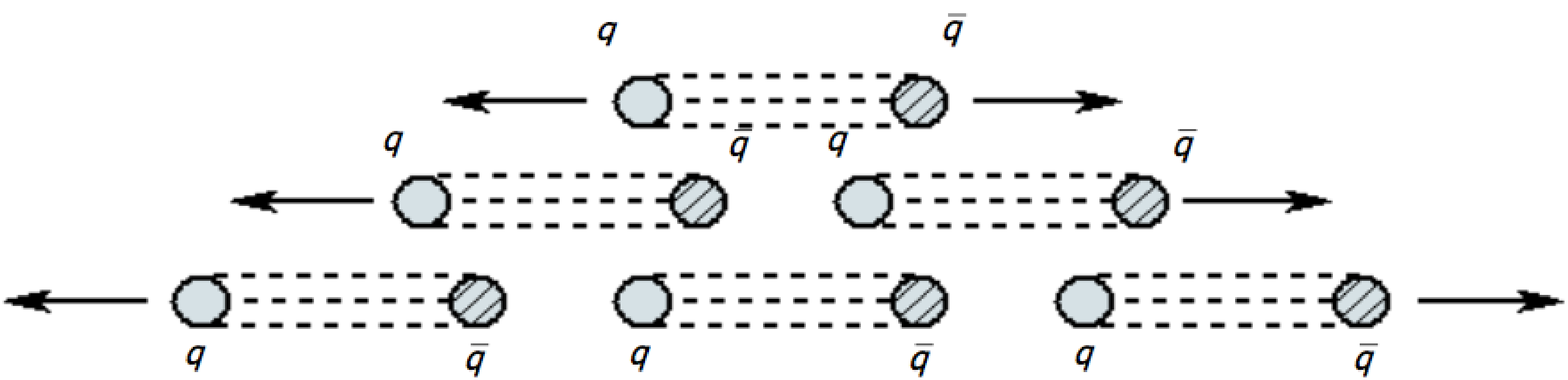
Dipole field for the Coulomb force between opposite electrical charges.



Dipole field between opposite-color quarks.

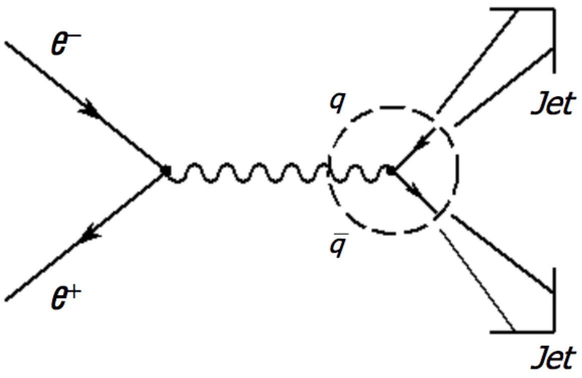
COLOR LINES AND HADRON PRODUCTION
WHY YOU CAN'T GET FREE QUARKS

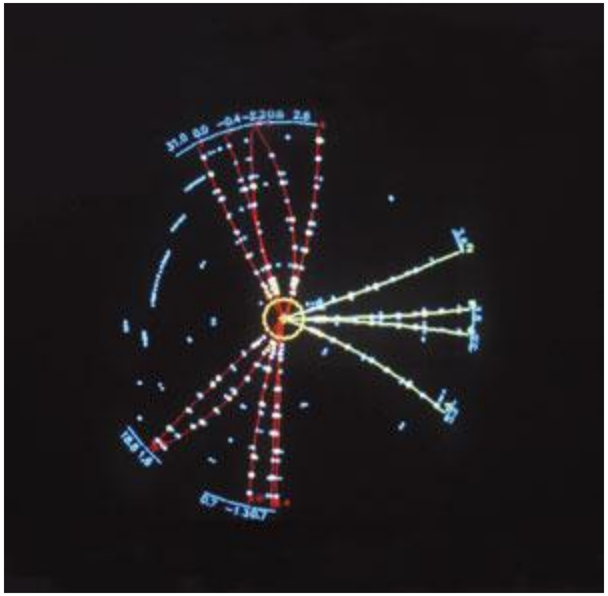
- Suppose we have a meson and we try to pull it apart. The potential energy in the quark-antiquark color field starts to increase.
- Eventually, the energy in the gluon field gets big enough that the gluons can pair-produce another quark-antiquark pair.
  - The new quarks pair up with the original quarks to form mesons, and thus our four quarks remain confined in colorless states.
- Experimentally, we see two particles!



HADRONIC JETS

- The process just described is observed experimentally in the form of hadron jets.
- In a collider experiment, two particles can annihilate and form a quark-antiquark pair.
- As the quarks move apart, the color lines of force are stretched until the potential energy can create another quark-antiquark pair.
- This process continues until the quarks' kinetic energy is low enough that clusters of colorless particles form.
- The experimentalist then detects several "jets" of hadrons, but never sees free quarks or gluons.





Jet formation at TASSO detector at PETRA

ASYMPTOTIC FREEDOM

- As mentioned earlier, perturbation theory can only be applied when the coupling constant α is small.
- At these lower energy regimes of jet formation, α_s is of the order of unity, and that means we can't ignore the many-vertex Feynman diagrams as we do in QED (we can't treat QCD perturbatively!).
- However, as we already saw, the coupling constant is actually not a constant at all, and depends on the energy of the interaction.
- As the energy increases, the coupling constant becomes smaller.
- In fact, at high enough energies, α_s gets so small that QCD can be dealt with as a perturbative theory (e.g. LHC high-energy collisions!)

ASYMPTOTIC FREEDOM

- Asymptotic freedom: as the energy of interactions goes up, QCD asymptotically approaches a regime in which quarks act like free particles.
  - Looking at quarks with a very high energy probe.
- D. Gross, H. Politzer, F. Wilczek (1970's): asymptotic freedom suggests that QCD can be a valid theory of the strong force.

The Nobel Prize in Physics 2004
David J. Gross, H. David Politzer, Frank Wilczek

David J. Gross
Prize share: 1/3

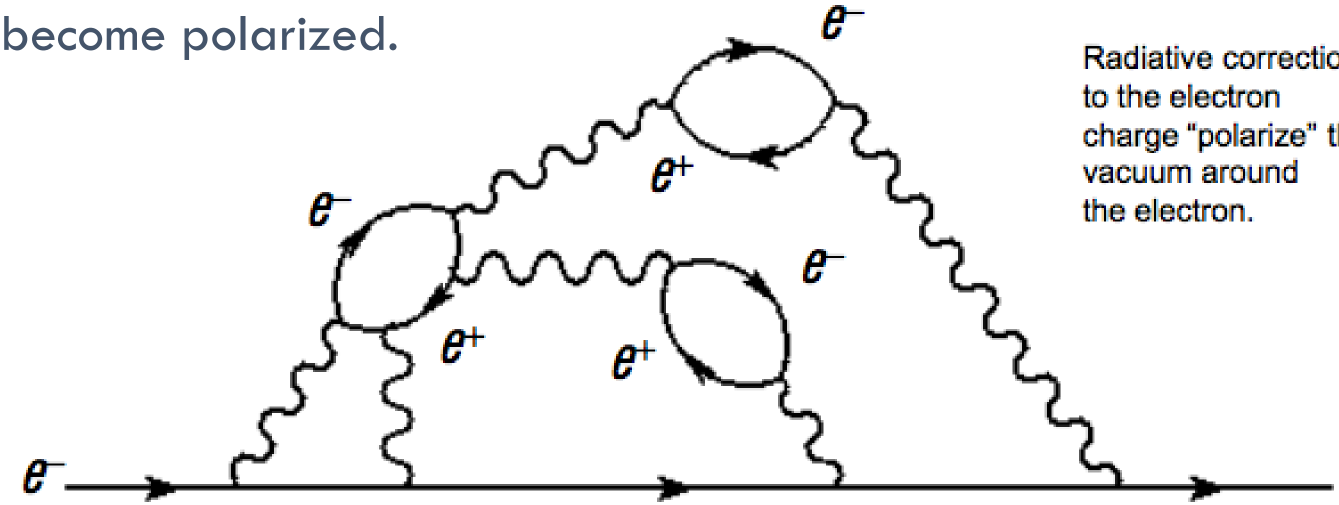
H. David Politzer
Prize share: 1/3

Frank Wilczek
Prize share: 1/3

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek *"for the discovery of asymptotic freedom in the theory of the strong interaction"*.

BACK TO QED
POLARIZATION OF THE VACUUM

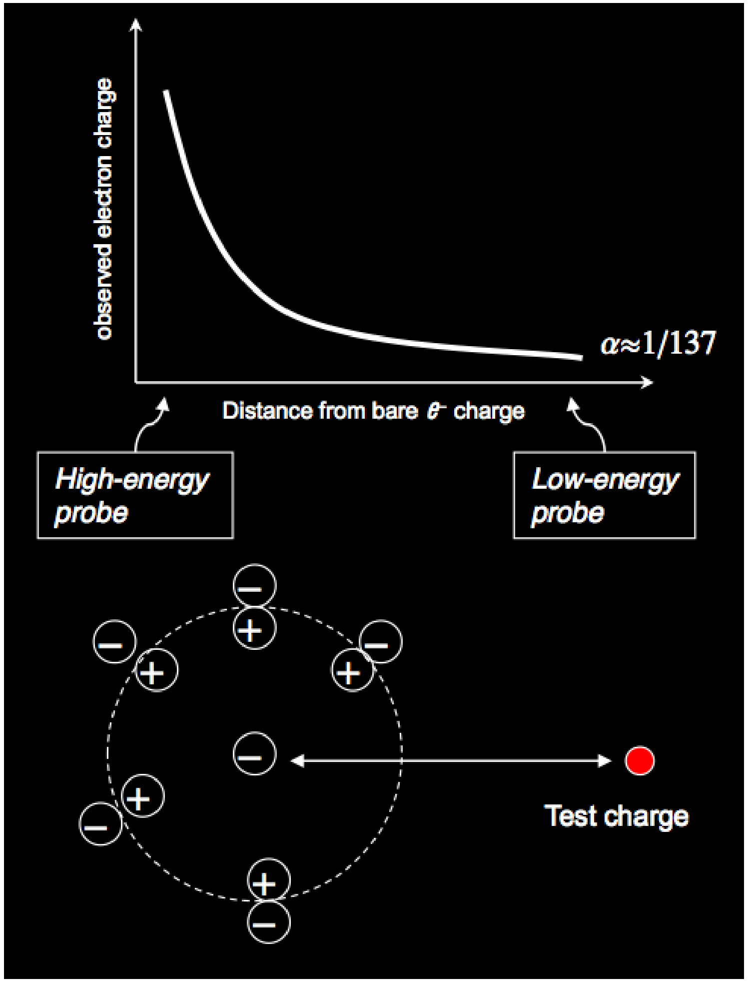
- The vacuum around a moving electron becomes populated with virtual e^+e^- pairs.
  - This is a purely quantum effect, and is allowed by Heinsenberg's Uncertainty Principle.
- Because opposite charges attract, the virtual positrons in the e^+e^- loops will be closer to the electron.
- Therefore, the vacuum around the electron becomes polarized (a net electric dipole develops), just like a dielectric inside a capacitor can become polarized.



Radiative corrections to the electron charge "polarize" the vacuum around the electron.

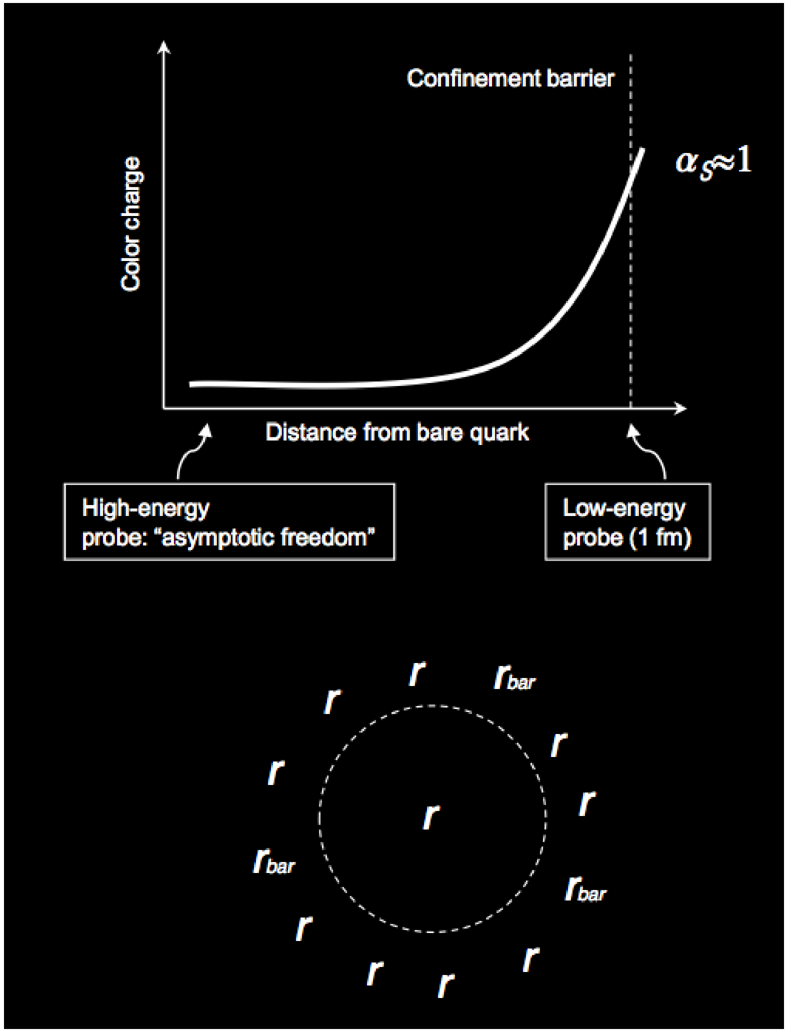
QED: CHARGE SCREENING

- Now, suppose we want to measure the charge of the electron by observing the Coulomb force experienced by a test charge.
- Far away from the electron, its charge is screened by a cloud of virtual positrons and electrons, so the effective charge is smaller than its **bare** charge.
- As we move closer in, fewer positrons are blocking our line of sight to the electron.
- Hence, with decreasing distance, the effective charge of the electron increases.
- **We can think of this as α increasing with energy.**



QCD: CHARGE ANTI-SCREENING

- In QCD, the additional gluon loop diagrams reverse the result of QED:
  - A red charge is preferentially surrounded by other red charges.
- By moving the test probe closer to the original quark, the probe penetrates a sphere of mostly red charge, and the measured red charge decreases.
- This is "antiscreening".
- **We can think of this as α_s decreasing with energy.**



RUNNING CONSTANTS

- As we probe an electron at increasingly higher energies, its effective charge increases.
- This can be rephrased in the following way: as interactions increase in energy, the QED coupling strength α between charges and photons also increases.
  - This should not really be a surprise; after all, the coupling strength of EM depends directly on the electron charge.
- Since α is not a constant, but a (slowly-varying) function of energy, it is called a **running coupling constant**.
- In QCD, the net effect is that the quark color charge and α_s decrease as the interaction energy goes up.

UNDERLYING SOURCE
SELF-INTERACTIONS OF THE MEDIATORS

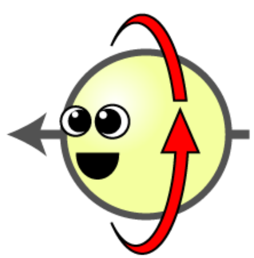
- Gluon self-interaction!
- W and Z (weak force mediators) also self-interact.
  - Similar behavior.
  - The weak coupling constant also decreases as the energy scale goes up.

PARTICLE HELICITY
ASIDE

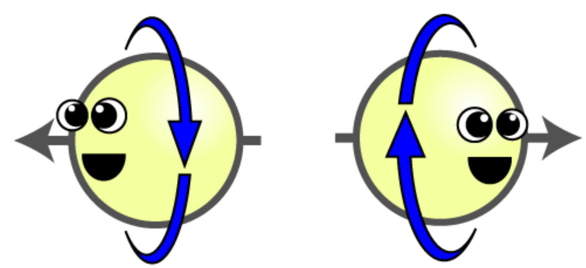
- Given that angular momentum is:
  - Conserved
  - Quantized
  - Constrained by yet another Heisenberg relation:
    - Only one of the x, y, z components of angular momentum can be measured with arbitrary precision

$$\Delta L_x \Delta L_y \geq \frac{\hbar}{2}$$

- Define helicity as the projection of the particle's angular momentum (e.g., spin) on its direction of travel

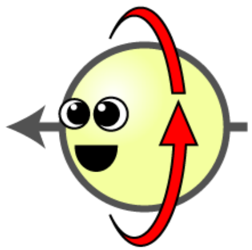


Right-handed:
Spin in same direction as motion



Left-handed:
Spin in opposite direction to motion

PARTICLE HELICITY
ASIDE



Right-handed:
Spin in same direction as motion

- Helicity flips when looked at in a mirror
  - Equivalent to inverting one of the space coordinates (z → -z)

WEAK INTERACTIONS

- Unlike electromagnetism and the strong interaction, the weak interaction is mediated by massive bosons:
  - $m_Z = 91.19$ GeV
  - $m_W = 80.39$ GeV
- This makes it extremely short-range

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

- And very weak at low energies

- Only left-handed particles* participate in weak interactions
  - Nature is different if looked at in a mirror!
  - * and right-handed anti-particles

WEAK INTERACTIONS

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- And very wea

$\geq \frac{\hbar}{2}$

Neutrinos **only** interact via the weak force: are there **right-handed** neutrinos at all?!

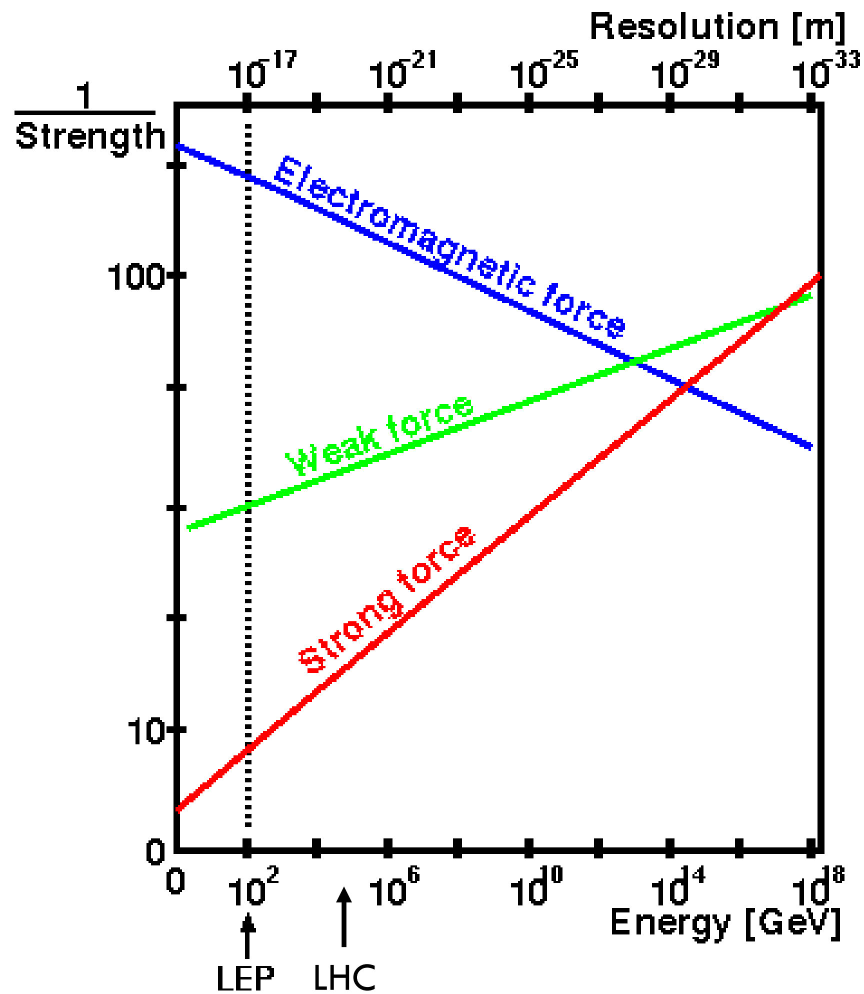
- Only left-handed particles* participate in weak interactions
  - Nature is different if looked at in a mirror!
  - * and right-handed anti-particles

WEAK INTERACTIONS

- At low energies, the effective weak coupling strength is 1000 times smaller than the electromagnetic force.
- As interaction energies start to approach the mass-energy of the W and Z particles (~100 GeV), the effective coupling rapidly approaches the intrinsic strength of the weak interaction α_W.
  - At these energies, the weak interaction actually dominates over electromagnetism.
- Beyond that, the effective weak coupling starts to decrease.

FORCE UNIFICATION
ASIDE

- At laboratory energies near M_W O(100) GeV, the measured values of the coupling constants are quite different.
- However, their "running" trends suggest that they approach a common value near 10^{16} GeV.
- This is an insanely high energy!
- The Standard Model provides no explanation for what may happen beyond this unification scale, nor why the forces have such different strengths at low energies.



PARTICLE/FIELD FORMULATION

PARTICLE/FIELD FORMULATION

- In particle physics, we define fields like φ(x,t) at every point in spacetime.
- These fields don't just sit there; they fluctuate about some minimum energy state.
- The oscillations combine to form wavepackets.
- The wavepackets move around in the field and interact with each other. We interpret them as elementary particles.
- Terminology: the wavepackets are called the **quanta** of the field φ(x,t).

PARTICLE/FIELD FORMULATION

- How do we describe interactions and fields mathematically?
- Classically,

  Lagrangian L = kinetic energy - potential energy

- Particle physics:
  - Same concept, using Dirac equation to describe free spin-1/2 particles:

$$L = \bar{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi$$

Field

Ψ = wavefunction

m = mass

γ^{μ} = μ^{th} gamma matrix

∂_{μ} = partial derivative

LAGRANGIAN MECHANICS

- Developed by Euler, Lagrange, and others during the mid-1700's.
- This is an energy-based theory that is equivalent to Newtonian mechanics (a force-based theory, if you like).

$$\frac{\partial \mathcal{L}}{\partial x_i} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_i}\right)$$

- **Lagrangian:** quantity that allows us to infer the dynamics of a system.

STANDARD MODEL LAGRANGIAN

Free Fields

Interaction

$$\mathcal{L} \quad = \quad \underbrace{\mathcal{L}_0}_{\text{Free Fields}} + \underbrace{\mathcal{L}'}_{\text{Interaction}}$$

STANDARD MODEL LAGRANGIAN

Free Fields

Interaction

$$\mathcal{L} \quad = \quad \mathcal{L}_0 + \mathcal{L}'$$

Gauge Bosons

$$\mathcal{L}_0 \quad = \quad -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi$$

Fermions

$$F_{\mu\nu}F^{\mu\nu} \quad = \quad G_{\mu\nu}G^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}$$

STANDARD MODEL LAGRANGIAN

Free Fields

Interaction

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

Gauge Bosons

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi$$

Fermions

$$\mathcal{L}' = e\bar{\psi}\gamma^\mu A_\mu\psi$$

Fermion-Boson Coupling

$$\begin{aligned} eA_\mu &= \frac{g_s}{2}\lambda_\nu G_\mu^\nu + \frac{g}{2}\vec{\tau}\,\vec{W}_\mu + \frac{g'}{2}YB_\mu \\ F_{\mu\nu}F^{\mu\nu} &= G_{\mu\nu}G^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu} \end{aligned}$$

STANDARD MODEL LAGRANGIAN

Free Fields

Interaction

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

Gauge Bosons

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi$$

Electroweak bosons and interaction

Fermions

$$\mathcal{L}' = e\bar{\psi}\gamma^\mu A_\mu\psi$$

Gluons and strong interaction

Fermion-Boson Coupling

$$eA_\mu = \boxed{\frac{g_s}{2}\lambda_\nu G^\nu_\mu} + \boxed{\frac{g}{2}\vec{\tau}\,\vec{W}_\mu + \frac{g'}{2}YB_\mu}$$

$$F_{\mu\nu}F^{\mu\nu} = \boxed{G_{\mu\nu}G^{\mu\nu}} + \boxed{W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}}$$

STANDARD MODEL LAGRANGIAN

- Encompasses all theory.
  - From the Lagrangian to cross-section predictions:

$$\boxed{\sigma \sim \langle\, f\,|\mathbf{S}|\,i\,\rangle^{\,2}}$$

Inelastic
Cross Section
[for |i⟩ ≠ |f⟩]

$$[\mathrm{Def.}:\ |\,t=+\infty\,\rangle \equiv \mathbf{S}|\,t=-\infty\,\rangle\,]$$

Time Evolution
From Schrödinger-Equation
[Dirac picture]

$$|\,t\,\rangle = |\,t_0\,\rangle - i\int_{t_0}^{t} \mathrm{d}t'\,\mathbf{H}'(t')|\,t'\,\rangle$$

$$\mathbf{H}'(t) = -\int \mathcal{L}'(x,t)\,\mathrm{d}^3x$$

Lagrangian
of Interaction

$$\langle\, f\,|\mathbf{S}|\,i\,\rangle \cong \delta_{fi} - i\int_{-\infty}^{\infty} \mathrm{d}t'\langle\, f\,|\mathbf{H}'(t')|\,i\,\rangle$$

➥ Feynman rules

SYMMETRIES AND INVARIANCE

- Noether's theorem (1915):
  - Every symmetry under some operation corresponds to a conservation law

symmetry	invariant
Space translation	momentum
Time translation	energy
Rotation	Angular momentum
Global phase; $\Psi \rightarrow e^{i\theta}\Psi$	Electric charge
Local phase; $\Psi \rightarrow e^{i\theta(x,t)}\Psi$	Lagrangian + gauge field ($\rightarrow$ QED)

SYMMETRIES AND INVARIANCE

- There are carefully chosen sets of transformations for Ψ which give rise to the observable gauge fields:
  - That is how we get electric, color, weak charge conservation!

QED FROM LOCAL GAUGE INVARIANCE

- Apply local gauge symmetry to Dirac equation:

$$\bar{\Psi} \to e^{i\theta(x,t)}\bar{\Psi}, \ \Psi \to e^{-i\theta(x,t)}\Psi$$

This type of transformation leaves quantum mechanical amplitudes invariant.

- Consider very small changes in a field:

$$\Psi \to \Psi + \delta\Psi = \Psi - i\theta(x,t)\Psi \quad \text{ie. } \delta\Psi = -i\theta(x,t)\Psi$$

- The effect on the Lagrangian is:

$$L = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi \Rightarrow \delta L = \bar{\Psi}\gamma^\mu \partial_\mu \theta(x,t)\Psi$$

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For the Lagrangian to remain invariant: $\delta L = 0$

QED FROM LOCAL GAUGE INVARIANCE

- To satisfy $\delta L=0$, we "engineer" a mathematical "trick":
  1. Introduce a gauge field A_μ to interact with fermions, and A_μ transform as: $A_\mu+\delta A_\mu= A_\mu+ 1/e\ \partial_\mu\theta(x,t)$
  2. In resulting Lagrangian, replace $\partial_\mu \to D_\mu= \partial_\mu+ ieA_\mu$

- In that case, L is redefined:

$$L = \overline{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

The new Lagrangian is invariant under local gauge transformations.

QED FROM LOCAL GAUGE INVARIANCE
ONE MORE THING…

- Need to add kinetic term for field (field strength):

Define $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Add term $-1/4 F_{\mu\nu}F^{\mu\nu}$ (Lorentz invariant, matches Maxwell's equations)

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Final lagrangian (for QED!):

$$L = -1/4 F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

- No mass term is allowed for A_μ, otherwise the Lagrangian is not gauge invariant
  - The gauge field is massless!

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  - The g…

We have mathematically engineered a quantum field that couples to fermions, obeys Maxwell's equations and is massless!

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The **photon**!

BONUS

INTEGRAL OVER "ALL POSSIBLE PATHS"

- Were does the integral notion come from?
- Recall, QM picture of free particle motion: there is some amplitude for a free electron to travel along any path from the source to some point p. Not just the straight, classical trajectory!
  - The word "path" here doesn't only refer to a x(y) path in space, but also the time at which it passes each point in space.
  - In 3D, a path (sometimes called wordline) is defined by three functions x(t), y(t) and z(t). An electron has an amplitude associated with a given path.
- The total amplitude for the electron to arrive at some final point is the sum of the amplitudes of all possible paths. Since there are an infinite number of paths, the sum turns into an integral.

THE QUANTUM MECHANICAL AMPLITUDE

- *Feynman: Each path has a corresponding probability amplitude. The amplitude ψ for a system to travel along a given path x(t) is:*

$$\psi[x(t)] = \text{const}\cdot\ e^{iS[x(t)]/\hbar}$$

where the object S[x(t)] is called the **action** corresponding to x(t).

- The total amplitude is the sum of contributions from each path:

$$\sum_{\text{over all paths}} \psi[x(t)]$$

THE QUANTUM MECHANICAL AMPLITUDE

$$\psi[x(t)] = \text{const} \cdot e^{iS[x(t)]/\hbar}$$

1. What is $e^{iS[x(t)]/\hbar}$?
2. What is the action $S[x(t)]$?

UNDERSTANDING THE PHASE

- You may not have seen numbers like $e^{i\theta}$, so let's review.
- Basically, $e^{i\theta}$ is just a fancy way of writing sinusoidal functions; from Euler's famous formula:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

- Note: those of you familiar with complex numbers (of the form $z=x+iy$) know that $e^{i\theta}$ is the phase of the so-called polar form of z, in which $z=re^{i\theta}$, with:

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

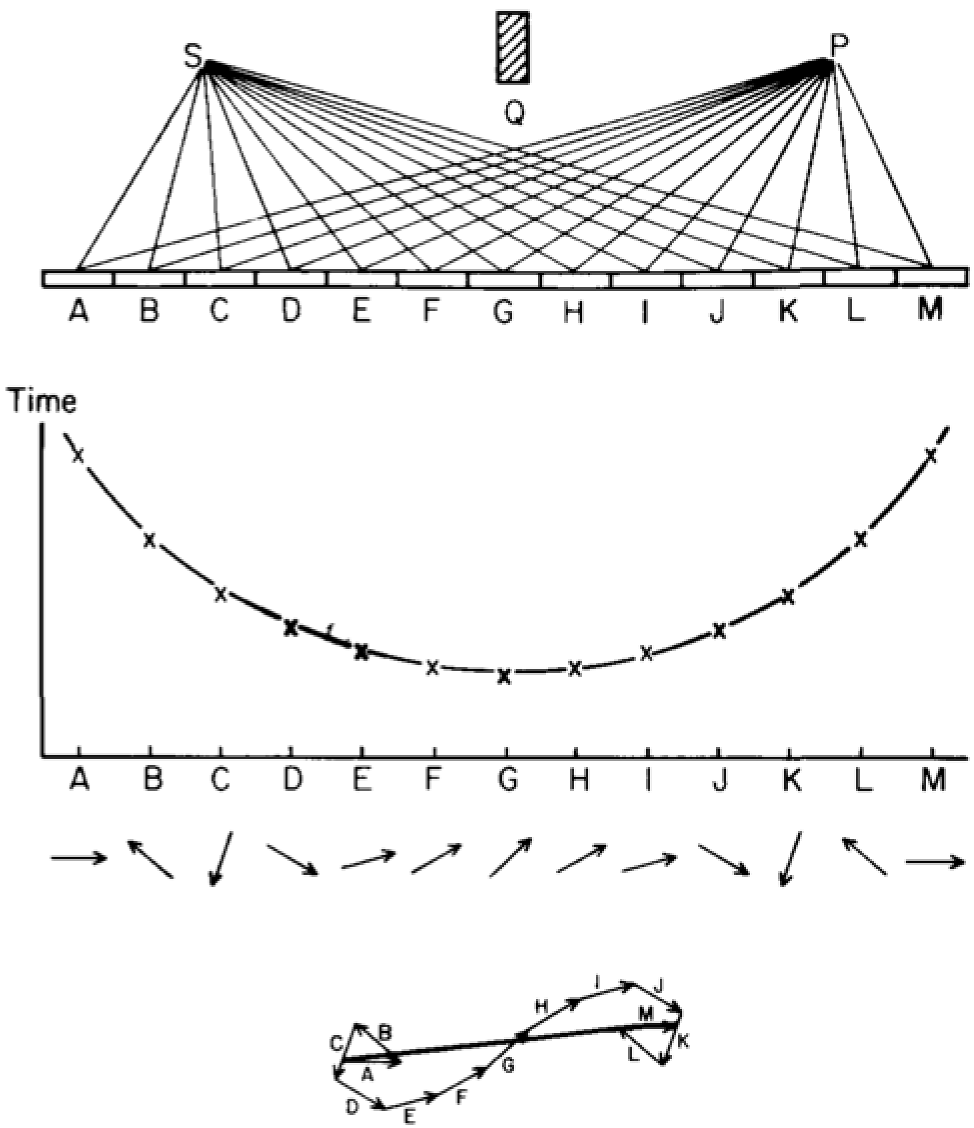
COMMENTS ON THE AMPLITUDE

- Now we can understand the probability amplitude $\psi[x(t)] \sim e^{iS[x(t)]/\hbar}$ a little better.
- The amplitude is a sinusoidal function – a wave – that oscillates along the worldline x(t). The frequency of oscillation is determined by how rapidly the action S changes along the path.
- The probability that a particle will take a given path (up to some overall multiplication constant) is:

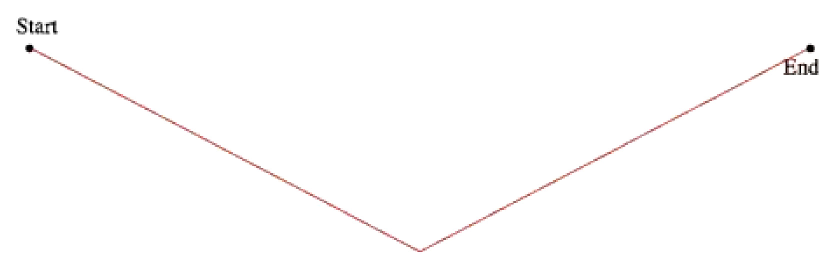
$$
\begin{aligned}
P &\propto |\psi|^2 = \psi^* \psi \\
&\propto e^{-iS[x(t)]/\hbar} e^{iS[x(t)]/\hbar} \\
&= e^{iS[x(t)]/\hbar - iS[x(t)]/\hbar} = e^0 \\
&= 1
\end{aligned}
$$

- This is the same for every worldline. According to Feynman, the particle is equally likely to take any path through space and time!
    - Contributions from “crazy” paths will likely be supressed by interference!

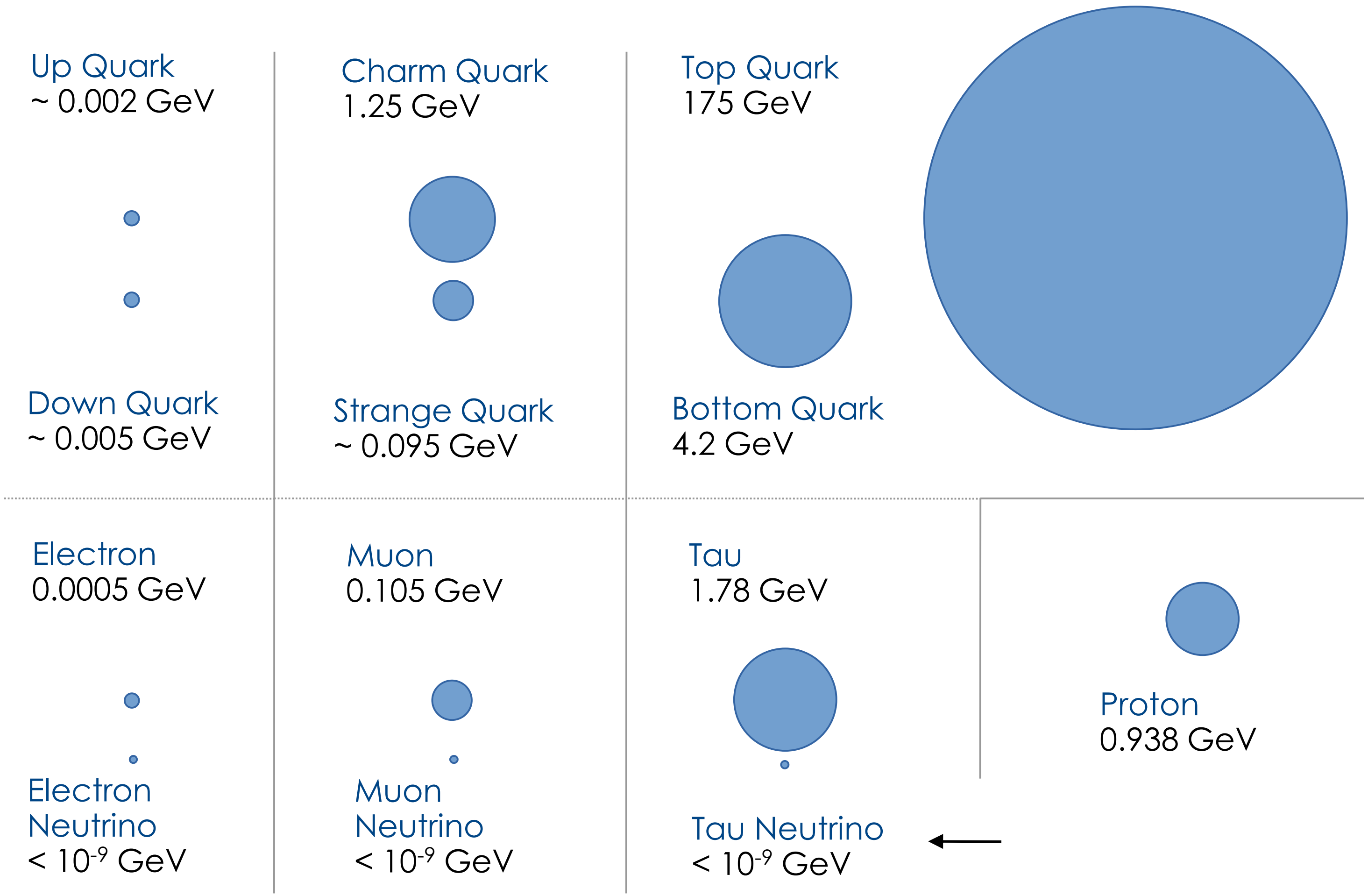
COMMENTS ON THE AMPLITUDE



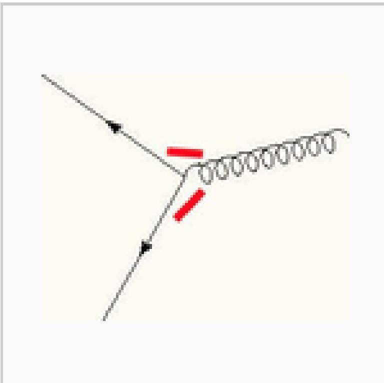
COMMENTS ON THE AMPLITUDE



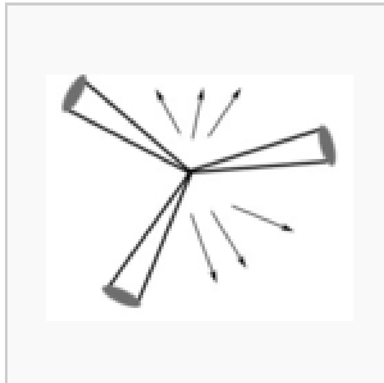
MASSES OF SM FERMIONS



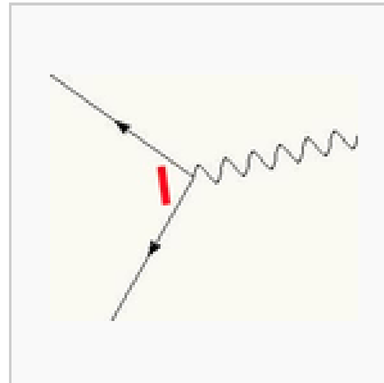
GLUON DISCOVERY AT TASSO, PETRA



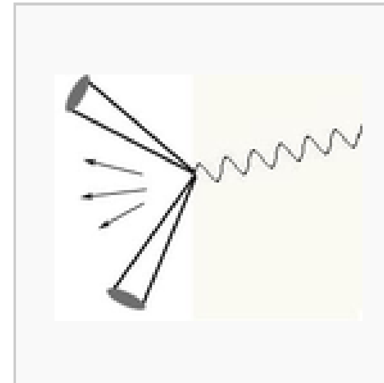
Two quarks (solid lines) and a gluon (curly line) fly apart, with the strings (red bars) primarily between the gluon and each quark.



As a result, three jets (cones) form, with extra hadrons (arrows) found where the strings formed.



For comparison, physicists looked at events with two quarks and a photon (wavy line). Here the string forms only between quarks.



Therefore extra hadrons are found only between the two jets, which is inconsistent with observations.