Particle Physics

Columbia Science Honors Program

Week 3: Special Relativity
October 7th, 2017

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Course Policies

- Attendance:
  - I will take attendance during class.
  - Up to four excused absences (two with notes from parent/guardian)
  - Send notifications of all absences to shpattendance@columbia.edu
- Valid excuses:
  - Illness, family emergency, tests or athletic/academic competitions, mass transit breakdowns
- Invalid excuses: sleeping in, missing the train
- Please no cell phones.
- Ask questions :)

Lecture Materials

- https://twiki.nevis.columbia.edu/twiki/bin/view/Main/ScienceHonorsProgram

Schedule

1. ~~Introduction~~
2. ~~History of Particle Physics~~
3. Special Relativity
4. Quantum Mechanics
5. Experimental Methods
6. The Standard Model - Overview
7. The Standard Model - Limitations
8. Neutrino Theory
9. Neutrino Experiment
10. LHC and Experiments
11. The Higgs Boson and Beyond
12. Particle Cosmology

Last week…

History of Particle Physics

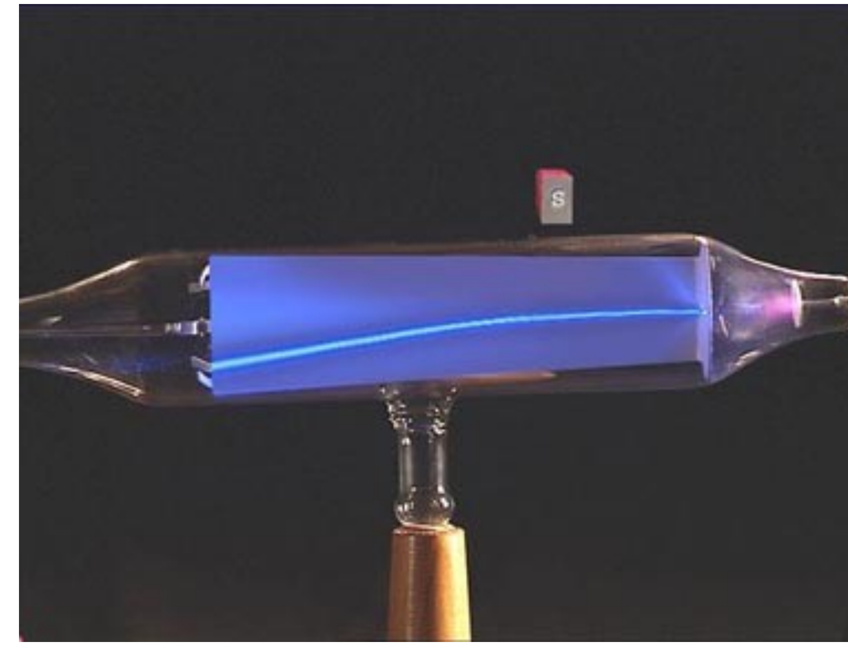
- Late 1800's to today:
  - Discoveries of Standard Model particles, particle properties, realization of fundamental symmetries, experimental triumphs, …
  - … but also a lot of puzzles, frustration, confusion…

From Classical Physics…
…to Modern Physics…
…and Particle Physics today.

Discovery of the electron (1897)

- For a number of years, scientists had generated "cathode rays" by heating filaments inside gas-filled tubes and applying an electric field.
  - Recall: we know cathode rays have electric charge, because they can be deflected by magnetic fields.
  - Question: are cathode rays some kind of charge fluid, or are they made of charge particles (like ions)?

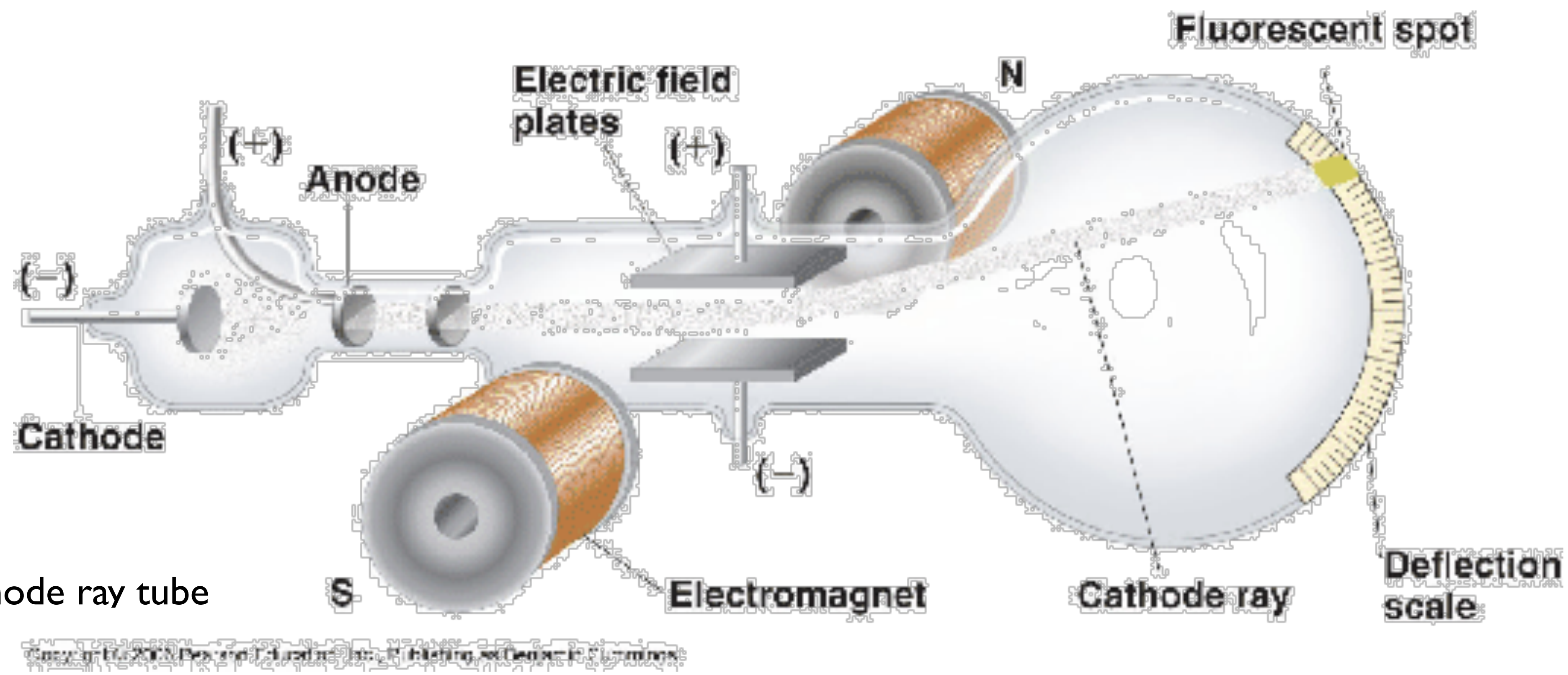
- In 1897, J.J. Thompson attempted a measurement of the charge / mass ratio of cathode rays to see if they were particles.



J. J. Thompson

Discovery of the electron (1897)

- Put a cathode ray into a known electric magnetic field.
- Measure the cathode ray's deflection.



Thompson's cathode ray tube

- If cathode rays are composed of discrete charges, their deflection should be consistent with the Lorentz Force Law:

$$\vec{F} = q\,(\vec{v} \times \vec{B} + \vec{E})$$

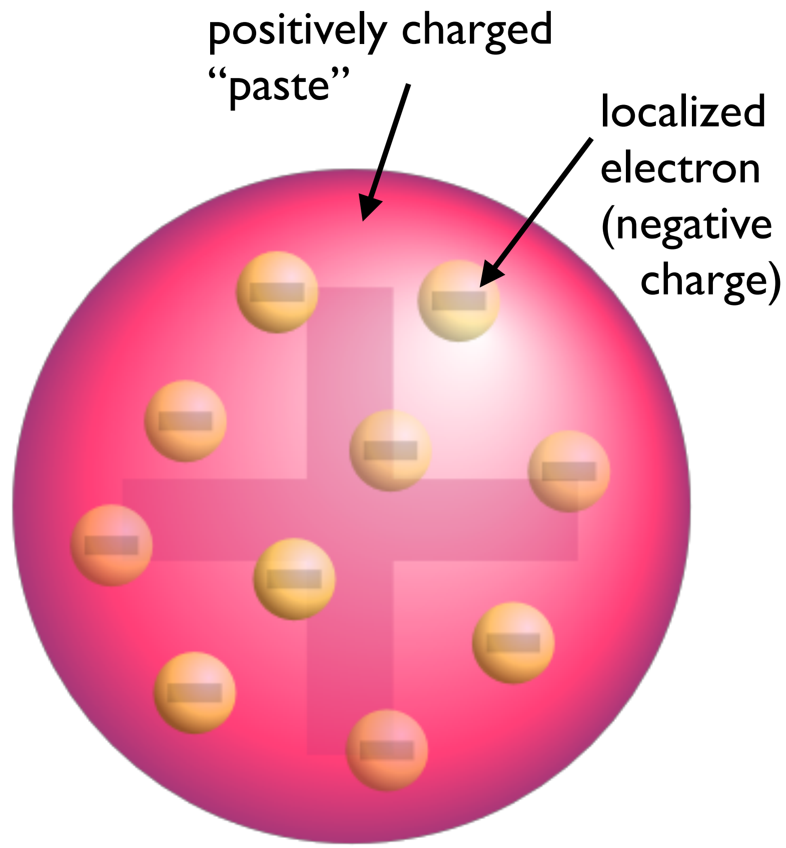
Discovery of the electron (1897)

- Thomson found that cathode ray deflections were consistent with the Lorentz Force, and could be particles ("corpuscules") after all.
- The charge to mass ratio e/m was significantly larger than for any known ion (over 1000x e/m of hydrogen). This could mean two things:
  1. **The charge e was very big.**
  2. **The mass m was very small.**
- Independent measurements of e (oil drop experiment) suggested that, in fact, cathode rays were composed of extremely light, negatively charged particles.
- Thomson called his corpuscle's charge the **electron** (from the Greek "amber"); eventually, this term was applied to the particles themselves, whose mass is:

$$\boxed{m_e = 0.511\ \mathrm{MeV}/c^2}$$

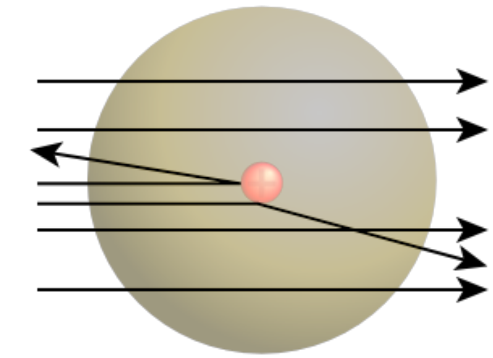
Discovery of the electron (1897)

- Thompson correctly believed that electrons were fundamental components of atoms (e.g. responsible for chemical behavior).
- Because atoms are electrically neutral, he concluded that the negatively charged point-like electrons must be embedded in a "gel" of positive charge such that the entire atoms is neutral.
- Thompson: electrons are contained in an atom like "plums in a pudding".

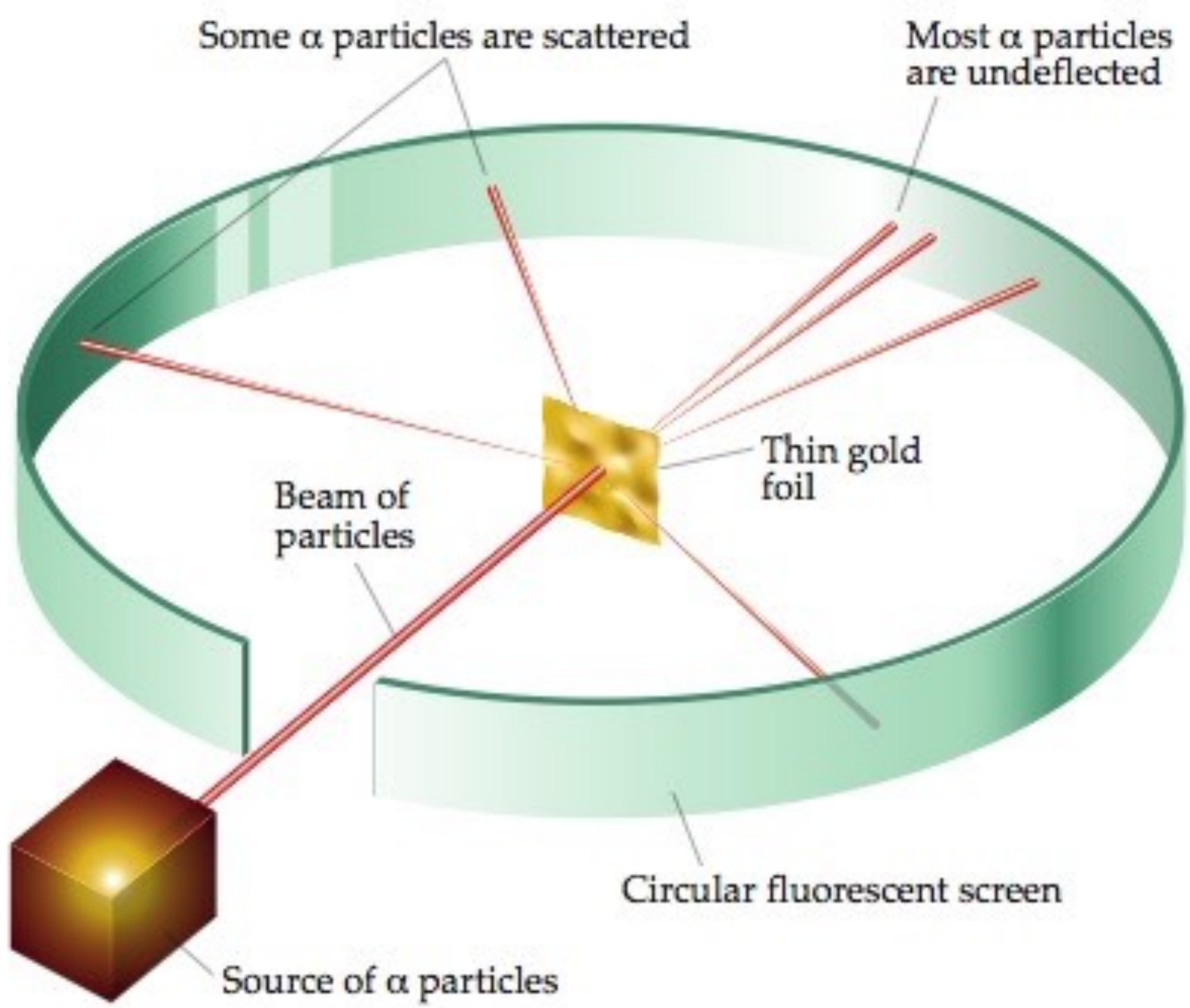


Thomson's plum-pudding model of the atom.

Rutherford Experiment

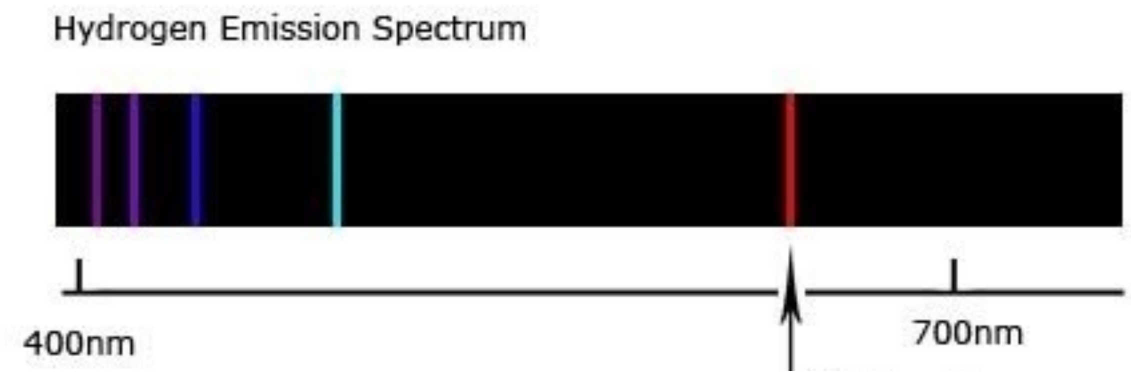


- Gold-foil experiment:



- Most α-particles were not scattered at all, but a few were scattered through angles of 90° or more!!
- Rutherford: large-angle scattering is exactly consistent with Coulomb repulsion of two small, dense objects.
- **Conclusion:** scattered particle beam is evidence of a dense, compact, positively-charged structure, located at the center of the atom.

The Bohr atom (1914)



- In 1914, N. Bohr developed a simple atomic model that perfectly explained the phenomenon of spectral lines.
- The three main ideas behind Bohr's semi-classical ansatz:

1. The electron moves in uniform circular motion, with the centripetal force provided by its Coulomb attraction to the nucleus:

$$F_{\text{centripetal}} = m_e \frac{v^2}{r} = \frac{e^2}{r^2} = F_{\text{Coulomb}}$$

2. The angular momentum of the electron in its orbit is quantized, satisfying the constraint:

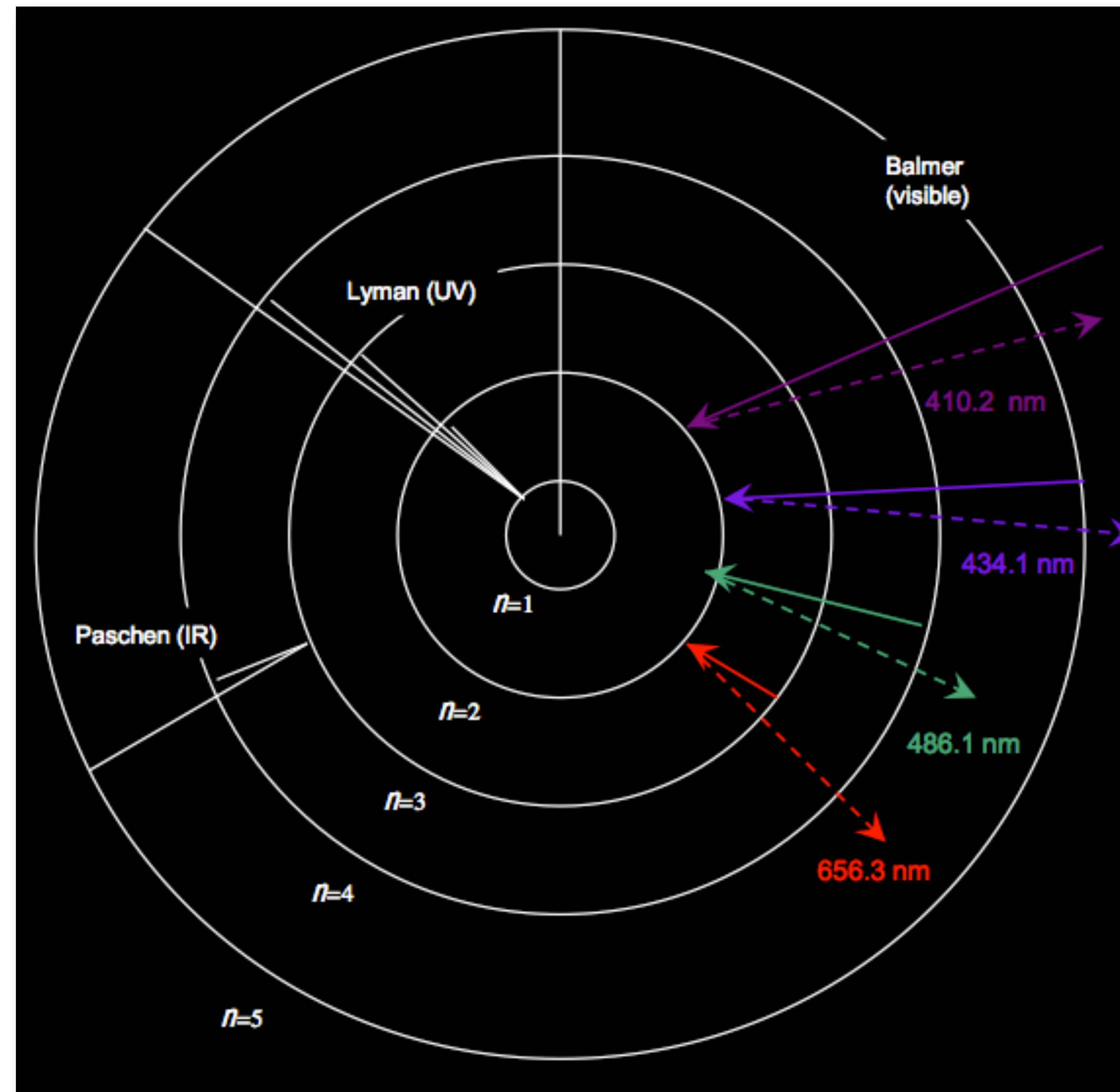
$$m_e v r = n\hbar, \ \ n \text{ is an integer}$$

3. Therefore, the electron can have only a discrete spectrum of allowed energies:

$$E = \frac{1}{2} m_e v^2 - \frac{e^2}{r} = -\frac{1}{n^2}\left(\frac{m_e e^4}{2\hbar^2}\right)$$

The Bohr hydrogen model

- In the context of the Bohr model, the discrete spectra seen in atomic spectroscopy make perfect sense.
- The electron occupies discrete orbits in the hydrogen atom.
- When hydrogen is excited in an electric field, the electron jumps into a higher energy orbit.
- Eventually, the electron will return to a lower energy state.
- Once this happens, light must be emitted to conserve the energy of the whole system.



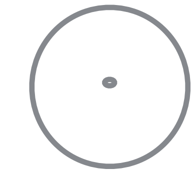
Emission spectrum of the hydrogen atom.

J. Chadwick
Image: NobelPrize.org

Discovery of the Neutron (1932)

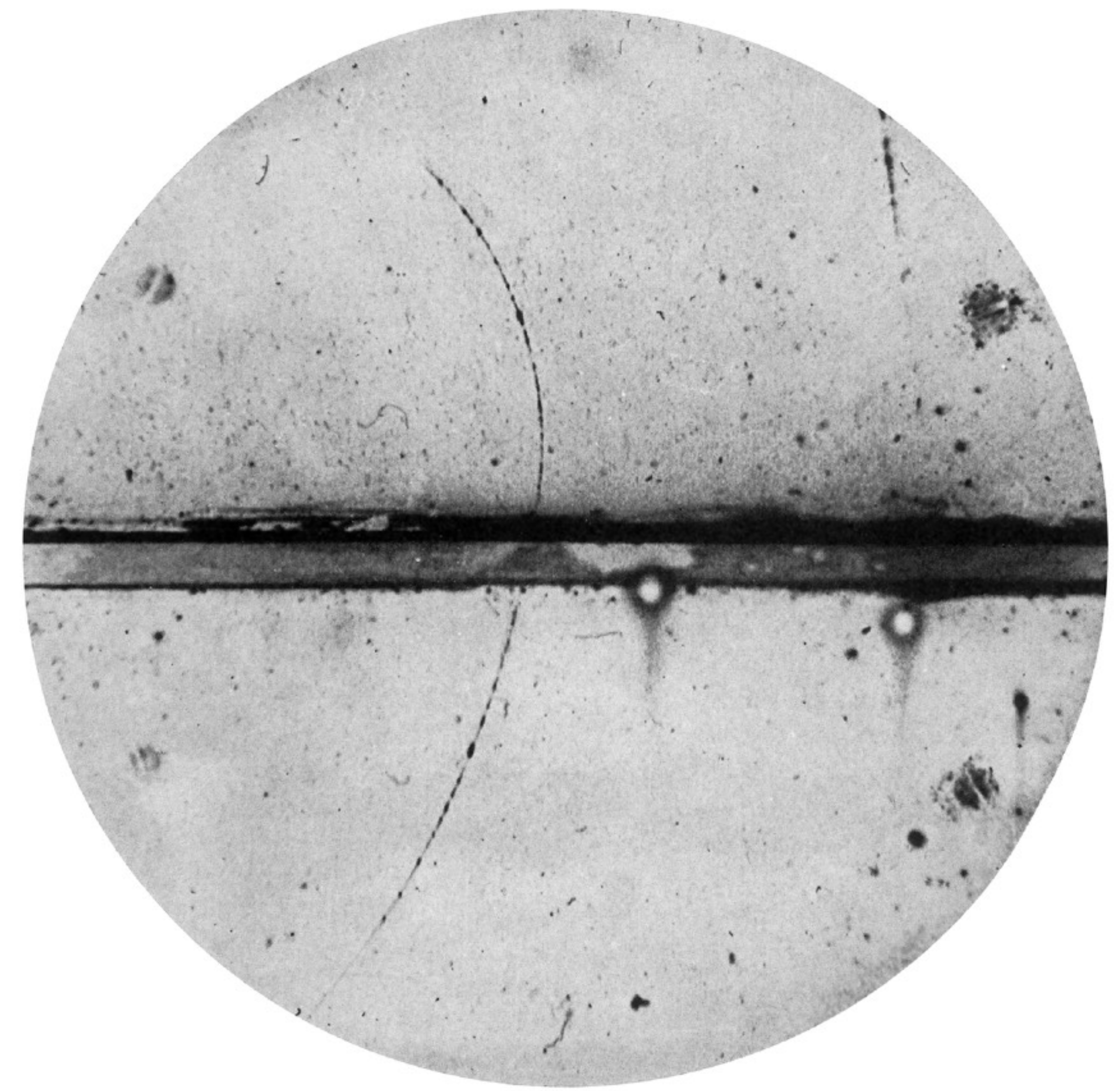
- In the Bohr atomic model, atoms consisted of just protons and electrons.
- However, there was a major problem: most elements were heavier than they should have been.
  - He charge is +2e, but weighs $4m_p$;
  - Li charge +3e, but weighs $7m_p$; etc.
- To account for the missing mass in heavier elements, nuclei had to contain other particles comparable in mass to the proton (1 GeV/c^2), but with no electric charge.
- The mysteries massive, neutral particle inside atomic nuclei eluded detection until 1932, when J. Chadwick observed the neutron in an α-Be **scattering experiment**.

Magnetic field



Discovery of antimatter (1932)

- In 1932, C. Anderson observed the anti-electron (positron), validating Dirac's theory.
- Feynman's explanation of negative energies: they are the positive energy states of anti-particles!
- Anti-matter is a universal feature of quantum field theory: all particles have matching anti-particles.
- Anti-particles have the same mass as their particle partners, but opposite quantum numbers (charge, lepton number, etc.).



Discovery of the positron in a cloud chamber by C. Anderson

Image: J. Griffiths, *Intro to Elementary Particles*

Notation: Particle e^-, p

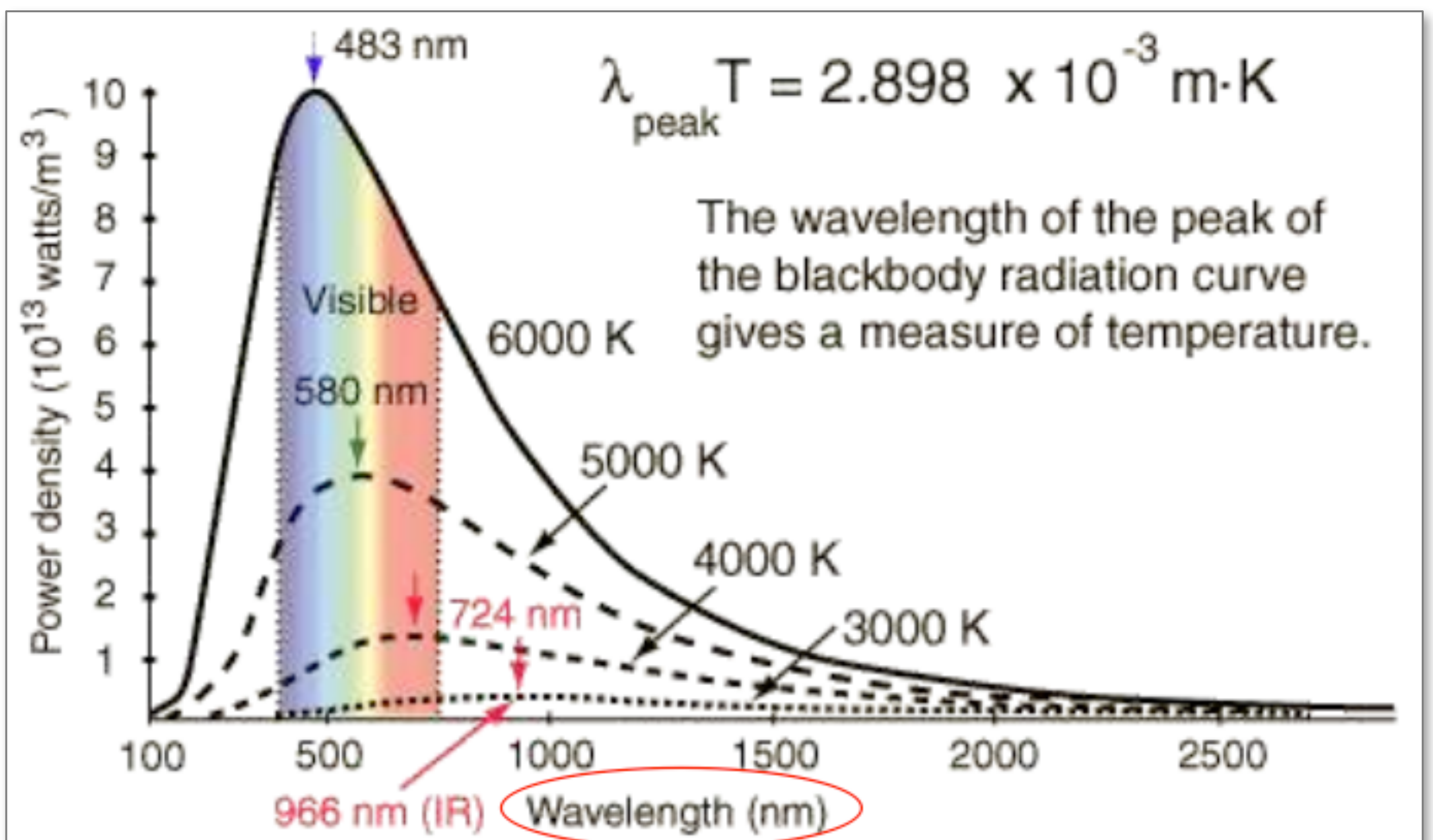
Antiparticle $e^+, \bar{e}, \bar{p}$

Meanwhile… (1900-1924)

- **A new particle, the electromagnetic field quantum**
- The discovery of the photon, the quantum of the electromagnetic field, marked a major departure from classical physics.
- As with the developing picture of the atom, it took several decades (and several incontrovertible experiments) before physicists accepted the existence of the photon.
- But, before we get into that, let's talk about what classical physics actually had to say about electromagnetism.

Failure of classical electrodynamics

- When light is emitted by hot objects, the intensity of the light always varies continuously with the wavelength - unlike atomic spectra - and the spectrum has a characteristic shape.



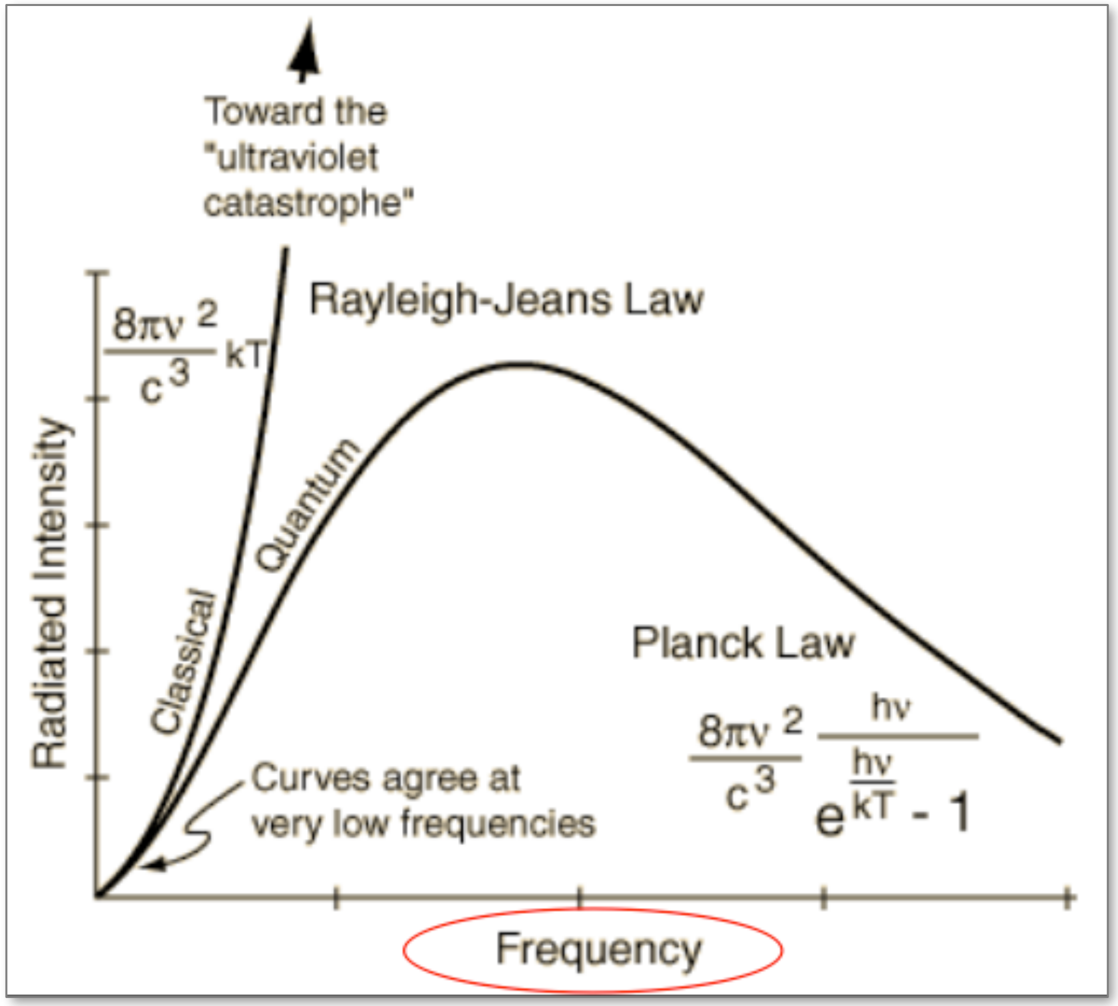
- This so-called **blackbody** spectrum always peaks at a wavelength that depends on the surface temperature of the body.
- Examples of blackbodies: stars, light filaments, toaster coils, the universe itself!

Failure of classical electrodynamics

"Ultraviolet catastrophe"

- A study of blackbody radiation with classical EM and statistical mechanics (the Rayleigh-Jeans Law) predicts that the emitted intensity varies with frequency and temperature as:

$$I_\nu(T) \propto \frac{k_B T}{c^3}\nu^2$$



- This means that as the light frequency increases into the UV, the intensity becomes infinite!
- This nonsensical answer was such an embarrassment for the theory that physicists called it the "ultraviolet catastrophe".

Planck's solution: light quanta

- In 1900, using arguments from statistical mechanics (the theory of bodies in thermal equilibrium), M. Planck derived a theoretical curve that fit the blackbody spectrum perfectly:

$$I_\nu(T) \propto \frac{h}{c^3} \frac{\nu^3}{e^{h\nu / k_B T} - 1}$$

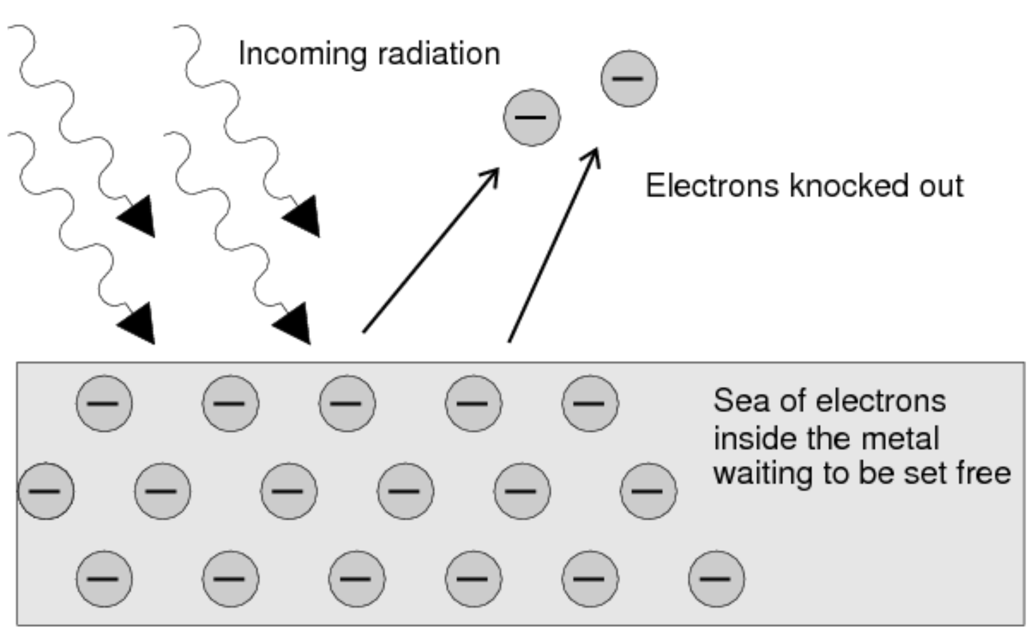
- However, to get this result, Planck had to assume that thermal radiation is quantized; that is, it's emitted in little "packets" of energy, **photons**, proportional to the frequency ν: $\boxed{E = h\nu}$

- The quantity h, called Planck's constant, was determined from the fit to the blackbody spectrum. It turned out to be a fundamental constant of nature, and has the value: $\boxed{h = 4.1357 \times 10^{-15} \text{ eV} \cdot \text{s}}$

Photoelectric effect (1905)

- In the 1800's, it was discovered that shining light onto certain metals liberated electrons from their surface.
- Experiments on this photoelectric effect showed odd results:

1. Increasing the intensity of the light increased the number of electrons, but not the maximum kinetic energy of the electrons.
2. Red light did not liberate electrons, no matter how intense it was!
3. Weak violet light liberated few electrons, but their maximum kinetic energy was greater than that for more intense long-wavelength beams!



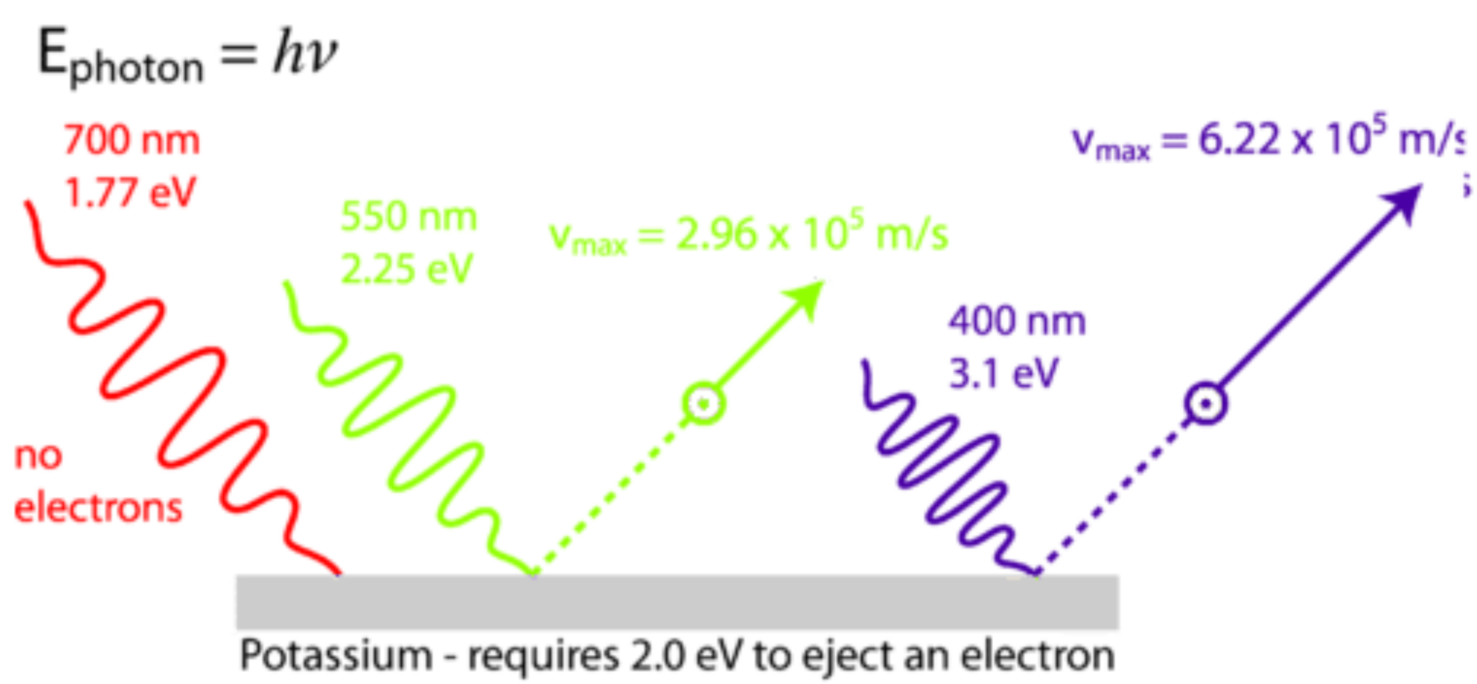
A. Einstein
NobelPrize.org

Photoelectric effect (1905)

- In 1905, A. Einstein showed that these results made perfect sense in the context of quantization of the EM field, where photon energy is proportional to frequency. If photons of energy $E=h\nu$ strike electrons in the surface of the metal, the freed electrons have a kinetic energy:

$$K = h\nu - \phi$$

- The work function ϕ is a constant that depends on the metal.

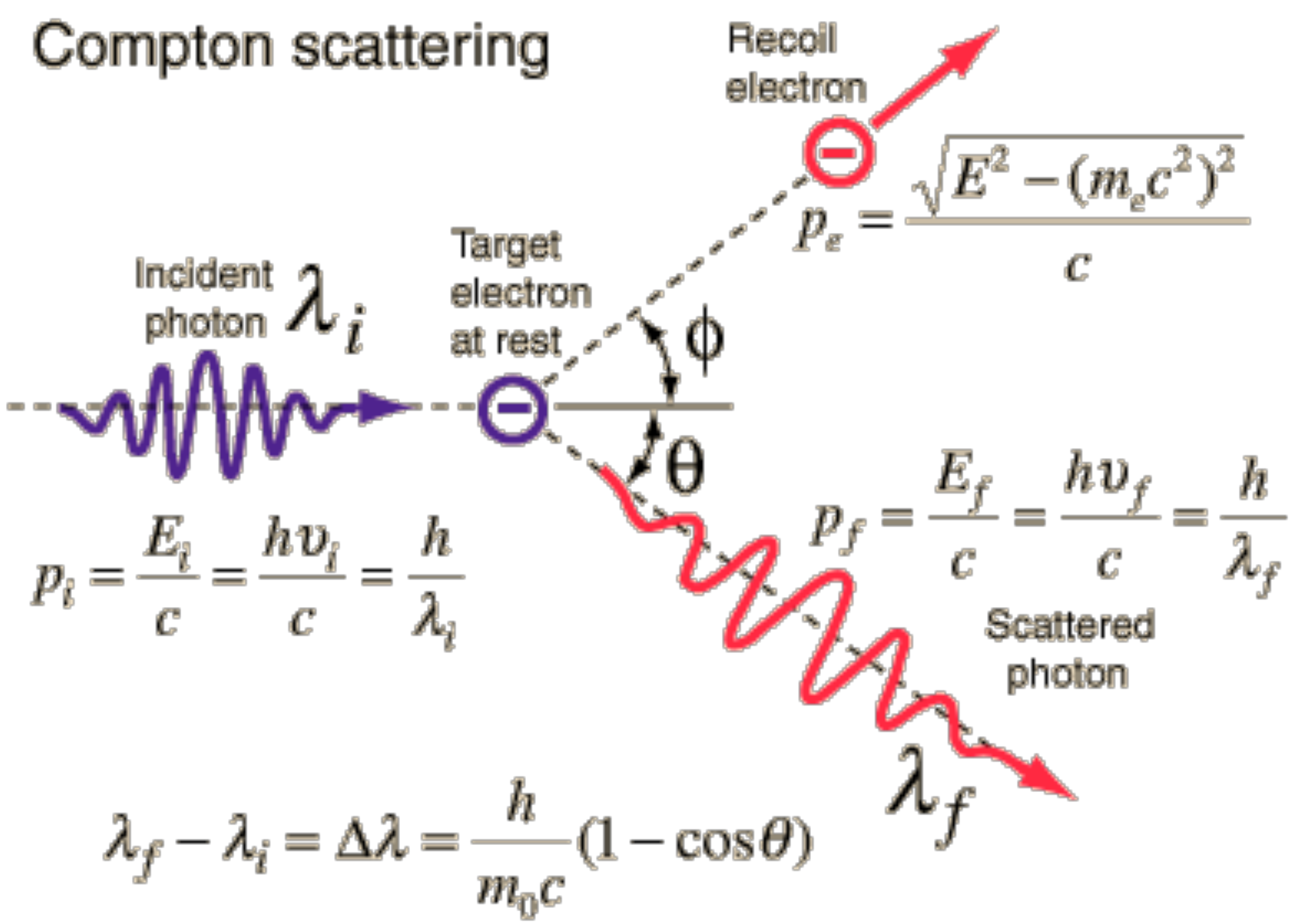


Compton Scattering (1923)

A. H. Compton
NobelPrize.org

- In 1923, A. H. Compton found that light scattered from a particle at rest is shifted in wavelength by an amount:

$$\lambda_f - \lambda_i = \lambda_c(1 - \cos\theta)$$



- Here, λc = h/mc is the Compton wavelength of the target mass m.
- There is no way to derive this formula if you assume light is a wave.
- If you treat the incoming light beam as a particle with energy E=hv, Compton's formula comes right out!

Discovery of neutrinos (1950s)

- By introducing neutrinos (symbol ν) to radioactive decay, conservation of energy was restored. Decay reactions started to look like this:

$$n \to p + e^- + \bar{\nu}$$
$$\pi \to \mu + \nu$$
$$\mu \to e + 2\nu$$

C. Cowan and F. Reines
Image: CUA

- By 1950, there was compelling theoretical evidence for neutrinos, but no neutrino had ever been experimentally isolated.
- Finally, in the mid-1950s, C. Cowan and F. Reines came up with a method to directly detect neutrinos using "inverse" β-decay: $\bar{\nu} + p \to n + e^+$
- **A difficult experiment.** Cowan and Reines set up a large water tank outside a commercial nuclear reactor, expecting to see evidence of the above reaction only 2 to 3 times per hour, which they did. **Conclusion: (anti-) neutrinos exist!**

Discovery of strange particles (1947)

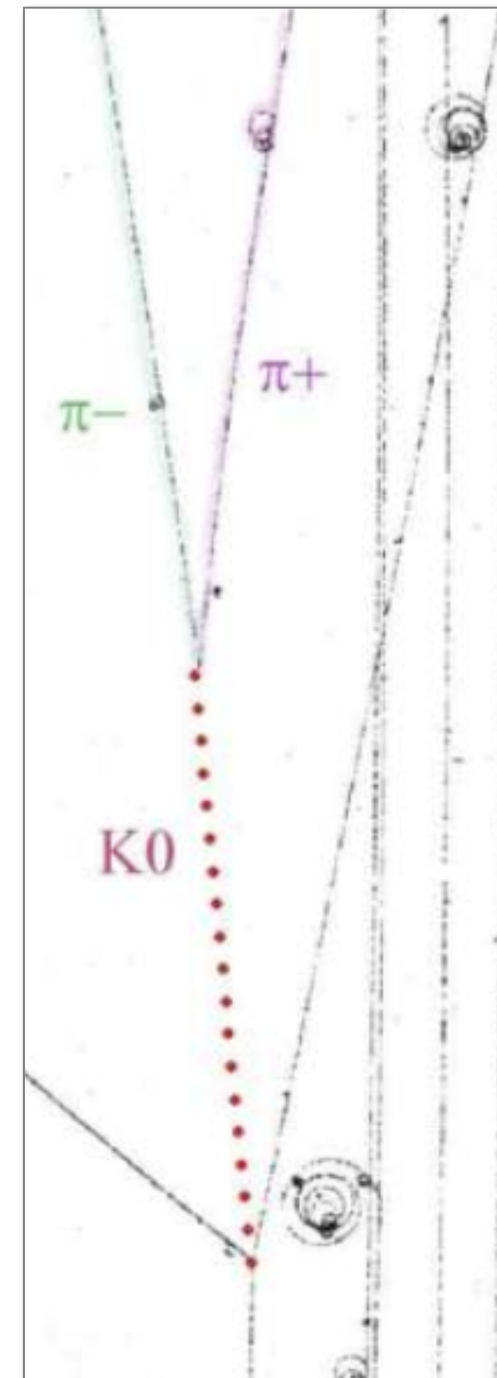
- By 1947, the catalog of elementary particles consisted of the p, n, π, μ, e, and the ν (and the anti-particles). The overall scheme seemed pretty simple.
- However, at the end of that year, a new neutral particle was discovered: the K0 ("kaon"):

$$K^0 \rightarrow \pi^+ + \pi^-$$

- In 1949, a charged kaon was found:

$$K^+ \rightarrow \pi^+ + \pi^+ + \pi^-$$

- The K's behaved somewhat like heavy π's, so they were classified as mesons ("mass roughly between the proton and electron mass").
- Over the next two decades, many more mesons were discovered: the η, the φ, the ω, the ρ's, etc.



Bubble chamber photo of kaon decay (U of Birmingham)

Summary of particle zoo (1960)

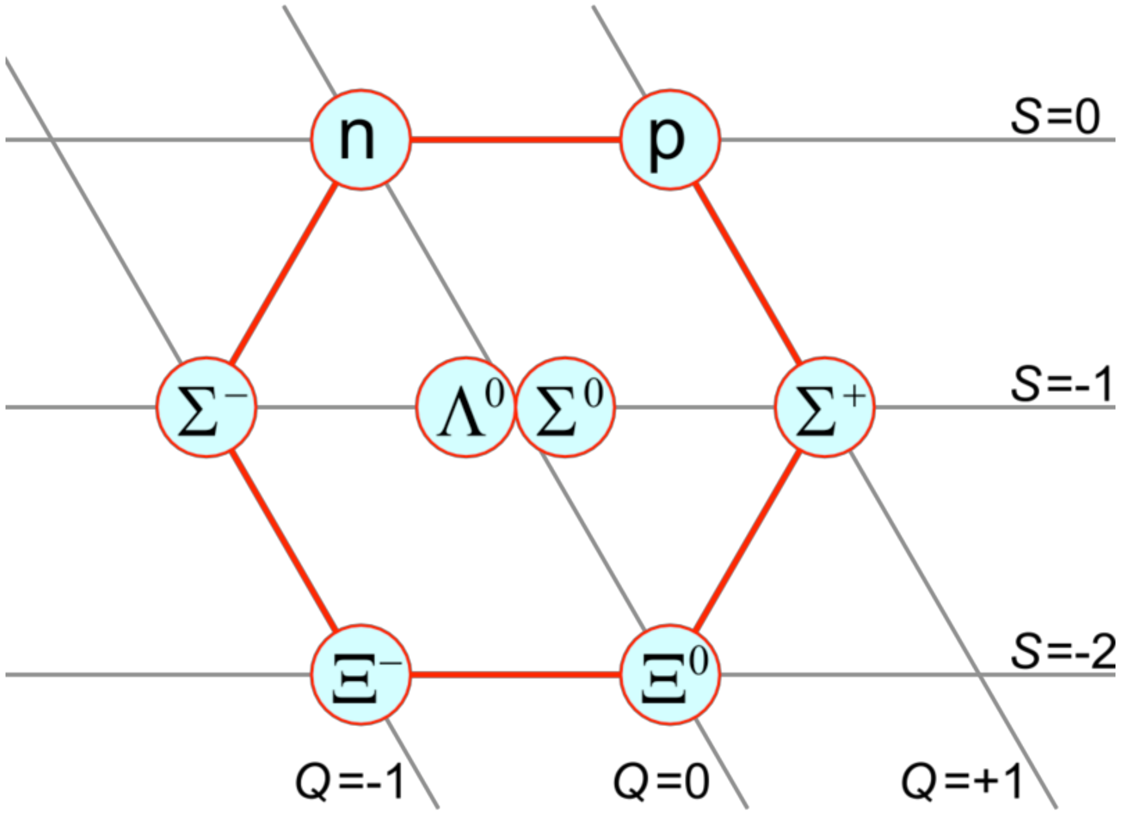
- **Leptons:** e, μ, ν_e, ν_μ. Lightest particles. Lepton number is conserved in all interactions.
- **Mesons:** π, η, φ, ω, ρ, … Middle-weight particles. There is no conserved "meson number".
- **Baryons:** p, n, Σ, Ξ, Λ, … Heaviest particles. Baryon number A is always conserved. Strangeness S is conserved sometimes (strong interactions) but not always (weak decays).

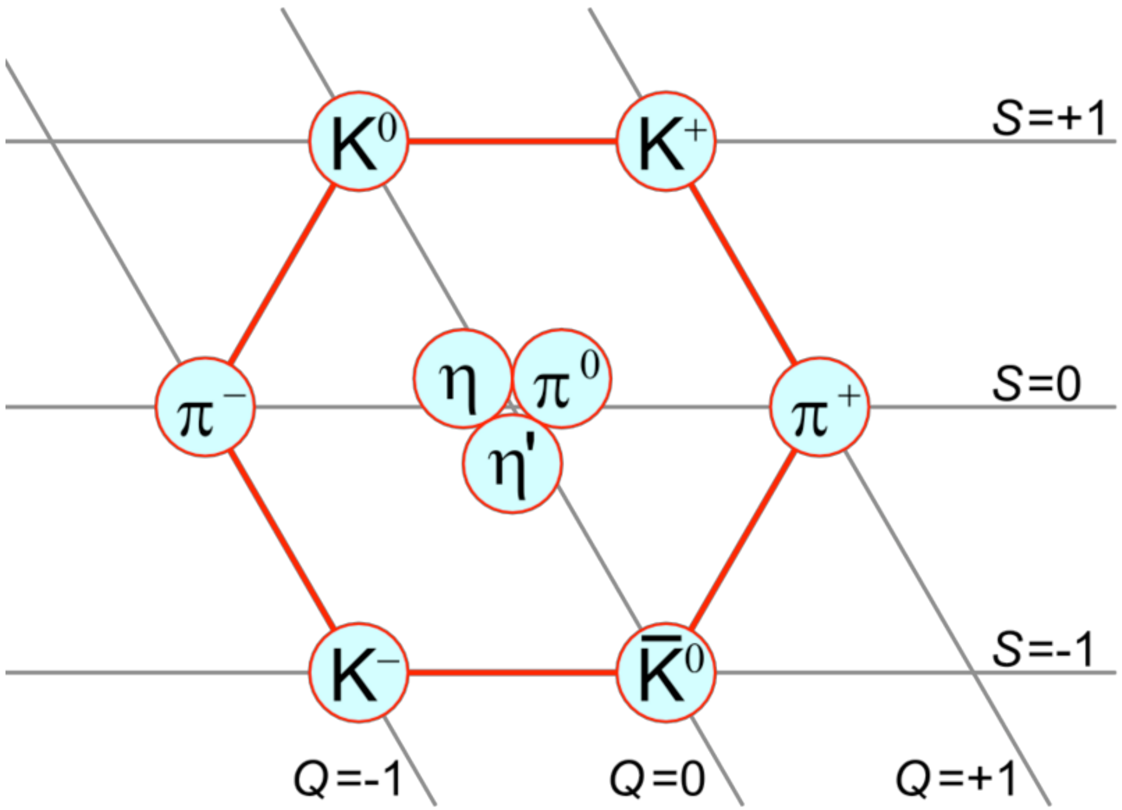
The Quark Era (1960 - 1978)

M. Gell-Mann
Image: NobelPrize.org

The Eightfold way

- Finally, in 1961, Gell-Mann brought some order to the chaos by developing a systematic ordering of the elementary particles.
- He noticed that if he plotted the mesons and baryons on a grid of strangeness S vs charge Q, geometrical patterns emerged.
- The lightest mesons and baryons fit into hexagonal arrays:



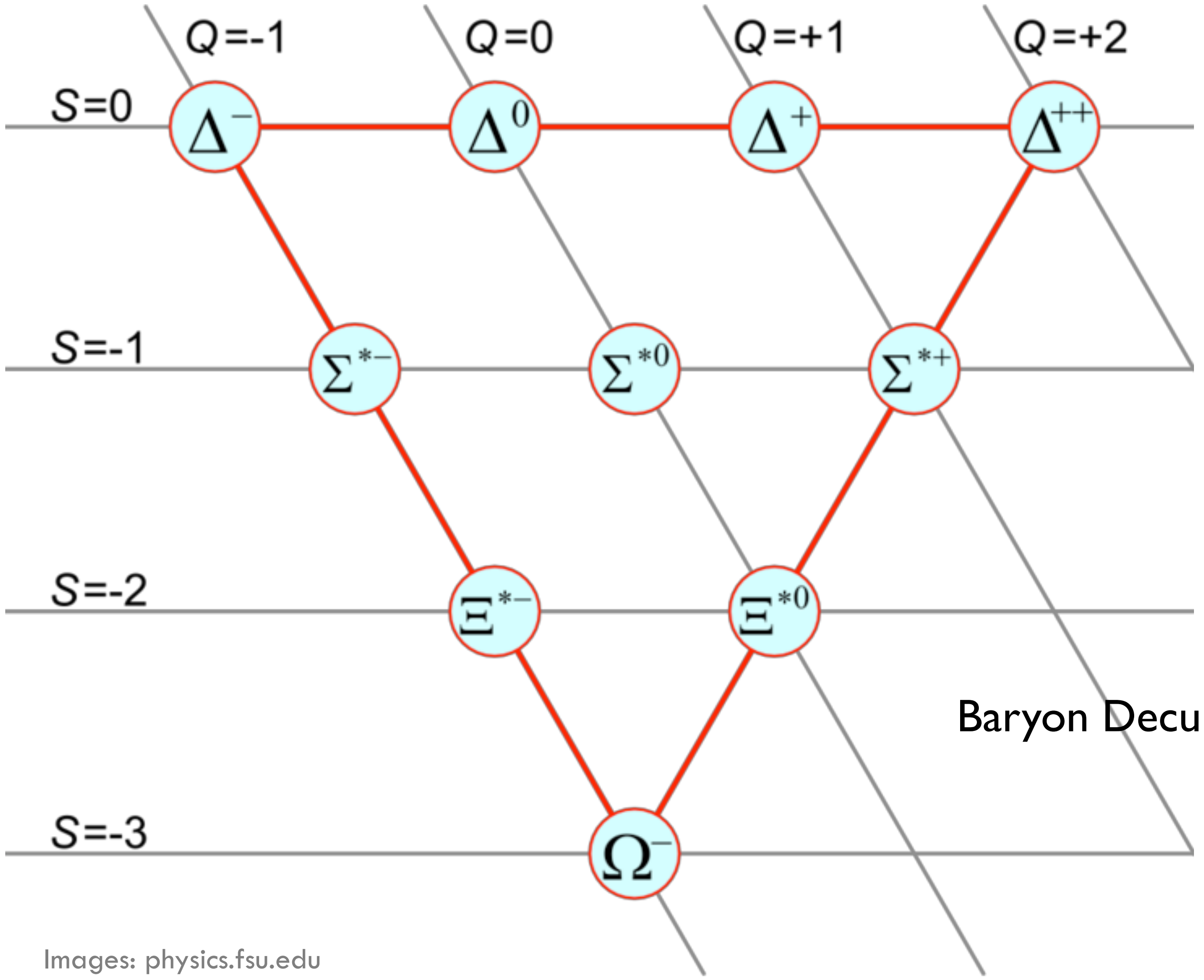


Images: physics.fsu.edu

M. Gell-Mann
Image: NobelPrize.org

The Eightfold way

- Gell-Mann called his organizational scheme the "Eightfold Way".
- Note that other figures were allowed in this system, like a triangular array incorporating 10 of the heavier baryons:



Baryon Decuplet

Images: physics.fsu.edu

Prediction of new baryons (1964)

- Like the Periodic Table of the elements, the Eightfold Way yields simple relations between the hadrons.
- Gell-Mann/Okubo mass formula: relates masses of the members of the baryon octet:

$$2\left(m_{p/n} + m_{\Xi}\right) = 3m_{\Lambda} + m_{\Sigma}$$

- Similarly, a mass formula for the baryon decuplet:

$$M_{\Delta} - M_{\Sigma^*} = M_{\Sigma^*} - M_{\Xi^*} = M_{\Xi^*} - M_{\Omega}$$

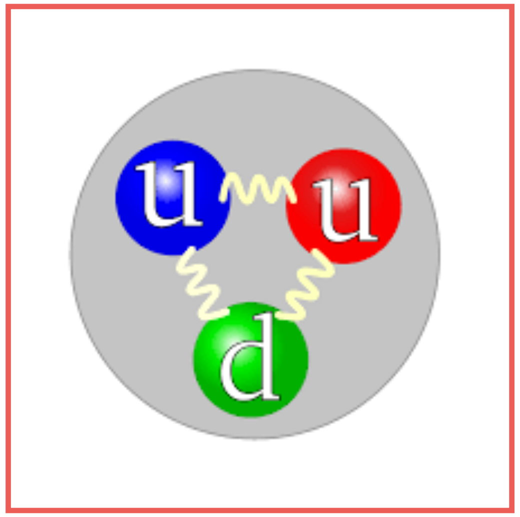
- Key point: in 1963, the Ω- had not yet been observed. Gell-Mann used the Eightfold Way to predict its mass, charge, and strangeness.
In 1964, the Ω- was found, and had exactly the properties predicted!

The quark model (1964)

- The patterns of the Eightfold Way evoke the periodicities of the Table of the Elements.
- In 1964, Gell-Mann and G. Zweig proposed an explanation for the structure in the hadron multiplets: all hadrons are composed of even more fundamental constituents, called quarks.
- According to their quark scheme, quarks came in three types, or "flavors": **up (u), down (d), and strange (s)**.
- To get the right hadronic properties, Gell-Mann gave his quarks fractional electric charge:

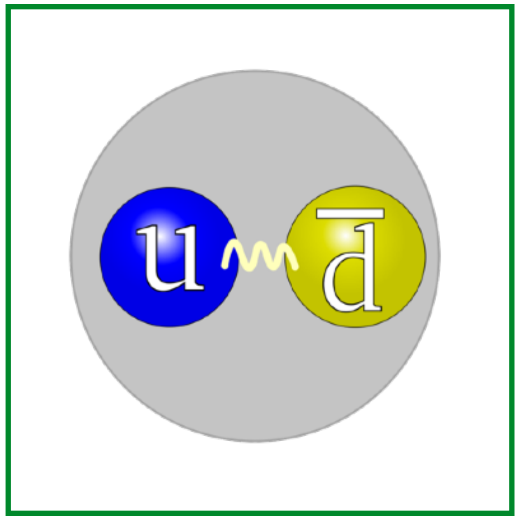
Quark Flavor	Charge (Q)	Strangeness (S)
Up: u	2/3	0
Down: d	-1/3	0
Strange: s	-1/3	-1

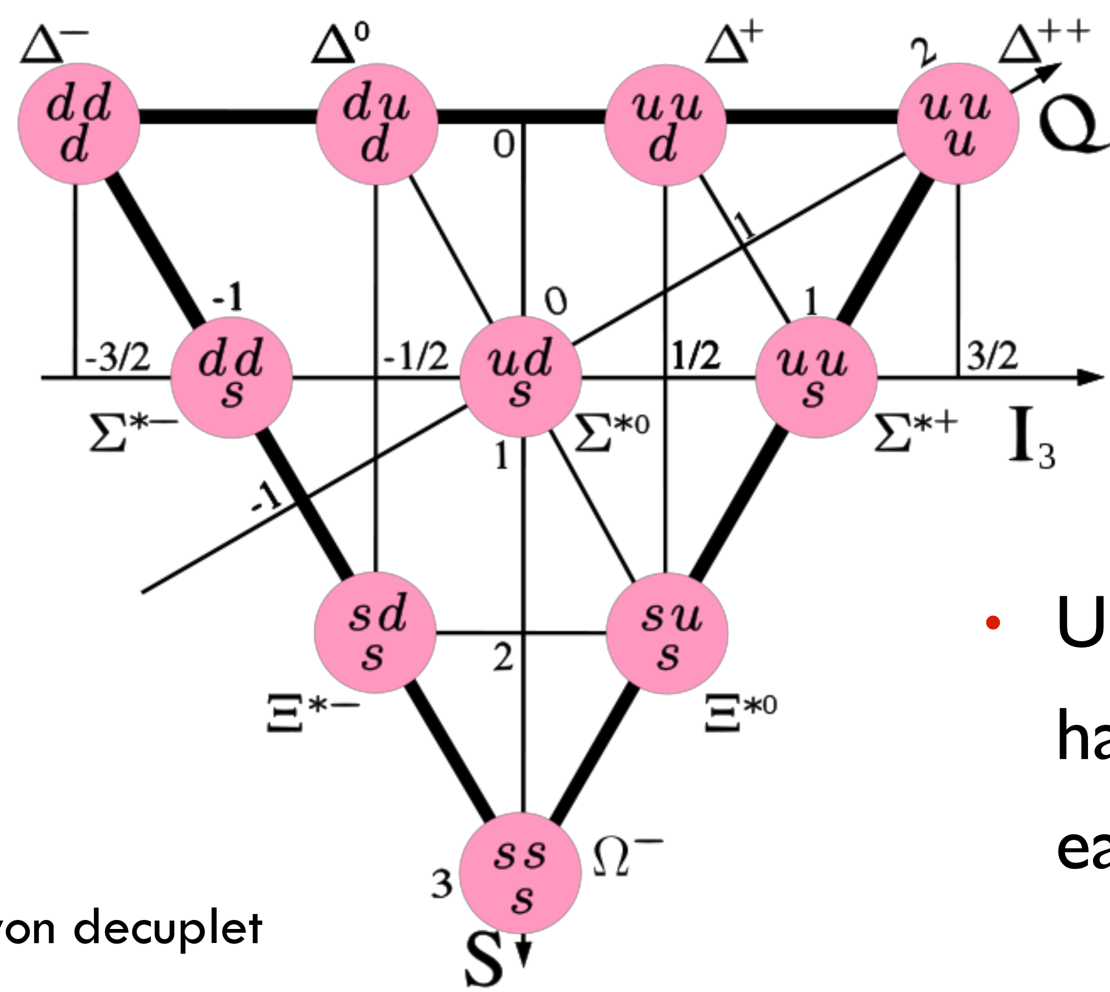
The quark model (1964)



- The quark model has the following conditions:

1. Baryons are composed of three quarks; antibaryons are composed of three antiquarks.
2. Mesons are composed of quark-antiquark pairs.





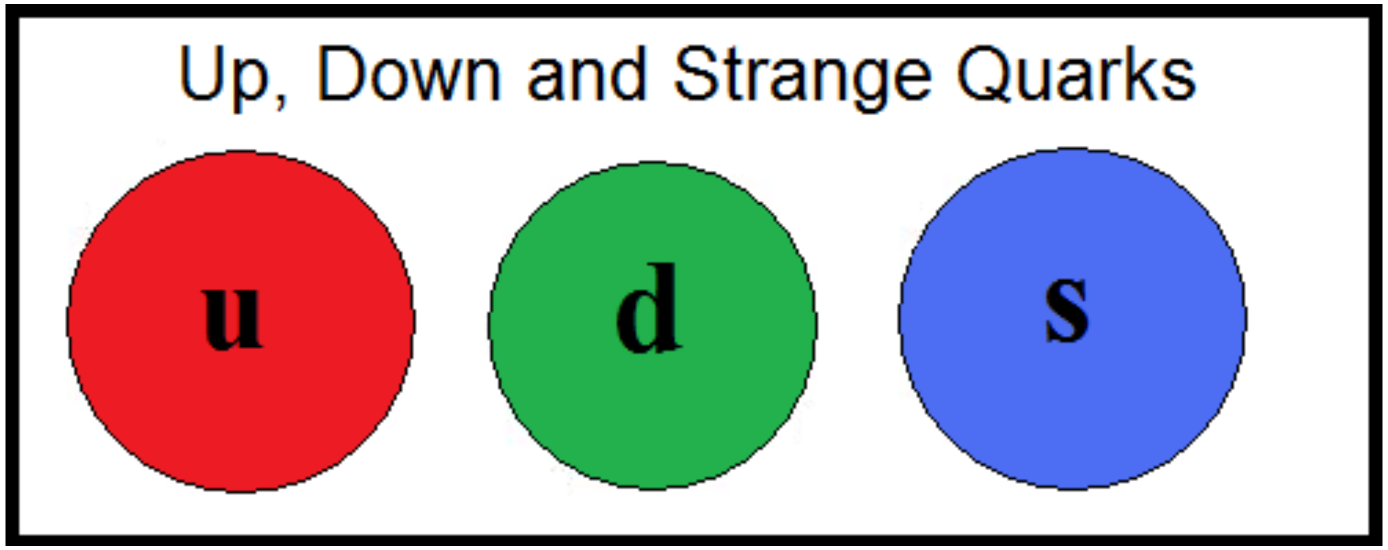
Baryon decuplet

- Using these rules, the hadronic multiplets are easily constructed…

The quark model (1964)

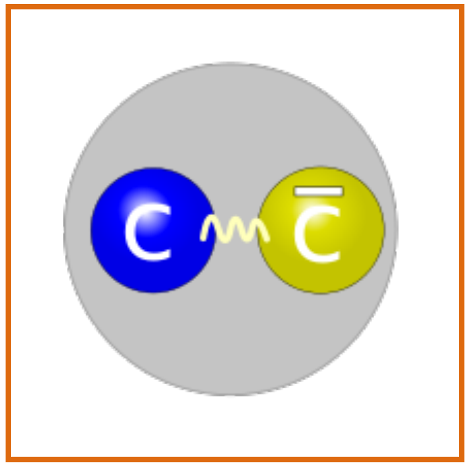
- Note: quarks have never actually been observed!
- There is no such thing as a free quark (more on this later…).

- However, scattering experiments indicate that hadrons do have a substructure (analogous to Rutherford scattering of atoms).

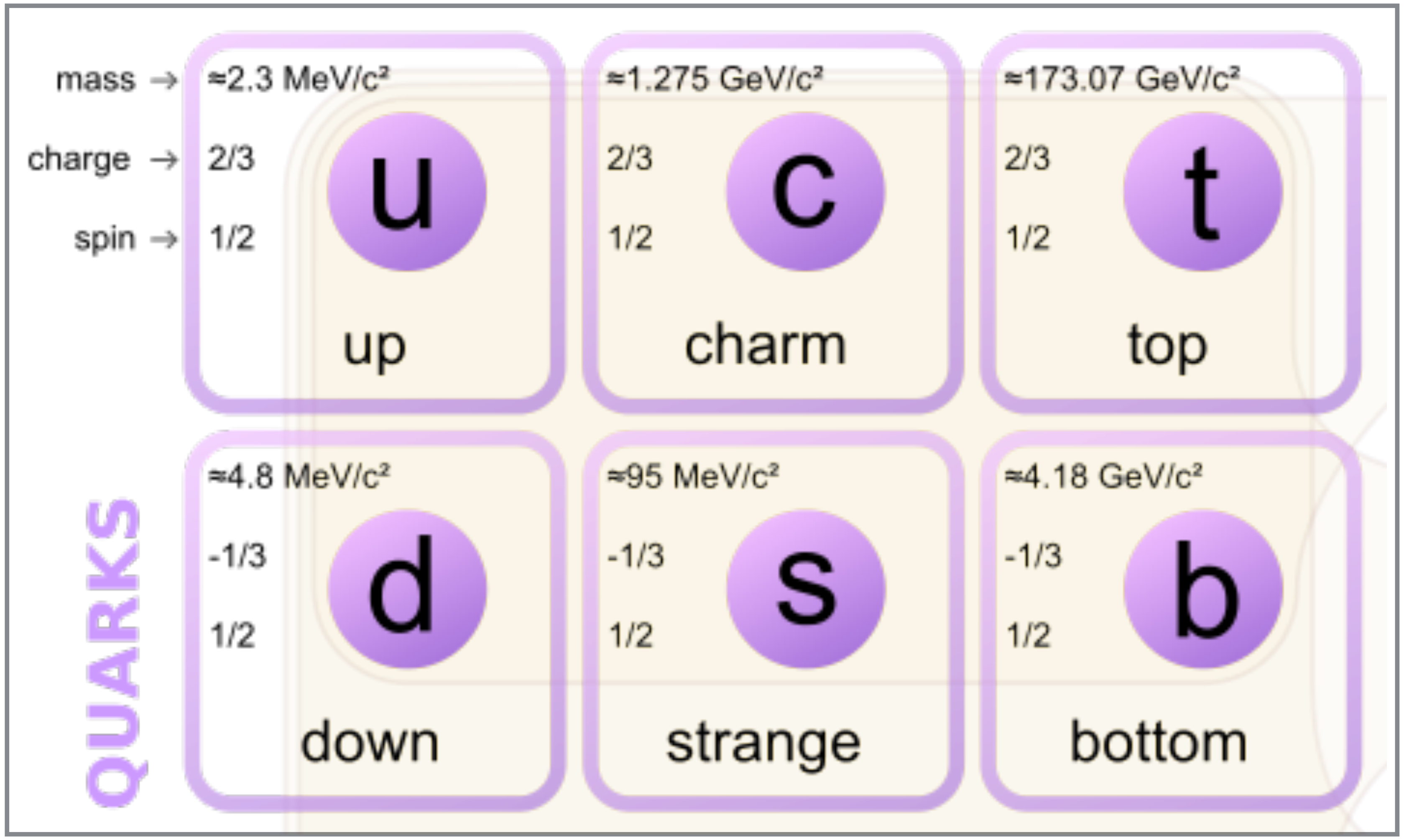


The quark model (1964)

- Until the mid-1970s, most physicists did not accept quarks as real particles.
- Then, in 1974, two experimental groups discovered a neutral, extremely heavy meson called the J/ψ.
  - The J/ψ had a lifetime about 1000 times longer than other hadrons in its mass range.
  - A simple way to explain its properties uses the quark model.
- A new quark, called charm (c), was introduced; and the J/ψ was shown to be a bound state of a charm-anticharm pair (sometimes called "charmonium").



The quark model



- **We have since discovered the bottom (beauty) quark, in 1977, and the top (truth) quark, in 1995.**

The quark model: btw…

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Aug 26, 2015

LHCb reports observation of pentaquarks

In 1964, Murray Gell-Mann and George Zweig independently predicted a substructure for hadrons: baryons would be comprised of three quarks, mesons of a quark–antiquark pair. They also said that baryons with four quarks and one antiquark were possible, as were mesons with two quarks and two antiquarks – dubbed, respectively, pentaquarks and tetraquarks, after the number of constituents. Since then, the picture for baryons and mesons has been thoroughly established within QCD, the theory of the strong interaction. Claims of the sighting of pentaquarks, meanwhile, have been thoroughly debunked. Nevertheless, their existence could cast important new light on QCD.

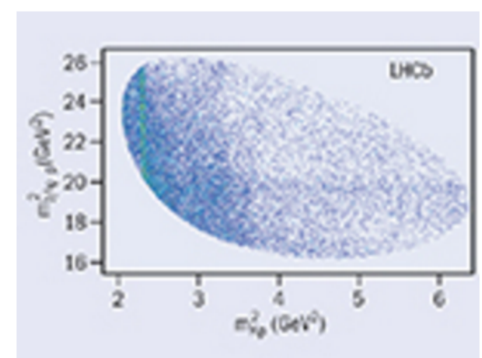
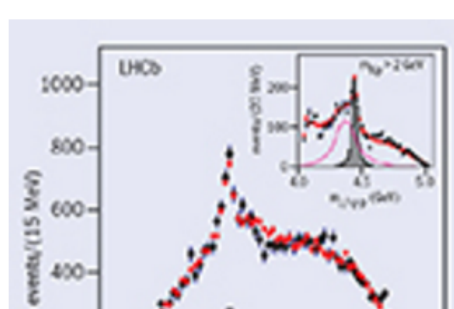


Fig. 1.



Now, the LHCb collaboration has announced the observation of two pentaquark states, P^+_c, in analysis of data collected during Run 1 of the LHC at CERN. The discovery was made during

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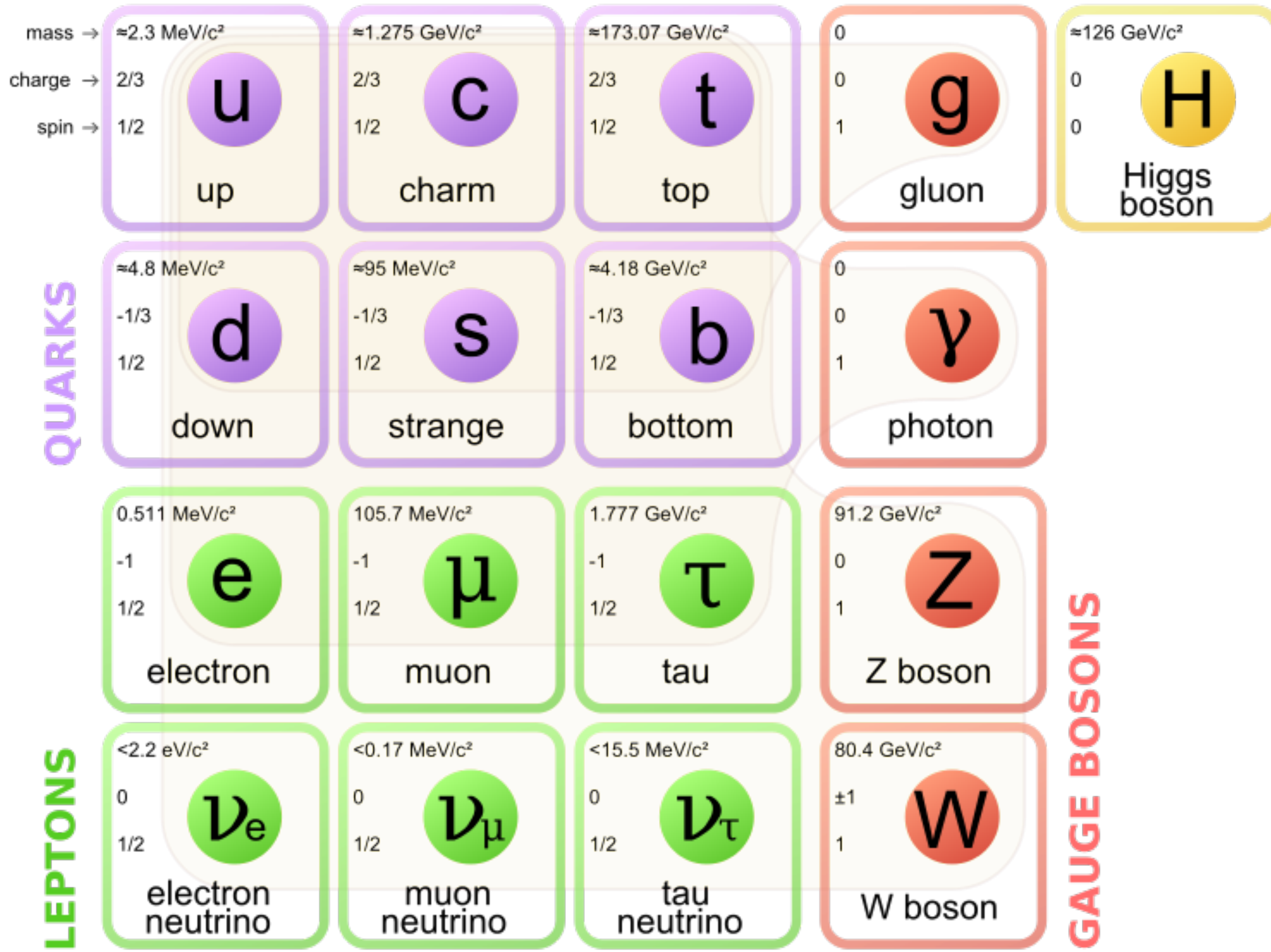
The Standard Model Now

The Standard Model now



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Special Relativity

The physics of "fast"

Relativistic mechanics

What's wrong with classical mechanics?

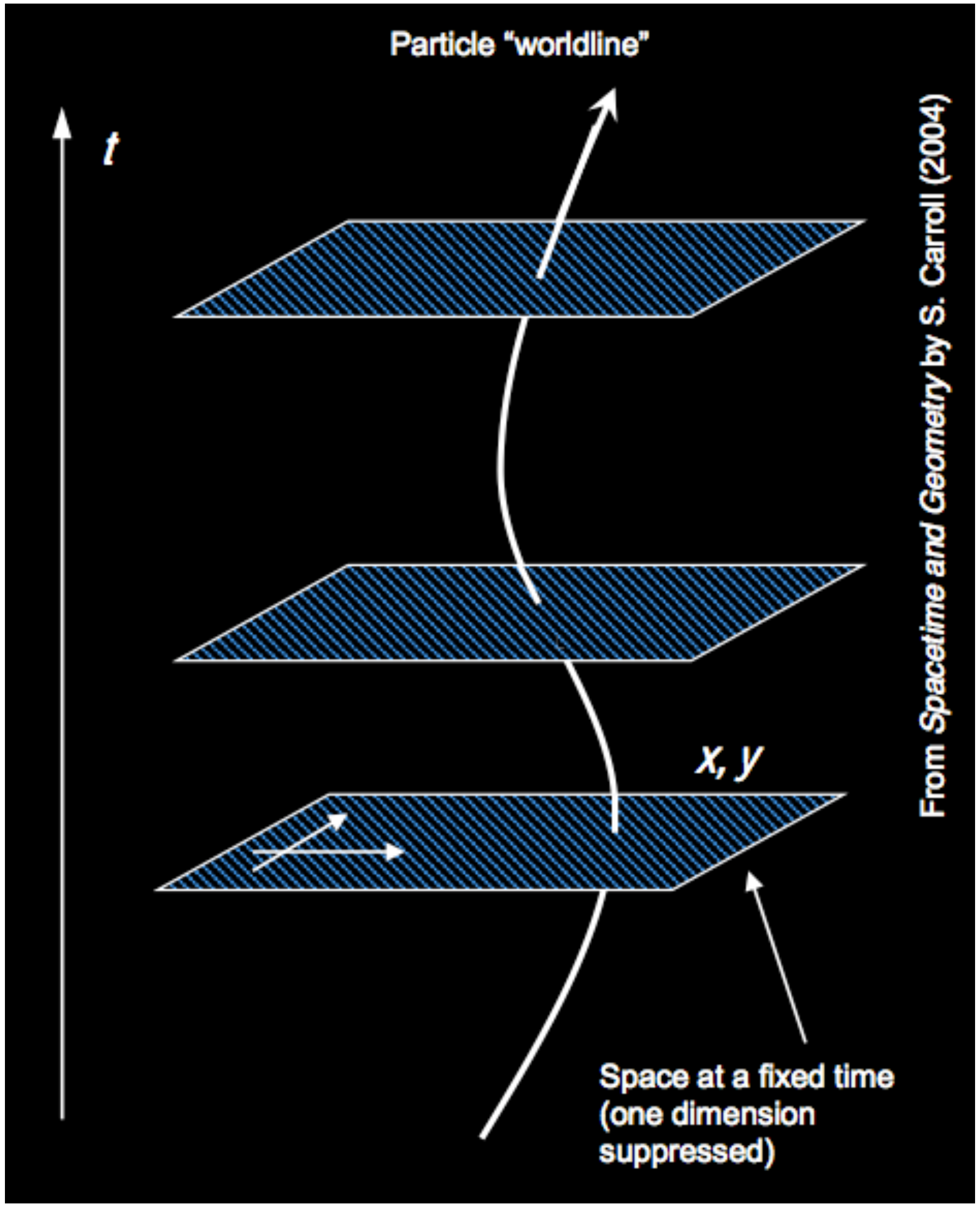
- We will see that classical mechanics is only valid in the limiting case where $v \to 0$, or $v << c$.
  - This is generally the case for everyday observables.
- However, this is not the case with particles traveling close to the speed of light.
  - In that case, classical mechanics fails to describe their behavior.
  - To properly describe particle kinematics, and particle dynamics, we need relativistic mechanics.

The notion of spacetime

Spacetime in Newtonian mechanics (v<<c)

"the world, as experienced by us"

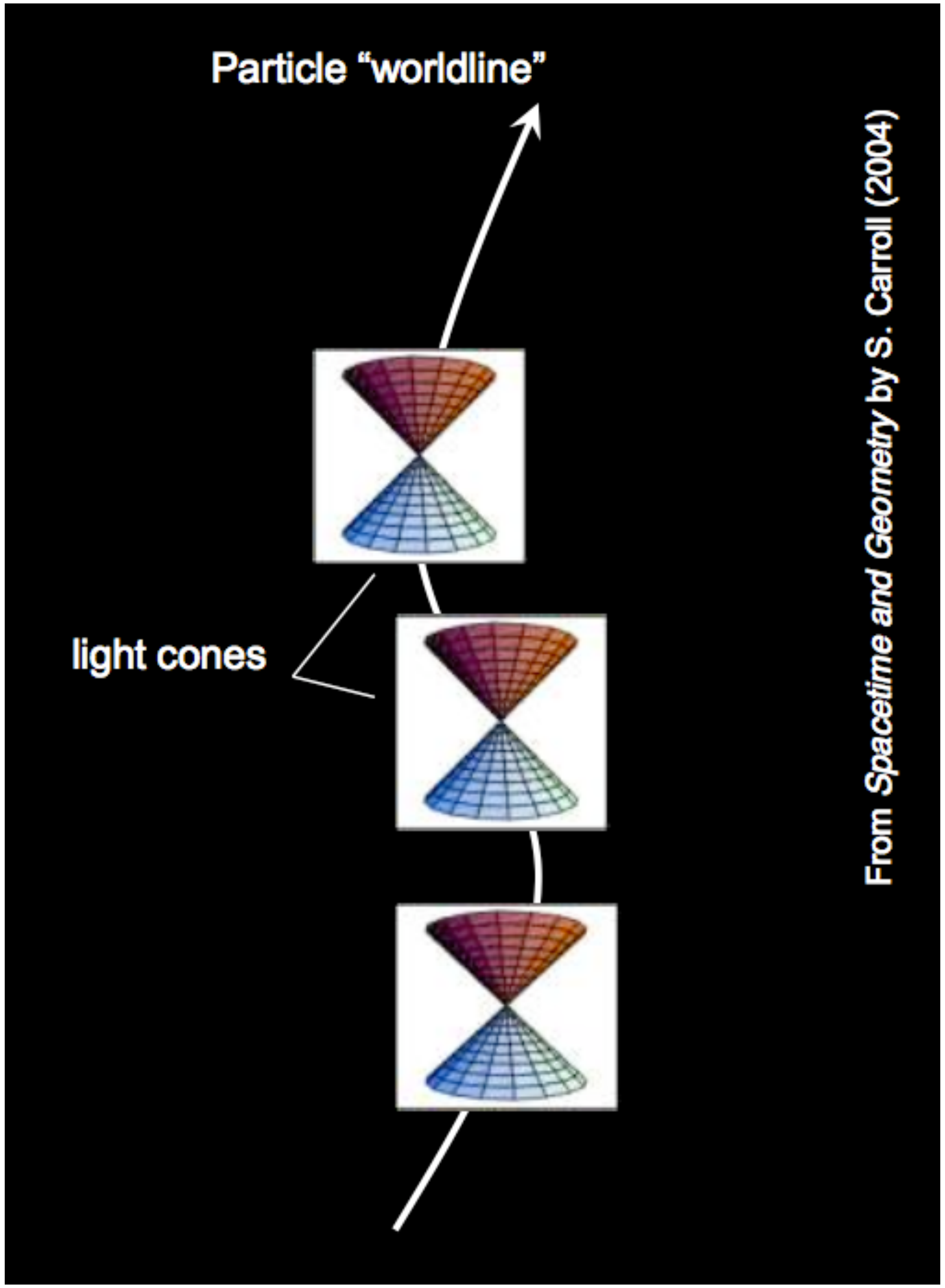
- Time is universal.
- Space can be cut into distinct "slices" at different moments in time.
- Particles must move forward in time, but can move through space in any direction.
- All observers agree whether two events at different points in space occur at the same moment of time.



The notion of spacetime

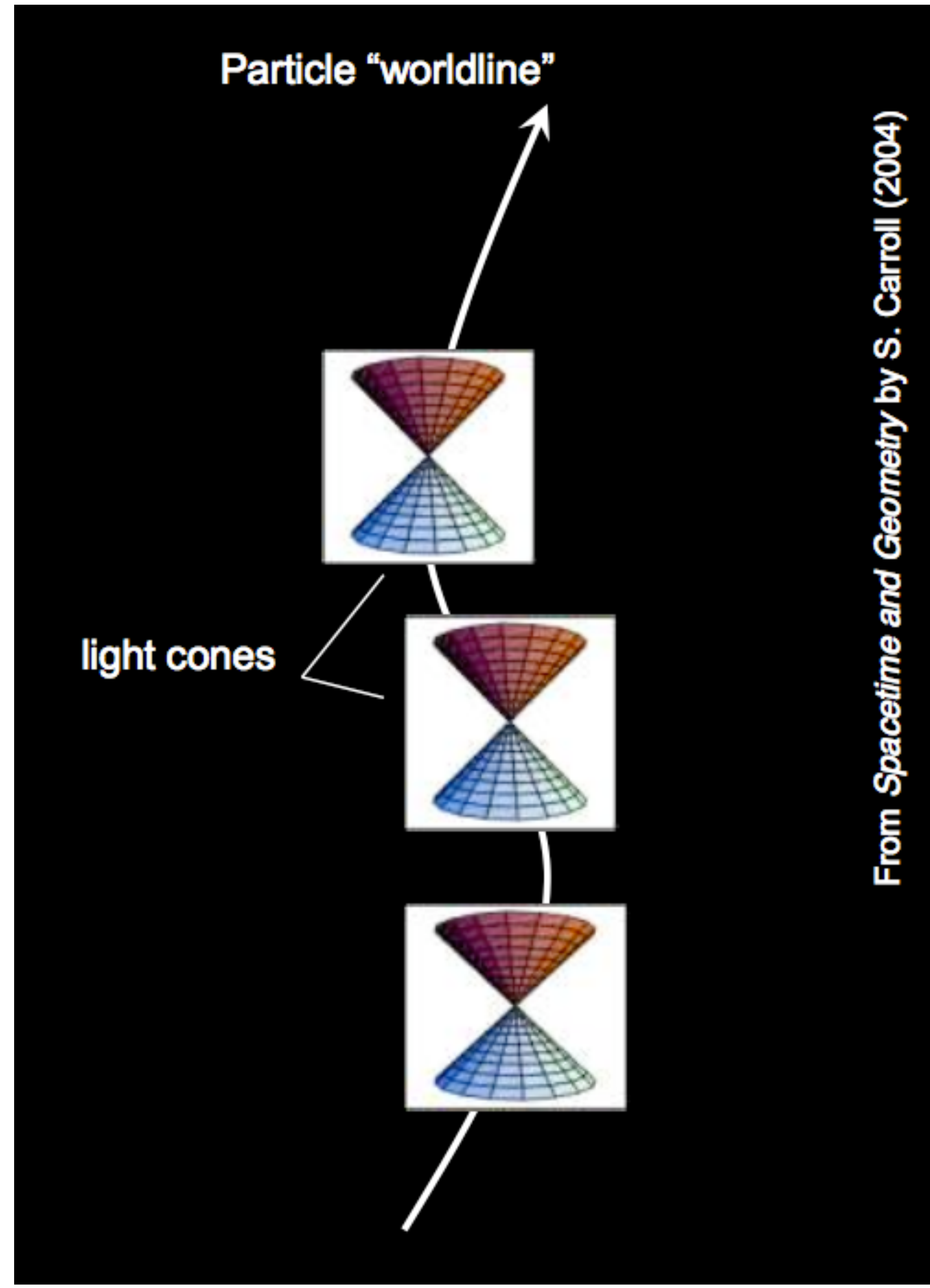
Spacetime in Special Relativity

- Time is local.
- Observers may not agree that two events occur at the same time.
- There is no absolute notion of all space at a moment in time.
- The speed of light is constant, and cannot be surpassed.
- Every event "exists" within a set of allowed trajectories (light cone).



Basic concepts

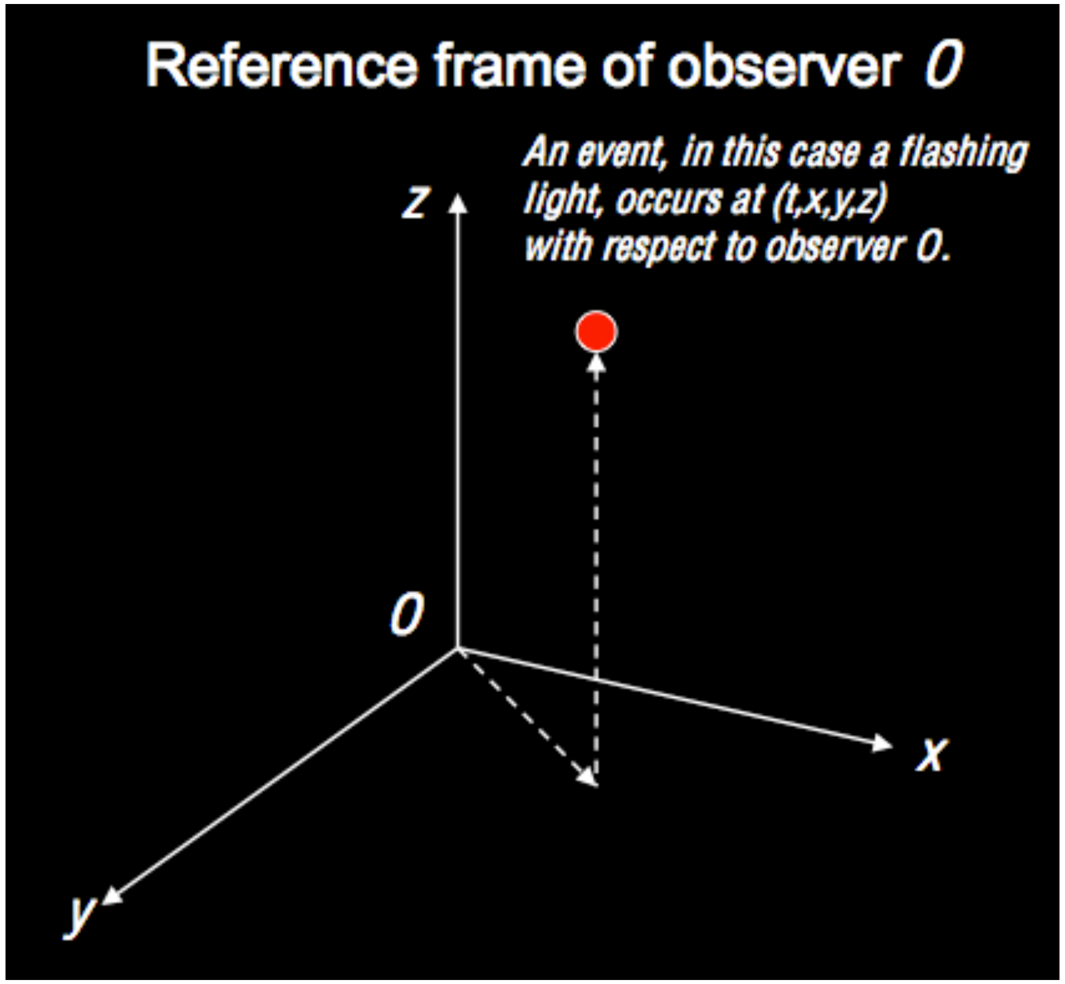
- **Event:** something that occurs at a specified point in space, at a specified time.
- **Observer**: someone who witnesses and can describe events (also known as a "frame of reference")
  - An observer describes events by using **"standard" clocks and rulers** which are at rest with respect to him/her.



Reference frames

What do we mean by "an observer is a frame of reference"?

- An observer O, in our sense of the word, sets up a Cartesian coordinate system for measuring positions *(x,y,z)*.
- O then places synchronized clocks at every point in space to measure time.
- Using the spatial coordinate system and clocks, O observes events and assigns each one a time stamp *t* and position *(x,y,z)*.



Inertial observer

Inertia: From Newton's first law of motion, an object not subject to any net external force moves at a constant velocity.

- **Isolated objects that are either at rest or move with constant velocity.**
- Hence, two inertial observers always move at constant velocity with respect to each other.
- All laws of physics, e.g. Newton's Second Law F=ma are valid for inertial observers.
  - E.g. for an observer moving at constant velocity V with respect to some "fixed" point:

$$v'(t) = v(t) - V$$

$$F = ma' = m\frac{\Delta v'(t)}{\Delta t} = m\frac{\Delta\left(v(t) - V\right)}{\Delta t} = m\frac{\Delta v(t)}{\Delta t} - 0$$

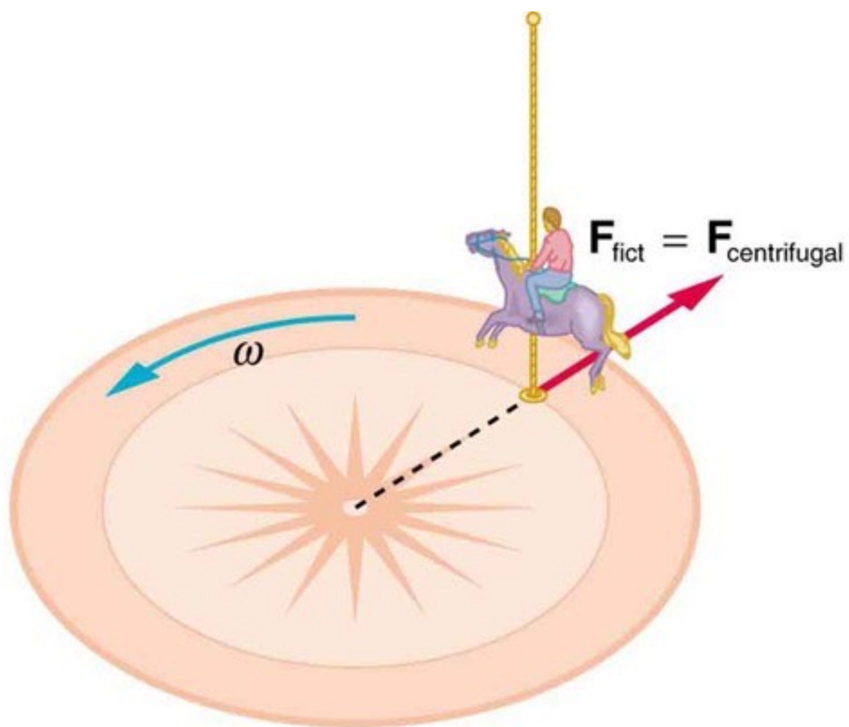
$$= ma$$

Non-Inertial observer

An observer undergoing acceleration is NOT inertial.

- Accelerating observers feel the influence of "pseudo-forces", resulting in changes to Newton's 2nd law.
- Example: an observer on a merry-go-round spinning at angular velocity ω will perceive that straight-line trajectories bend, and conclude that objects in his/her reference frame are affected by a Coriolis force:

$$\vec{F} = -2m(\vec{\omega} \times \vec{v})$$



Merry-go-round's rotating frame of reference

Postulates of special relativity

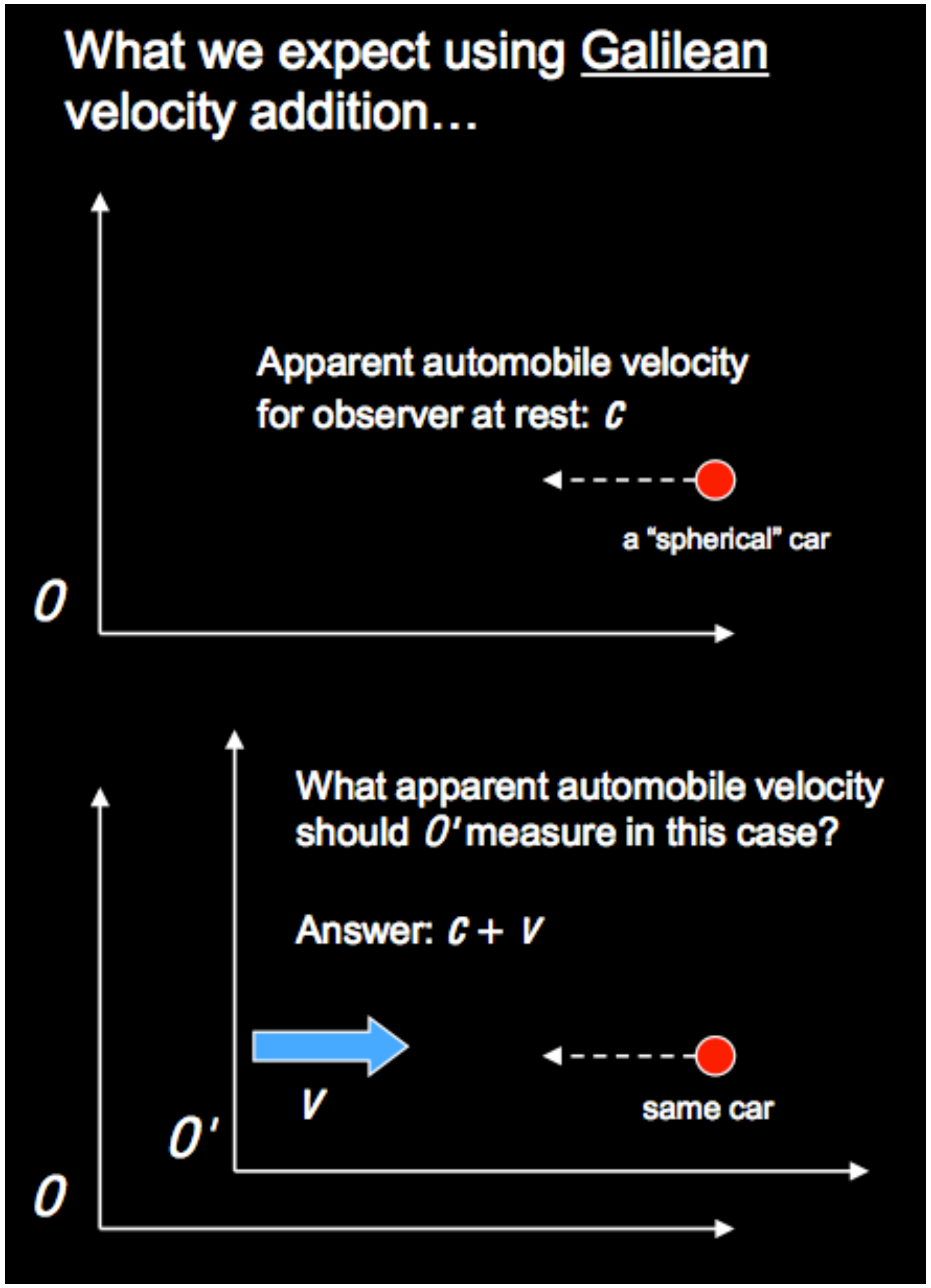
- In 1905, A. Einstein published two papers on special relativity, as well as a paper on the photoelectric effect (Nobel Prize 1921) and Brownian motion (the physics of particles suspended in a fluid).

- All of Einstein's conclusions in special relativity were based on only two simple postulates:

1. The laws of physics are the same in all inertial reference frames (old idea, dates to Galileo).
2. All inertial observers measure the same speed c for light in a vacuum, independent of the motion of the light source.

Postulates of special relativity

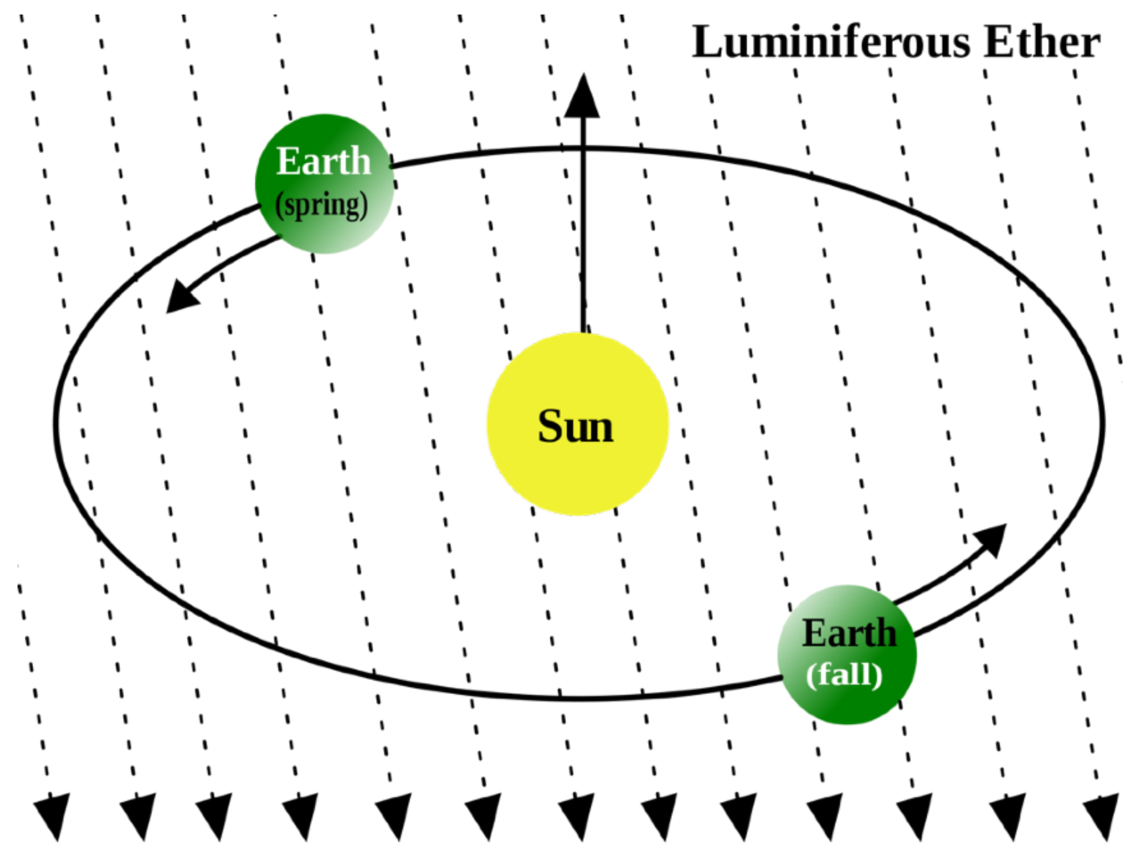
- The constancy of the speed of light is counter-intuitive, because this is not how "ordinary" objects behave.
- Example: imagine observing an oncoming car that moves at speed *c*.
  - We expect a moving observer to measure a different value for *c* than a stationary one.
  - According to SR, however, we always measure *c* for light, regardless of our motion!



Constancy of speed of light

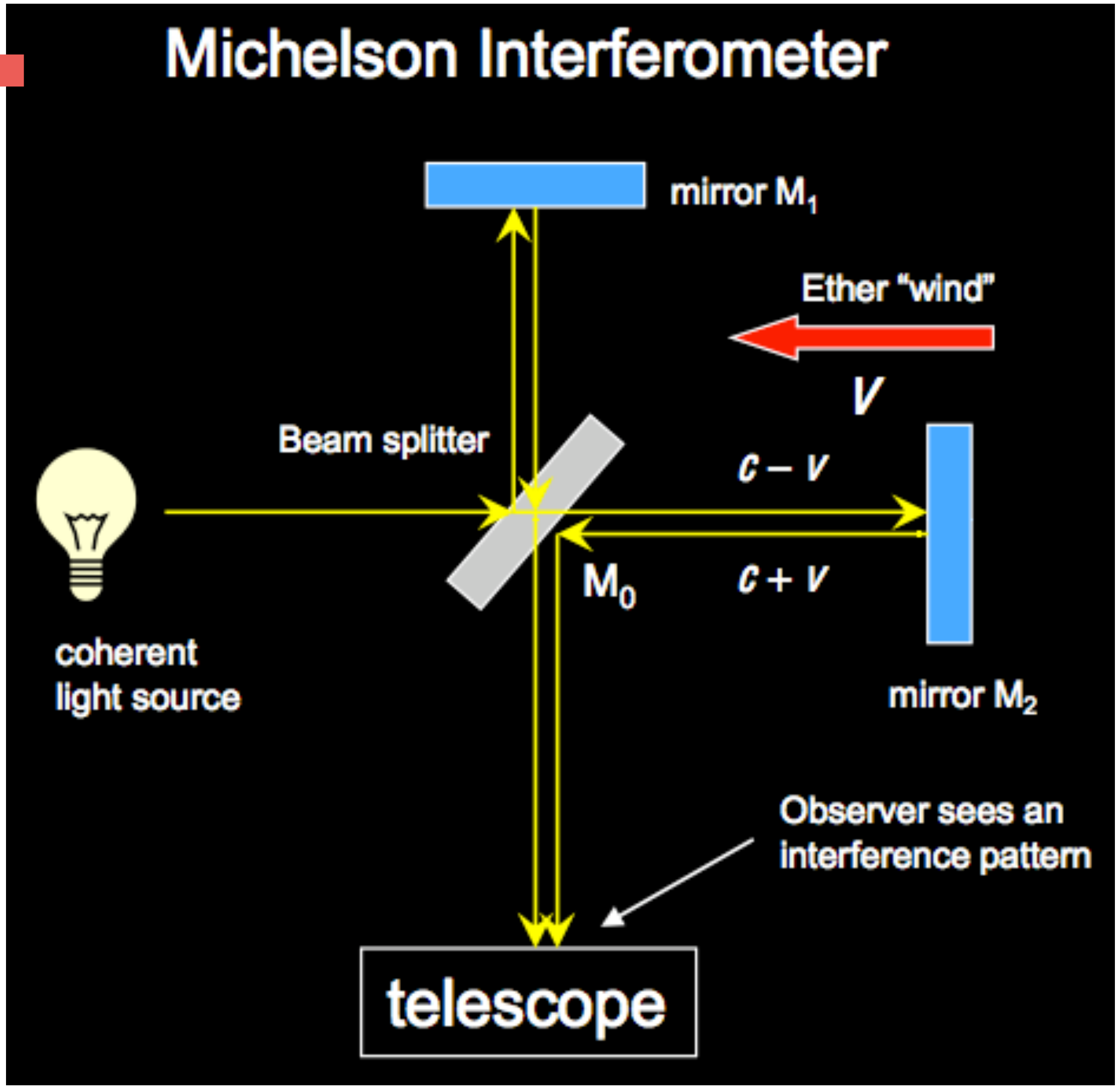
- The universality of *c* was first determined experimentally in 1887 by A. Michelson and E. Morley.

- At the time, it was believed that light propagated through a medium called the *ether* - physicists didn't think light self-propagated through empty space.
- Using an interferometer, Michelson and Morley expected to see the effect of changes in the speed of light relative to the ether velocity.



Constancy of speed of light

Hypothesis:
Earth's motion through the ether creates an "ether wind" of speed v.
Light moving "upwind" should have a speed c-v, and "downwind" c+v.
By rotating the interferometer, we should observe a change in the light beams' interference pattern due to the changing beam speed.



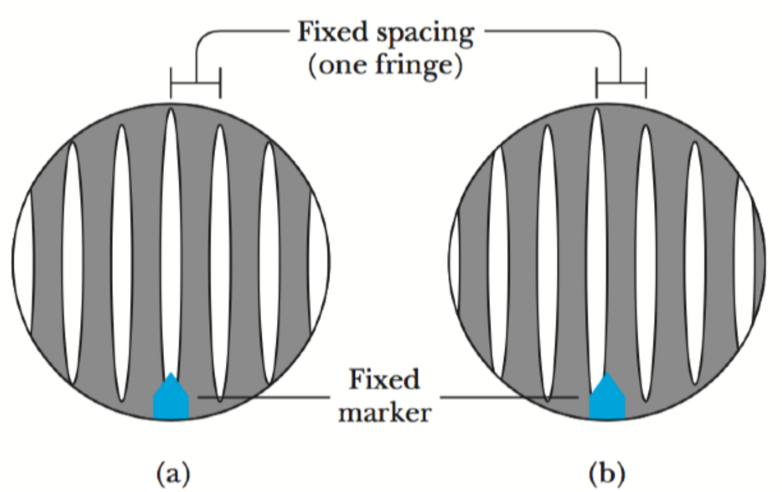


Figure 1.6 Interference fringe schematic showing (a) fringes before rotation and (b) expected fringe shift after a rotation of the interferometer by 90°.

Constancy of speed of light

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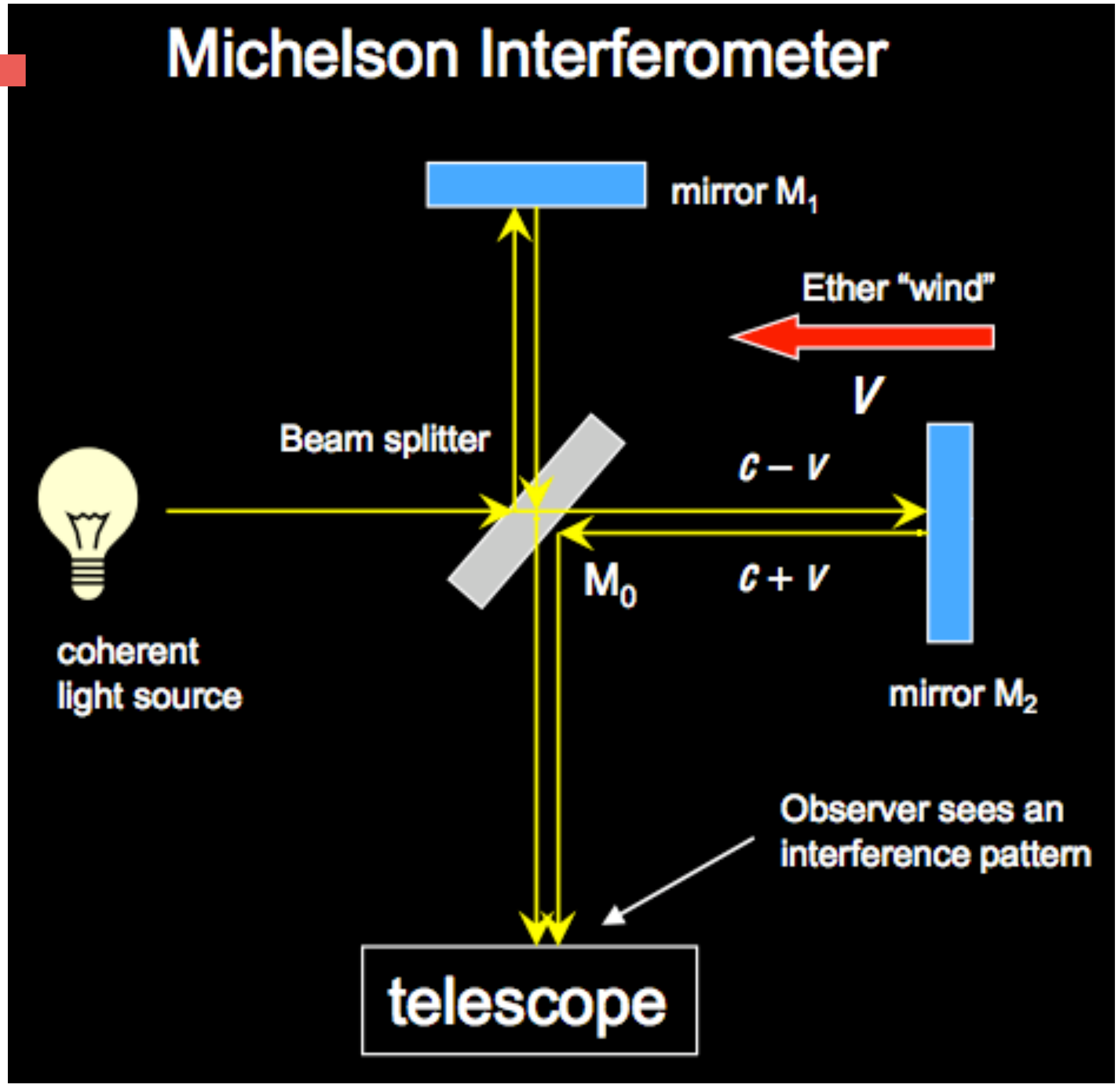

- In fact, they saw no such effect during repeated trials over several years.
- The simplest way to explain the result is to assume that there is no ether, and *c* is constant in all inertial frames.

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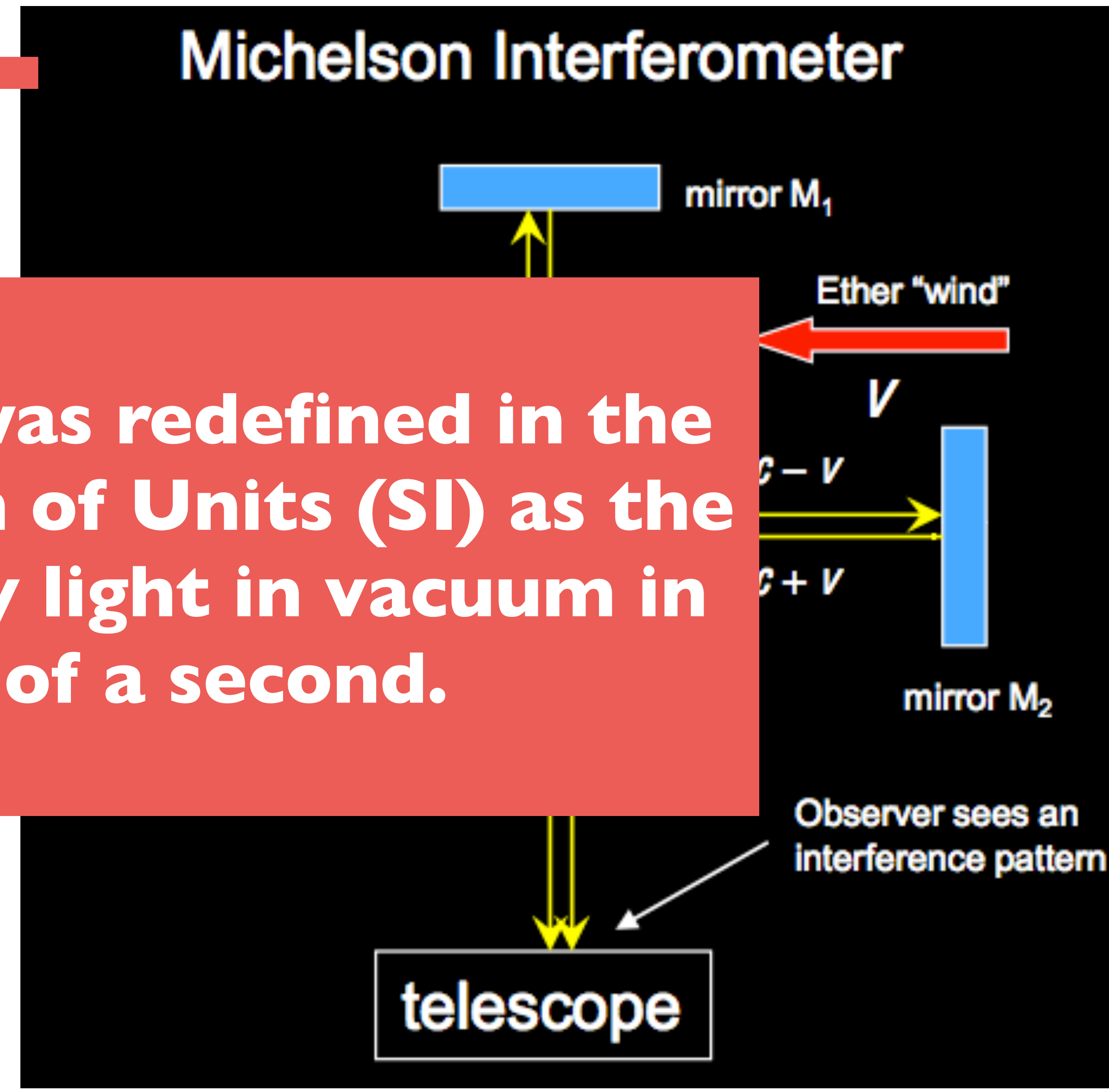


In 1983, the meter was redefined in the International System of Units (SI) as the distance travelled by light in vacuum in 1299,797,458 of a second.

- In fact, they saw no such effect during repeated trials over several years.
- The simplest way to explain the result is to assume that there is no ether, and *c* is constant in all inertial frames.

Implications of the postulates

- Einstein developed a series of "thought experiments" that illustrate the interesting consequences of the universality of c. These can be summarized as:

1. The illusion of simultaneity
2. Time dilation
3. Lorentz (length) contraction
4. Velocity addition

As we go through Einstein's examples, keep in mind that these results may seem a little counterintuitive.
You have to get rid of your Newtonian way of thinking!

(The relativity of) Simultaneity

- An observer O calls two events simultaneous if they occur at the same time in his / her coordinates.
- Interestingly, if the two events do not occur at the same *position* in frame O, then they will not appear simultaneous to a moving observer O'.
- In other words, events that are simultaneous in one inertial system are not necessarily simultaneous in others.
- **Simultaneity is not an absolute concept, but one that depends on the state of motion of the observer.**
- Again, this follows from the fact that *c* is the same in all inertial frames…

(The relativity of) Simultaneity

- A demonstration: Einstein's thought experiment.

"A boxcar moves with uniform velocity and two lightning bolts strike the ends of the boxcar, leaving marks on the boxcar and ground. The marks on the boxcar are labeled A' and B'; on the ground, A and B. The events recorded by the observers are the light signals from the lightning bolts. The two light signals reach observer O at the same time."

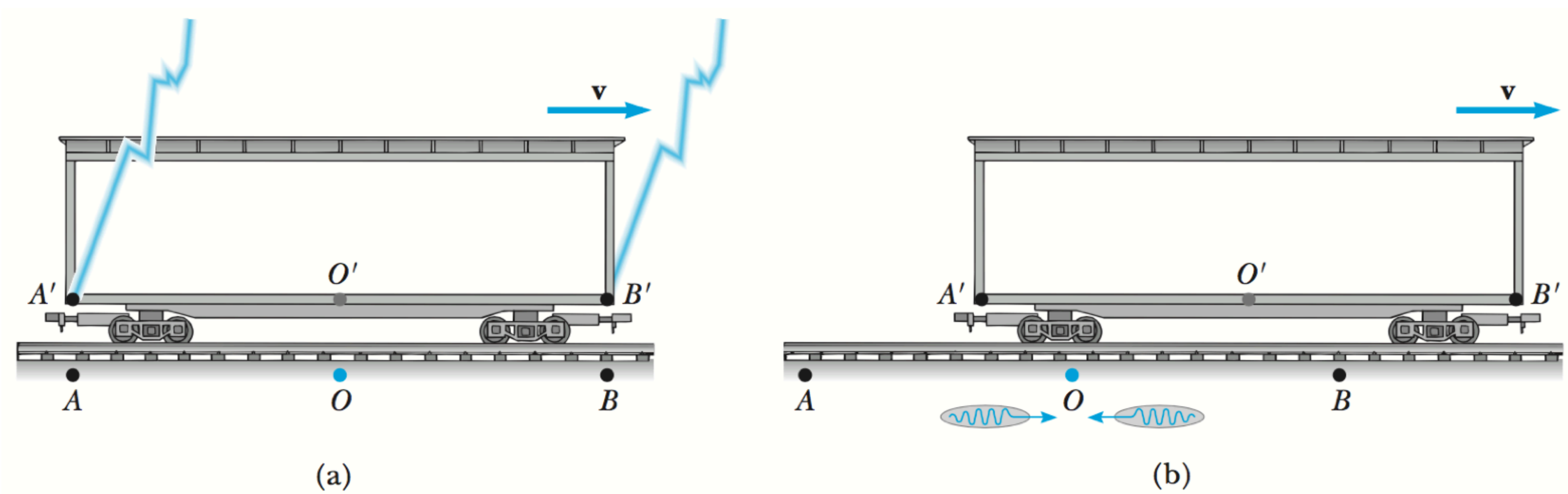
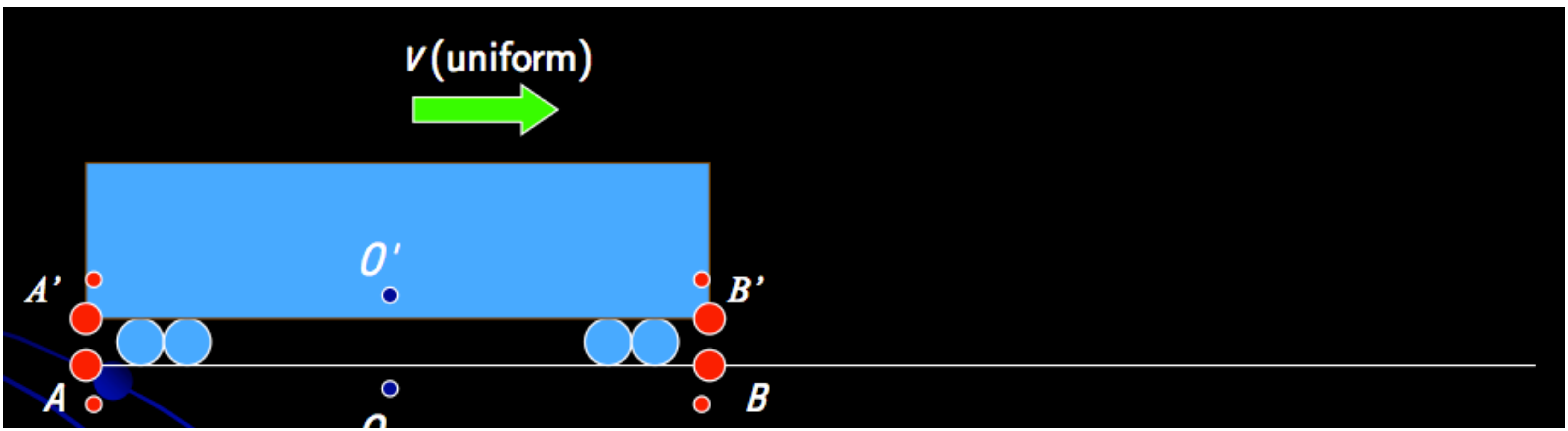


Figure 1.9 Two lightning bolts strike the ends of a moving boxcar. (a) The events appear to be simultaneous to the stationary observer at O, who is midway between A and B. (b) The events do not appear to be simultaneous to the observer at O', who claims that the front of the train is struck *before* the rear.

(The relativity of) Simultaneity

- A demonstration: Einstein’s thought experiment.

Flashed light from two ends of a moving boxcar is viewed by two observers. One sits inside the boxcar, and the other is stationary (outside). The lights are set up such that sources A and A’ flash at the same time, and sources B and B’ also flash at the same time.

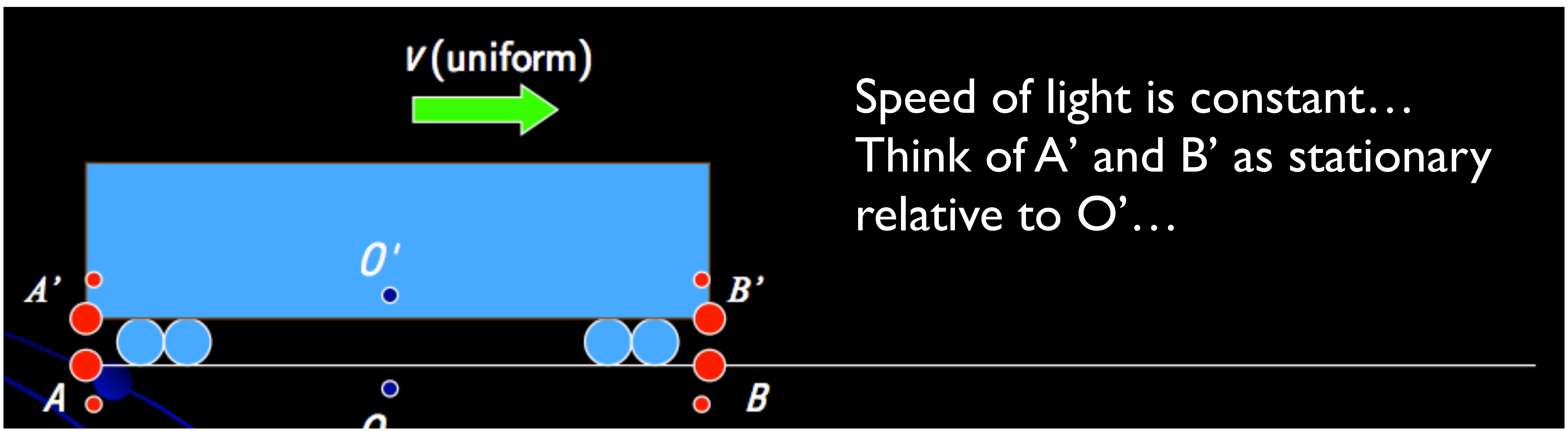


- Suppose the flashes from A and B appear simultaneous to O. Do the A' and B' flashes appear simultaneous to O’?

(The relativity of) Simultaneity

- A demonstration: Einstein's thought experiment.

Flashed light from two ends of a moving boxcar is viewed by two observers. One sits inside the boxcar, and the other is stationary (outside). The lights are set up such that sources A and A' flash at the same time, and at B and B' flash at the same time.



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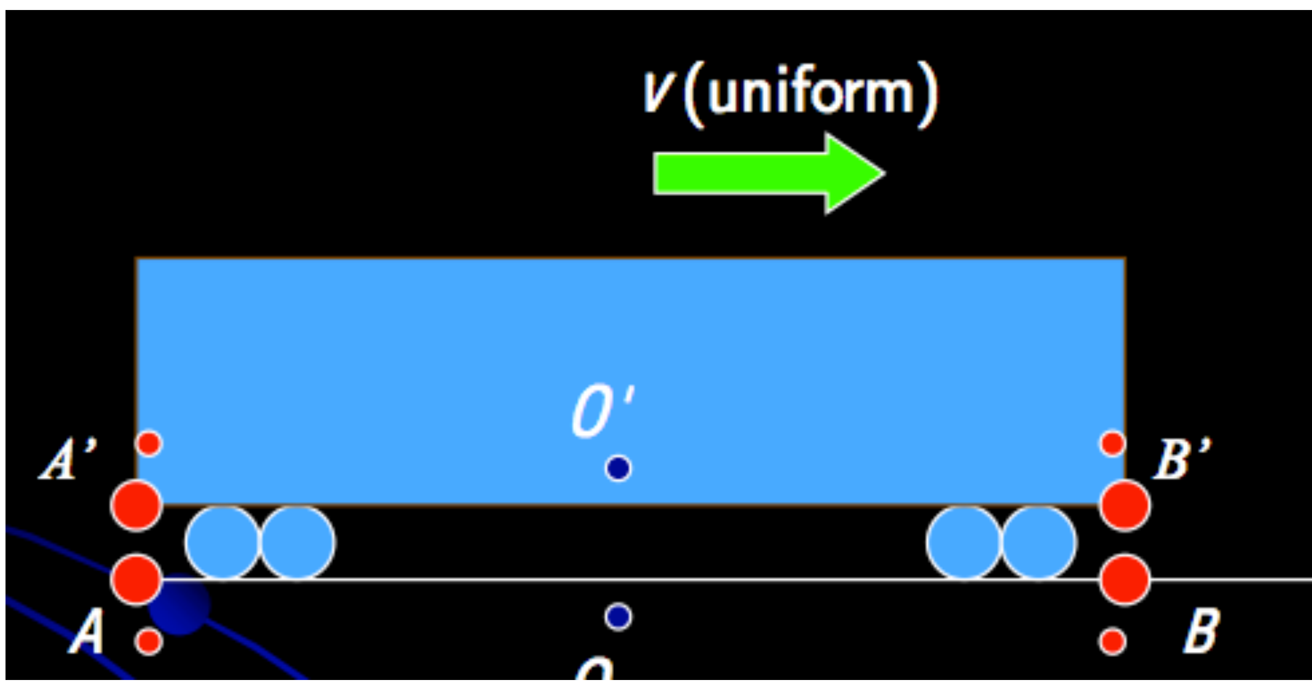
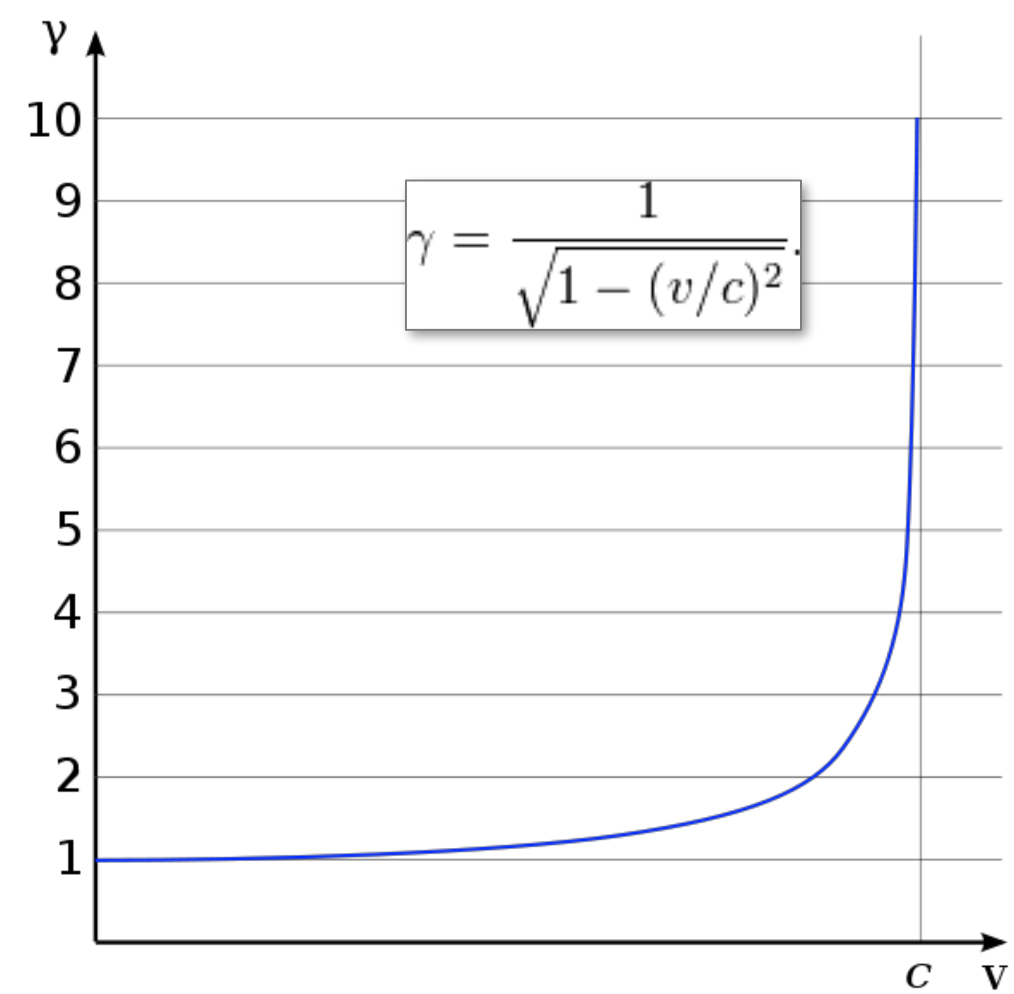


In this case, observer O' must see the light from B' first, since the flash emitted by the moving sources A' and B' must travel at the same speed as that emitted by A and B.

- This is not what Galilean / Newtonian physics predicts.

Time dilation

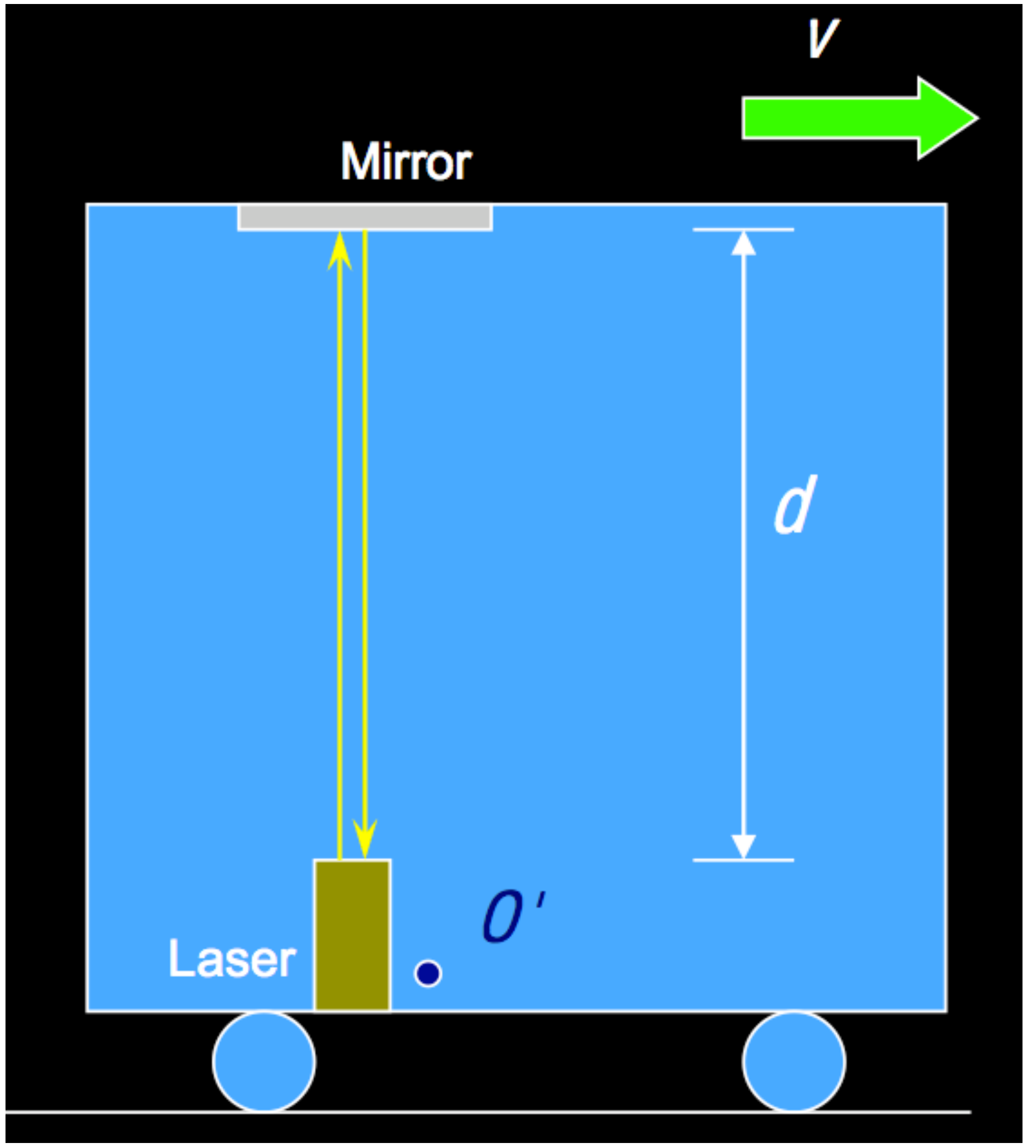
- Time dilation reflects the fact that observers in different inertial frames always measure different time intervals between a pair of events.
- Specifically, an observer O at rest will measure a longer time between a pair of events than an observer O' in motion, i.e., moving clocks tick more slowly than stationary clocks!
- The amount by which the observer at rest sees the time interval "dilated" with respect to the measurement by O' is given by the factor called Lorentz factor γ:



$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2 / c^2}} = \gamma \Delta t'$$

Time dilation

- Another thought experiment.
- Suppose an observer O' is at rest in a moving vehicle. She has a laser which she aims at a mirror on the ceiling.
- According to O', how long does it take the laser light to reach the ceiling of the car and bounce back to the ground?



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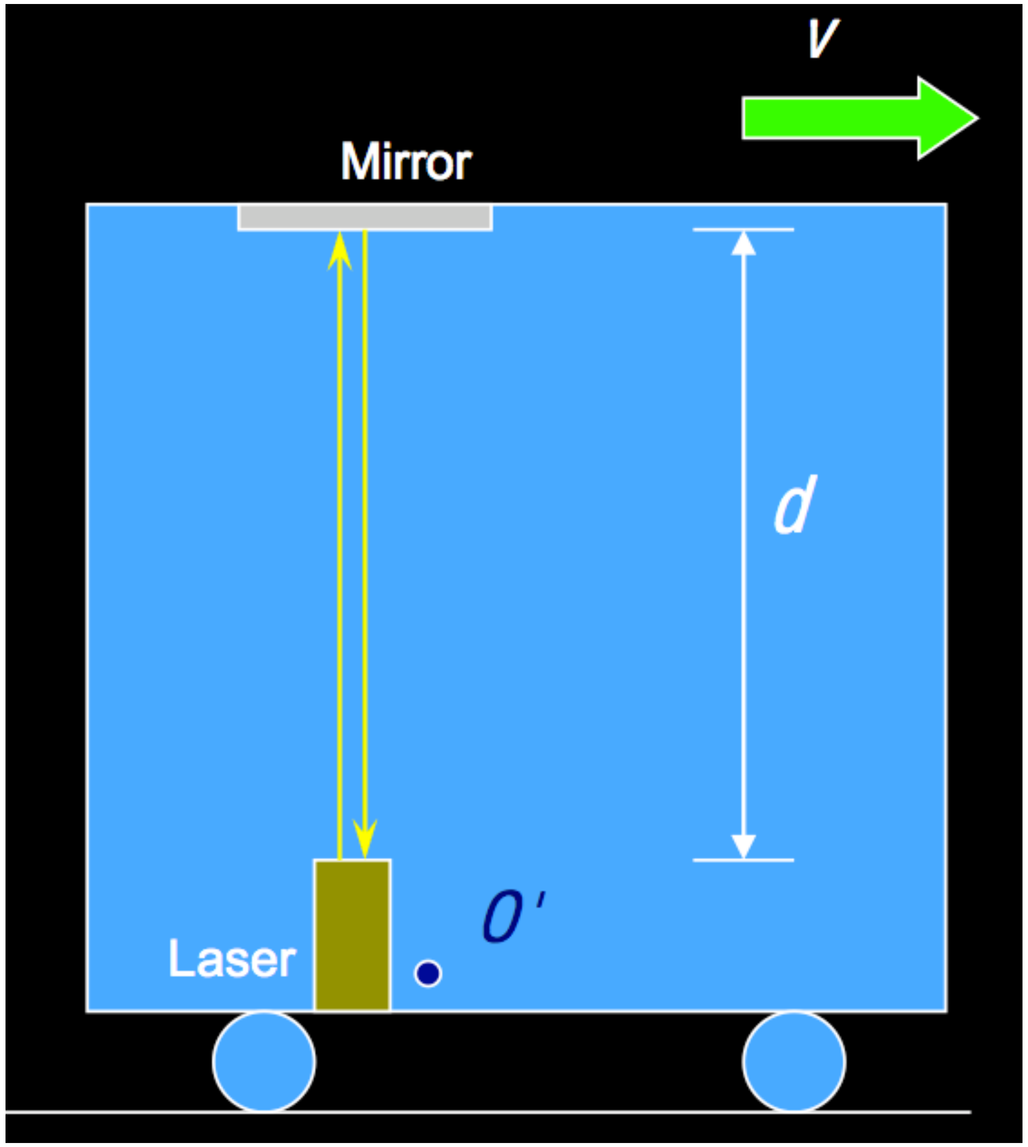
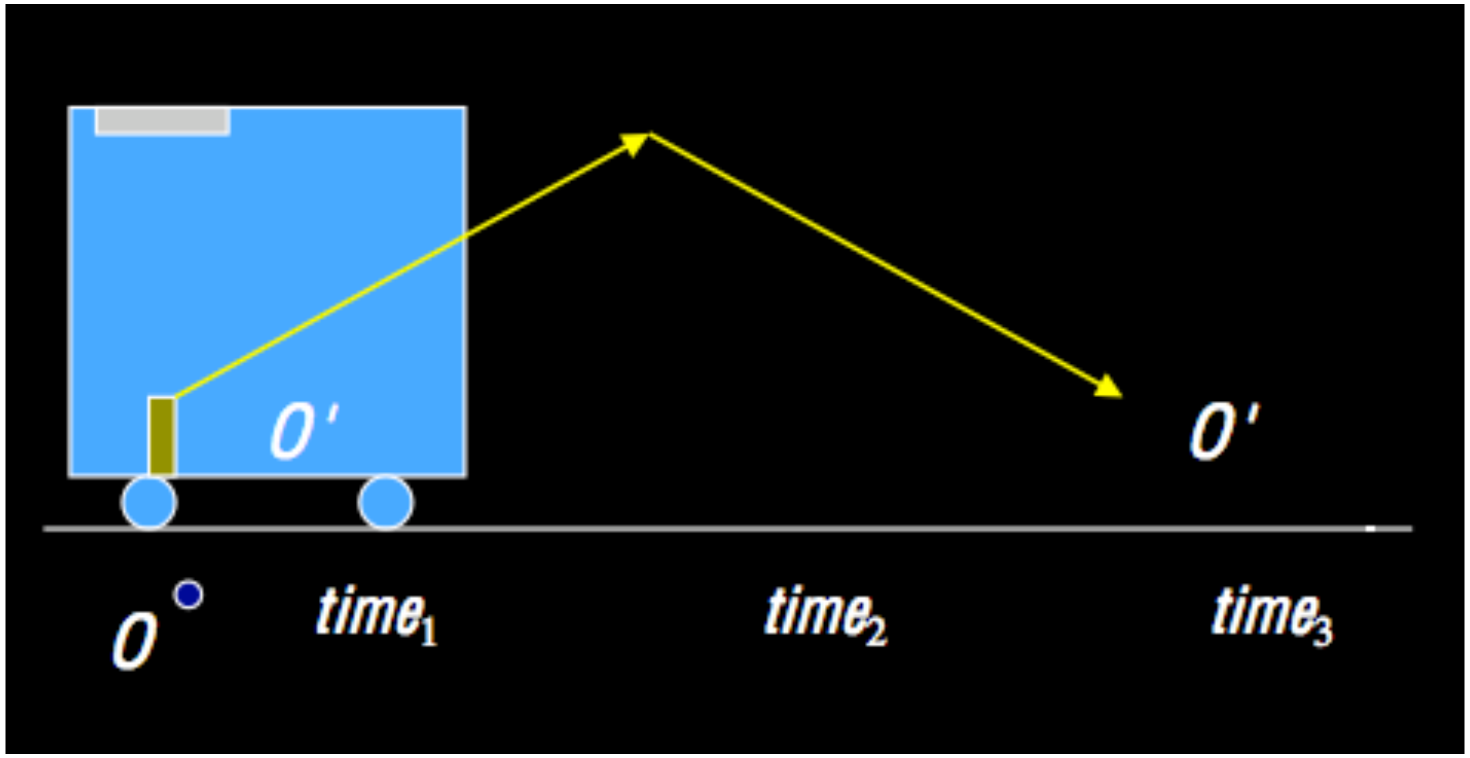
$$\Delta t' = \frac{\text{distance traveled}}{\text{speed}} = \frac{2d}{c}$$



Time dilation

- An observer O outside the car sees that it takes time Δt for the laser light to hit the mirror and come back.
- In that time, the car will have moved a distance $v\Delta t$ according to O.
- In other words, due to the motion of the vehicle, O sees that the laser light must leave the laser at an angle if it is to hit the mirror.
- Show that $\Delta t=\gamma\Delta t'$.



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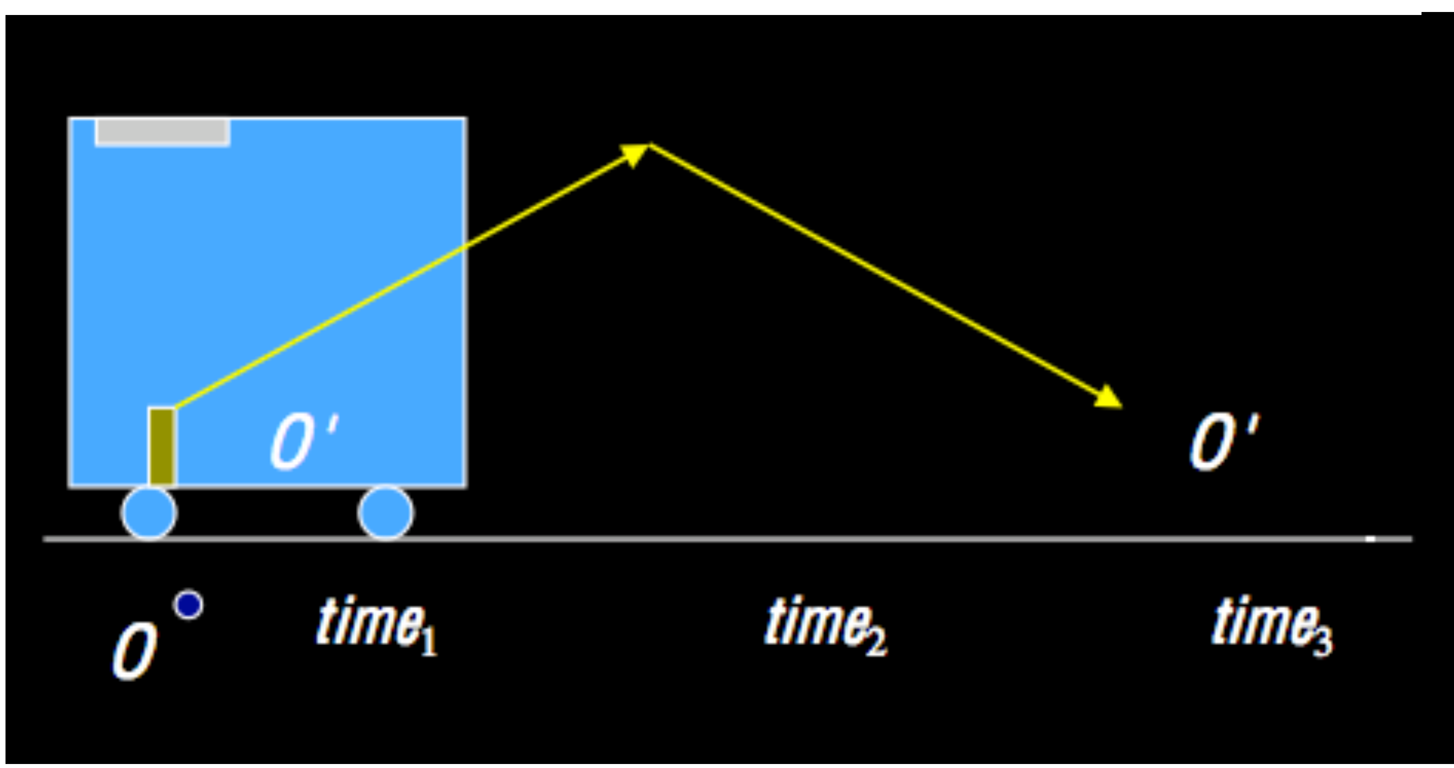

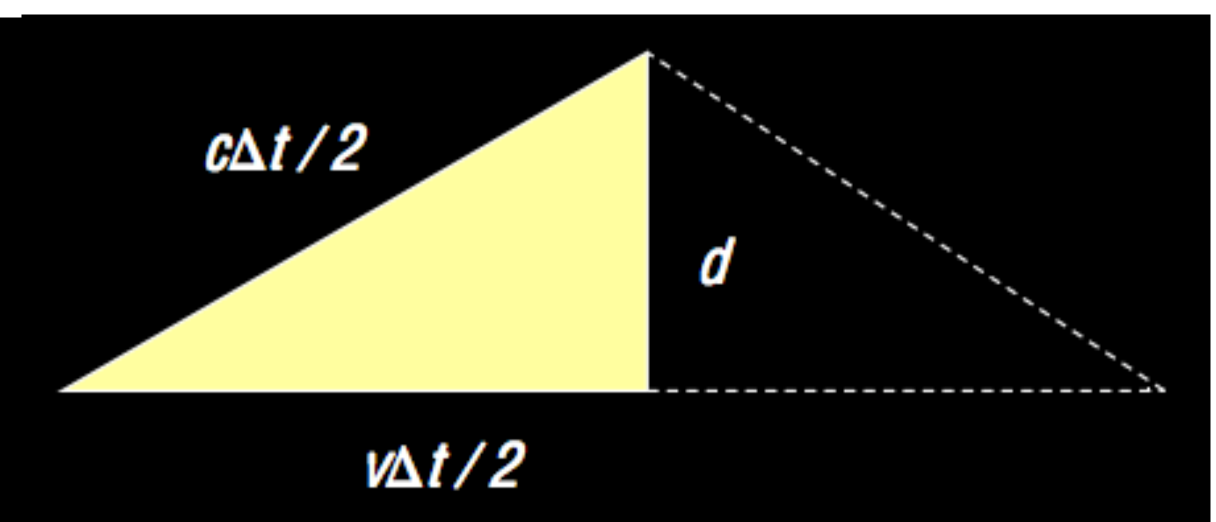

Hint: Use right triangle

Time dilation

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- In that time, the car will have moved a distance Δt according to O.
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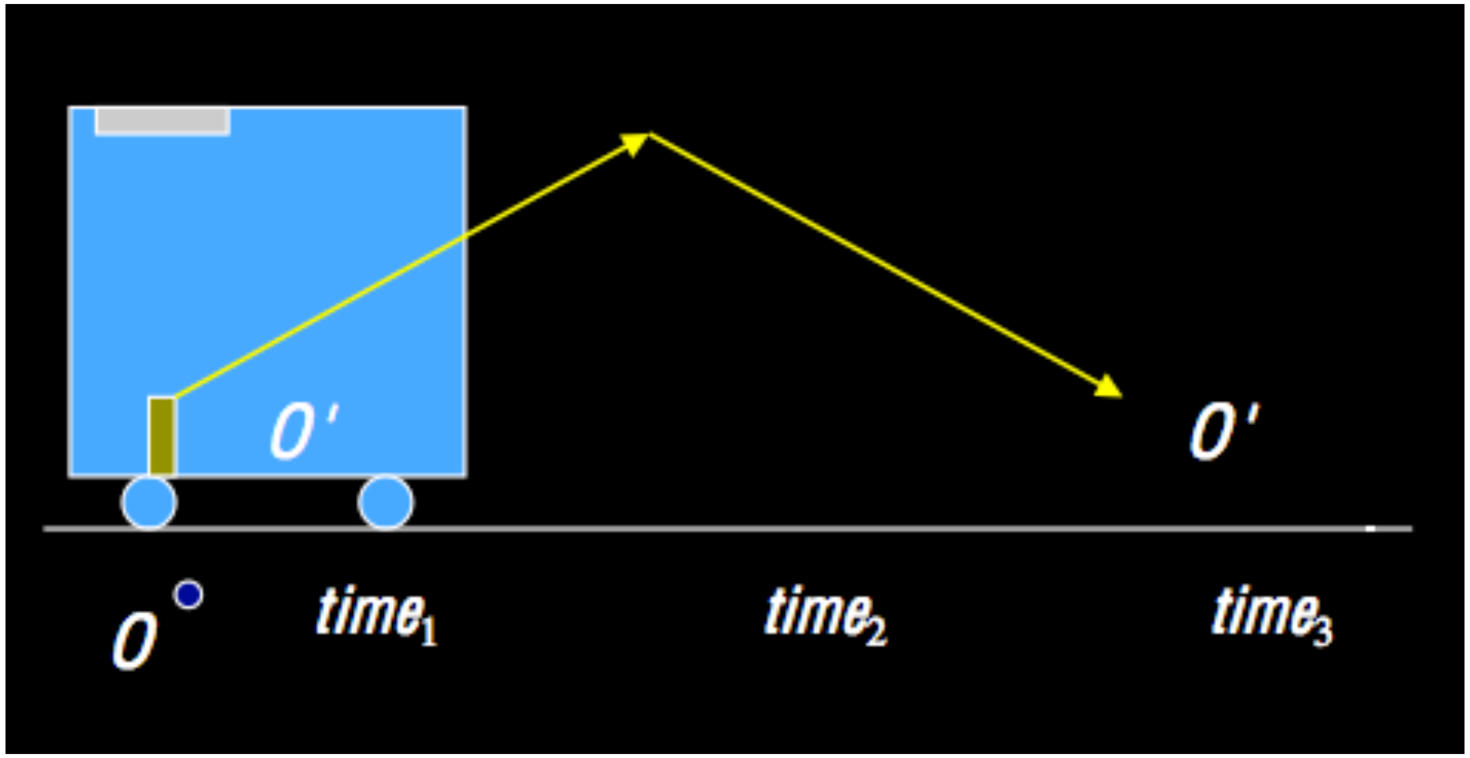
- Show that Δt=γΔt'.



Solution

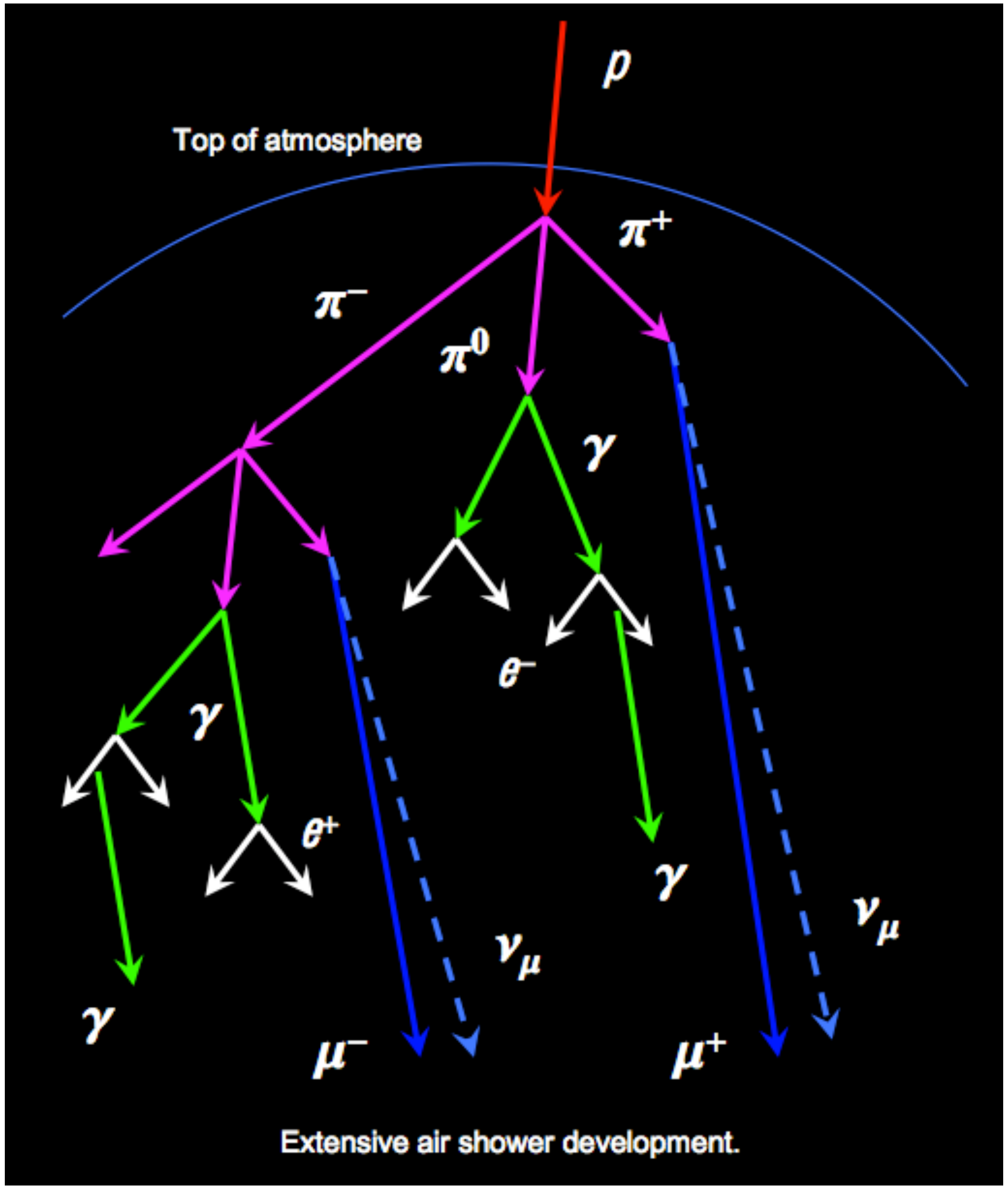
$$\left(\frac{c\Delta t}{2}\right)^2 = \left(\frac{v\Delta t}{2}\right)^2 + d^2$$

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d/c}{\sqrt{1 - v^2/c^2}}$$

$$\Delta t = \gamma \Delta t'$$

Time dilation in practice

- Recall our mention of cosmic ray air showers…
- Relativistic nuclei strike the atmosphere, causing a huge cascade of high energy decay products.
  - Many of these are detected at Earth's surface.
- However, most of them (like π's and μ's) are very unstable and short-lived.
- How do they make it to Earth's surface?



Time dilation in practice

Naively:

- The mean lifetime of the muon (in its rest frame) is 2.2 microseconds.
- Most air shower muons are generated high in the atmosphere (~8 km altitude).
- If they travel at 99.9% of the speed of light *c*, should they make it to Earth from that altitude?

Time dilation in practice

Naively:

- The mean lifetime of the muon (in its rest frame) is 2.2 microseconds.
- Most air shower muons are generated high in the atmosphere (~8 km altitude).
- If they travel at 99.9% of the speed of light *c*, should they make it to Earth from that altitude?

$$\begin{aligned}\text{Muon range} &= (\text{lifetime}) \times (\text{speed}) \\ &= (2.2 \times 10^{-6}\ \mathrm{s}) \times (0.999c) \\ &\approx 660\ \mathrm{m}\end{aligned}$$

This suggests that muons should not be able to make it to Earth's surface. But we detect them. Where did the calculation go wrong?

Time dilation in practice

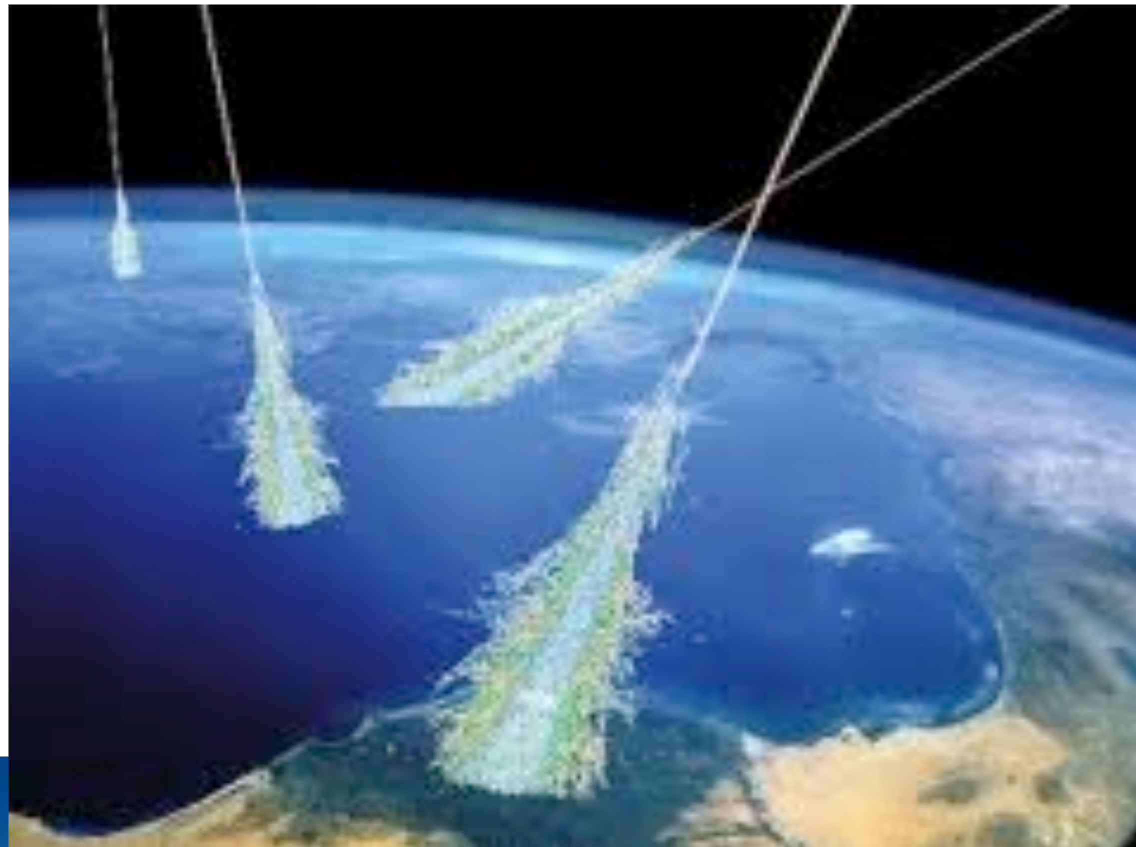
Accounting for relativity:

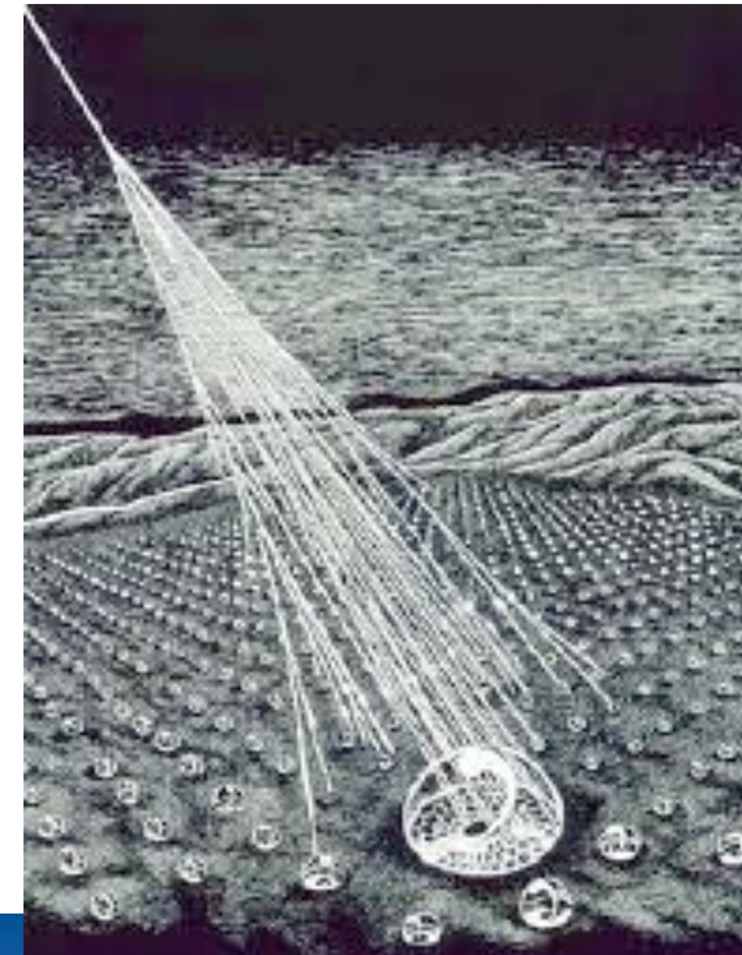
- In the lab (the stationary frame), the muon's lifetime undergoes time dilation (a muon's clock ticks slower…).
- Therefore, we have an effective lifetime to deal with:

$$\begin{aligned}\text{Muon range} &= \gamma \times (\text{lifetime}) \times (\text{speed}) \\ &= \left(\frac{1}{\sqrt{1-(0.999c/c)^2}} \right) \times (2.2 \times 10^{-6}\ \text{s}) \times (0.999c) \\ &\approx 14.7\ \text{km}\end{aligned}$$

So the muon can certainly make it to the ground, on average, when we account for relativistic effects.

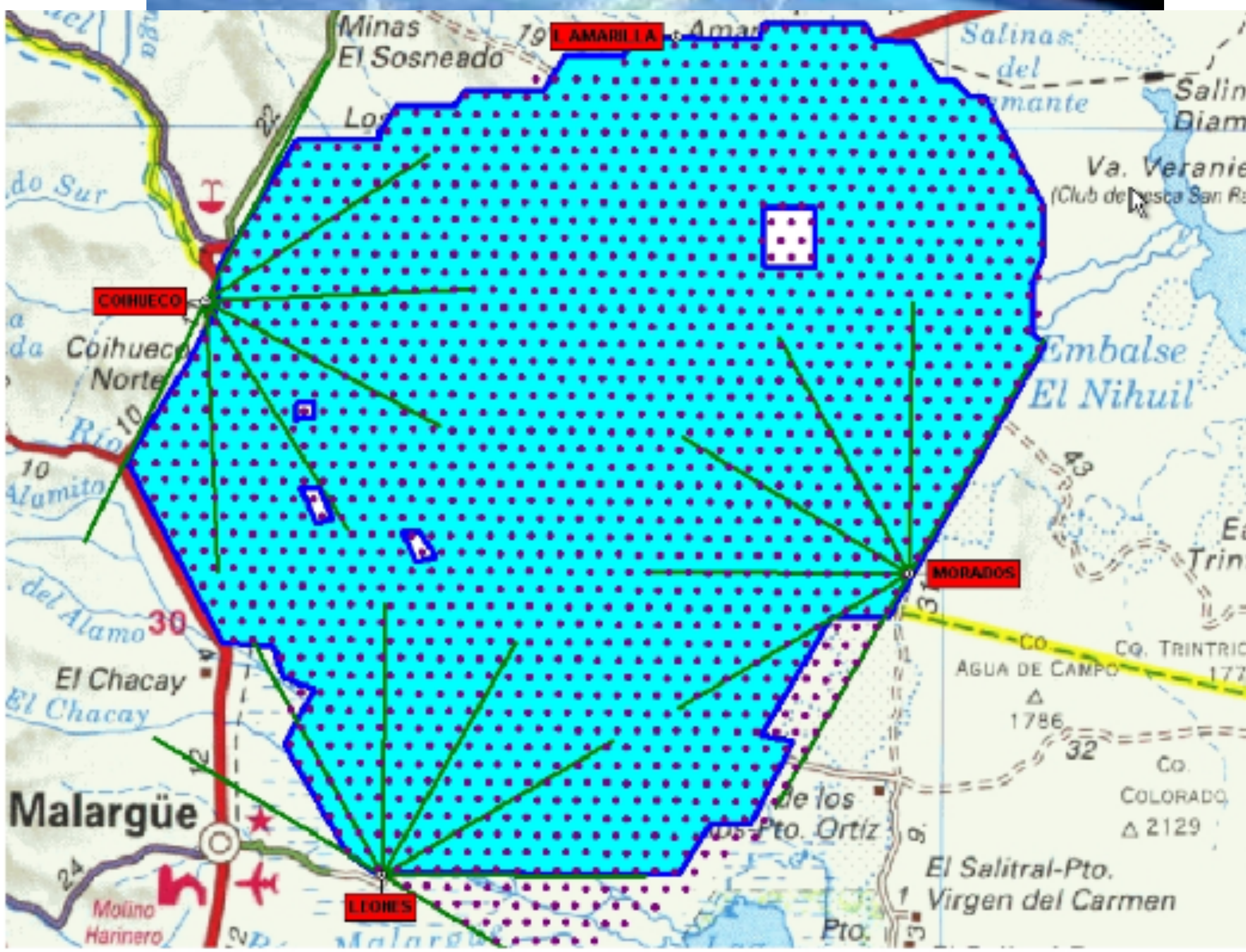
Cosmic ray experiments



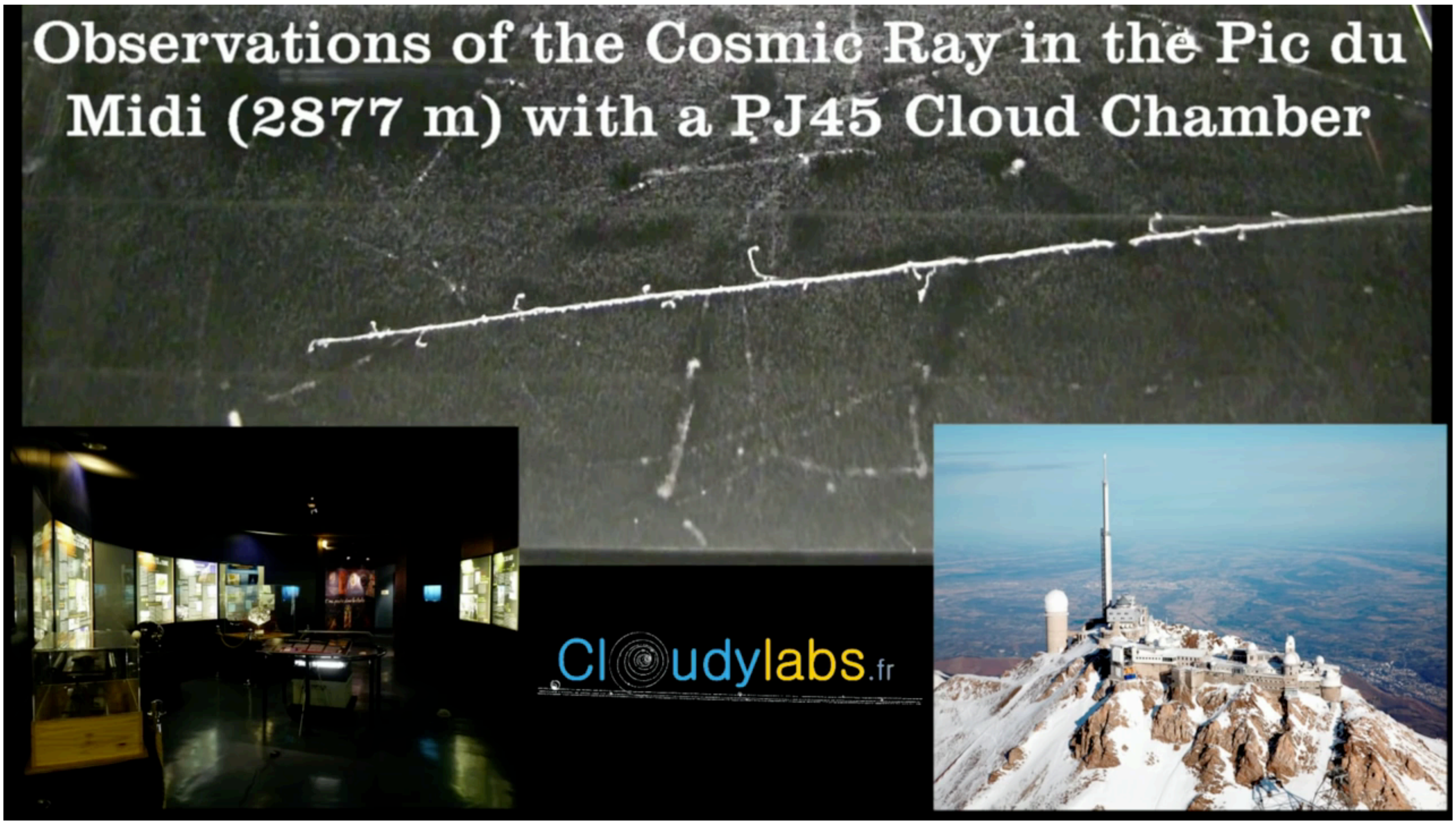


Cosmic ray experiments

The Pierre Auger Observatory:
Mendoza province, Argentina



Cosmic ray experiments



Transformation between reference frames

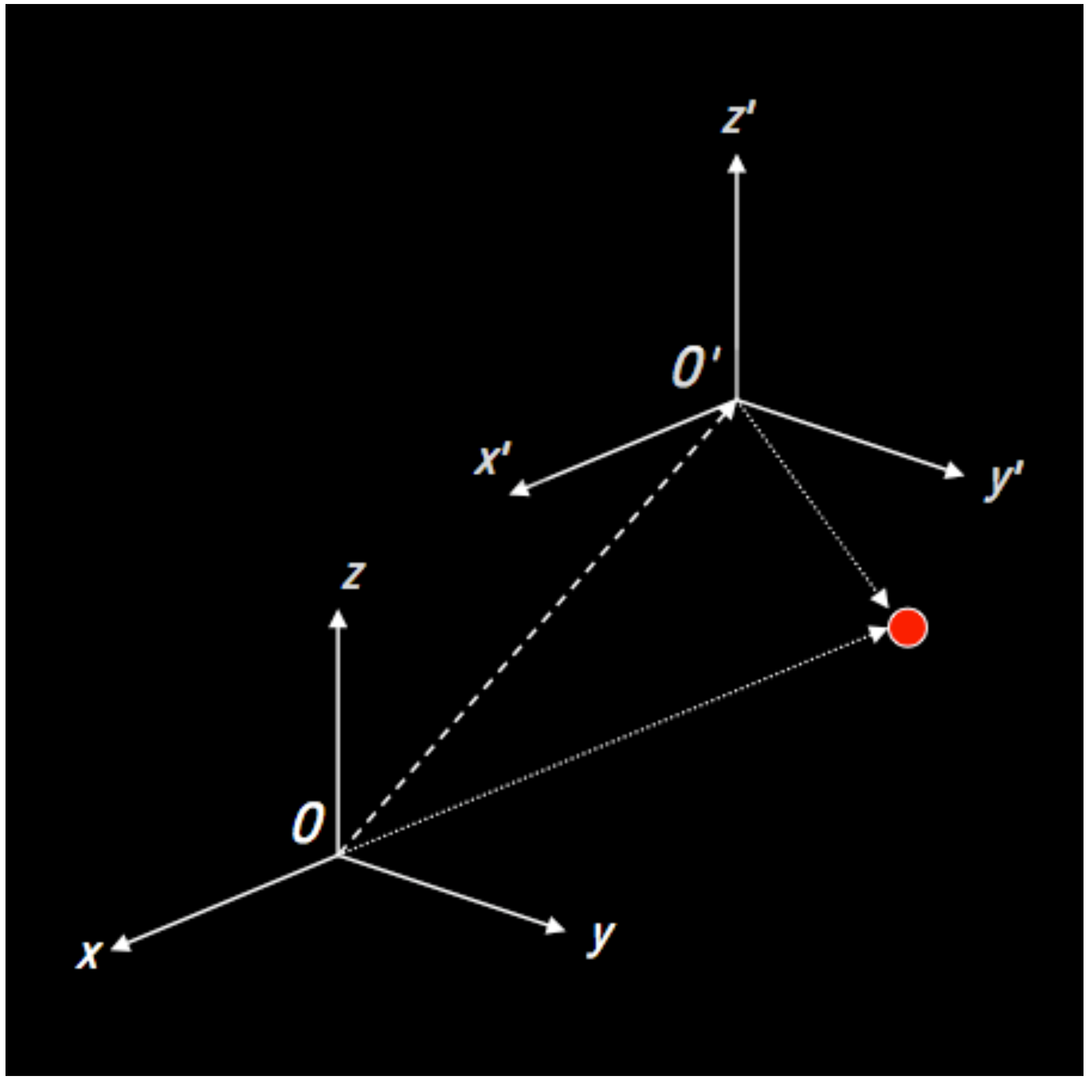
- Using the postulates of Special Relativity, we can start to work out how to transform coordinates between different inertial observers.
  - What is a transformation? It's a mathematical operation that takes us from one inertial observer's coordinate system into another's.
- The set of possible transformations between inertial reference frames are called the **Lorentz Transformations**.
- They form a group (in the mathematical sense of "group theory").
- The possible Lorentz Transformations:
  - Translations
  - Rotations
  - Boosts

Translations (fixed displacements)

- In fixed translations, the two observers have different origins, but don't move with respect to each other.
- In this case, the observers' clocks differ by a constant b_0 and their positions differ by a constant vector b:

$$\vec{x}' = \vec{x} - \vec{b}$$

$$t' = t - b_0$$

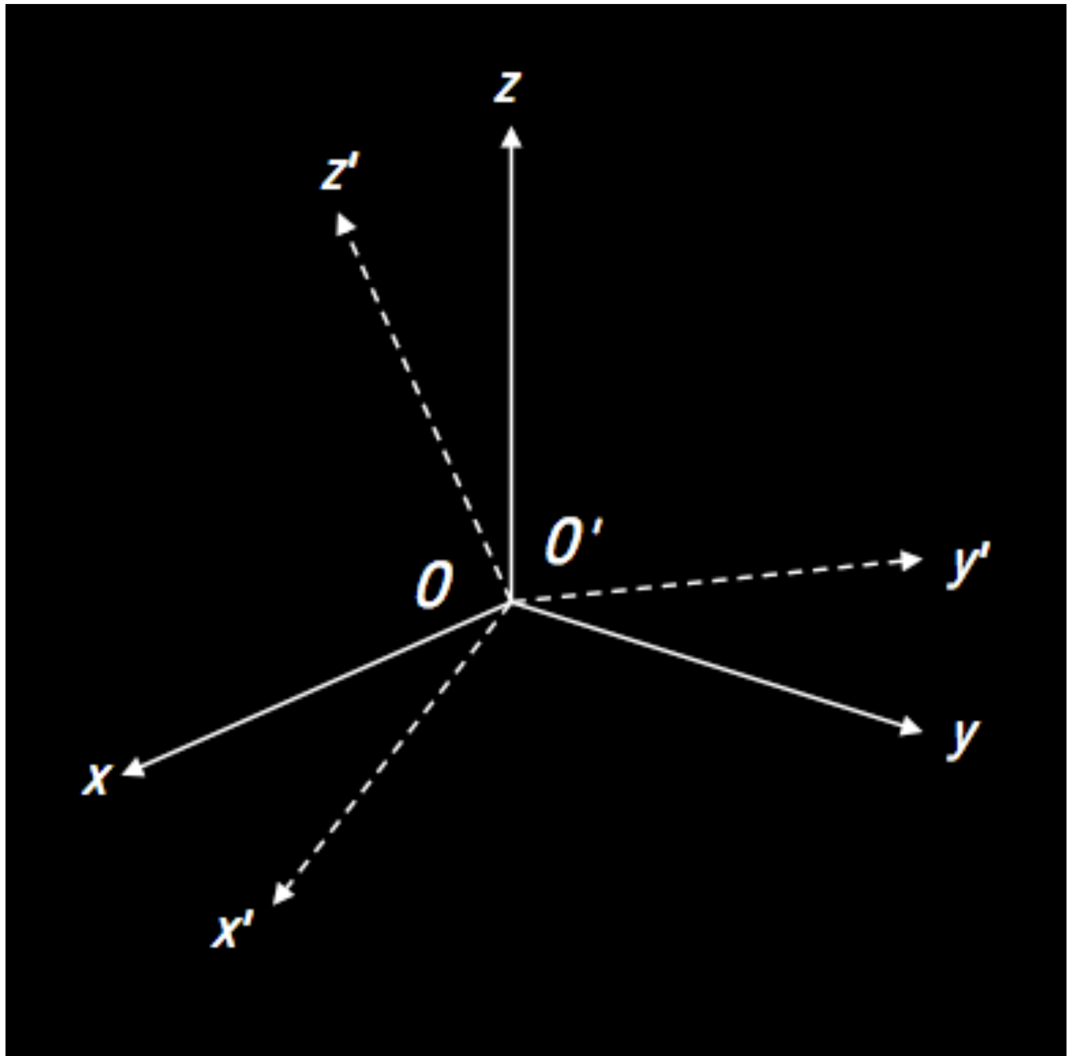


Rotations (fixed)

- In fixed rotations, the two observers have a common origin and don't move with respect to each other.
- In this case, the observers' coordinates are rotated with respect to each other.
- The spatial transformation can be accomplished with a rotation matrix; measured times are the same:

$$\vec{x}' = \overleftrightarrow{R} \cdot \vec{x}$$

$$t' = t$$



One coordinate system rotated with respect to another.

Fixed rotation example

- Consider two observers. They share a common origin and z-axis, but the x-y plane of O' is rotated counterclockwise by an angle of of φ relative to O.
- Their unit vectors are related by:

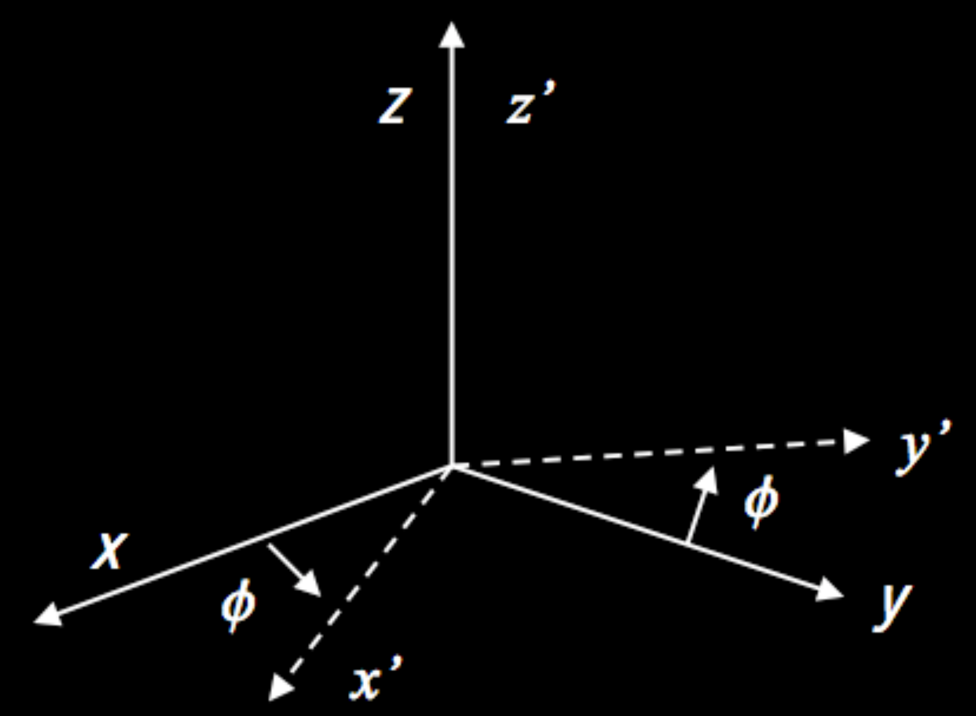
$$\begin{pmatrix} \hat{x}' \\ \hat{y}' \\ \hat{z}' \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix},$$

or

$$\hat{x}' = \hat{x}\cos\phi + \hat{y}\sin\phi$$

$$\hat{y}' = -\hat{x}\sin\phi + \hat{y}\cos\phi$$

$$\hat{z}' = \hat{z}$$

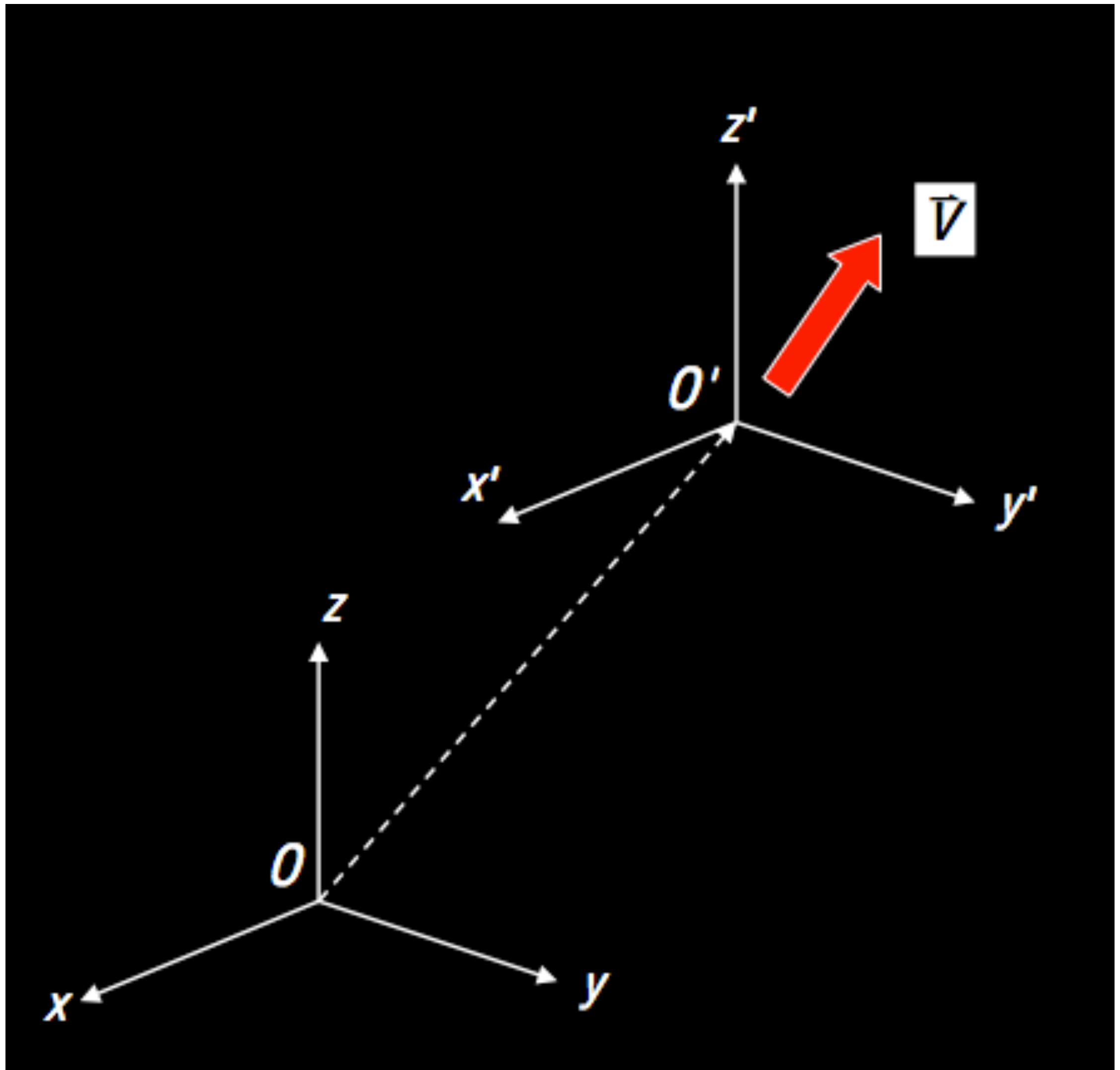


A rotation about the z axis. The unit vectors of the rotated coordinates are related to the original coordinates by a rotation matrix.

Try this yourself, using matrix multiplication rules!

Boosts

- In boosts, the two frame axes are aligned, but the frames move at constant velocity with respect to each other.
- The origins are chosen here to coincide at time t=0 in both frames.
- The fact that the observers' coordinates are not fixed relative to each other makes boosts more complex than translations and rotations.
- It is in boosts that the constancy of the speed of light plays a big role.



Observers' axes are aligned, but one moves at constant velocity v with respect to the other.

Boosts: Galileo vs Lorentz

- Suppose we have two observers O and O'. O is at rest, and O' moves along the x direction with constant velocity v.
- According to Galileo, the transformation between the coordinates of O and O' is pretty simple.
- According to Lorentz and Einstein, we get complicated expressions with many factors of c involved: the so-called Lorentz transformations.
- If an event occurs at position (x,y,z) and time t for observer O, what are the space-time coordinates (x',y',z') and t' measured by O'?

Galileo

$$\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}$$

Lorentz

$$\begin{aligned} x' &= \gamma\left(x - vt\right) \\ y' &= y \\ z' &= z \\ t' &= \gamma\left(t - vx/c^2\right) \end{aligned} \quad , \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \geq 1$$

Note the Lorentz factor γ in the Lorentz boosts.

Lorentz (length) contraction

- Suppose a moving observer O' puts a rigid "meter" stick along the x' axis: one end is at $x'=0$ and the other at $x'=L'$.
- Now an observer O at rest measures the length of the stick at time $t=0$, when the origins of O and O' are aligned. What will O measure for x'?
- Using the first boost equation $x'=\gamma(x-vt)$ at time $t=0$, it looks like the lengths are related by:

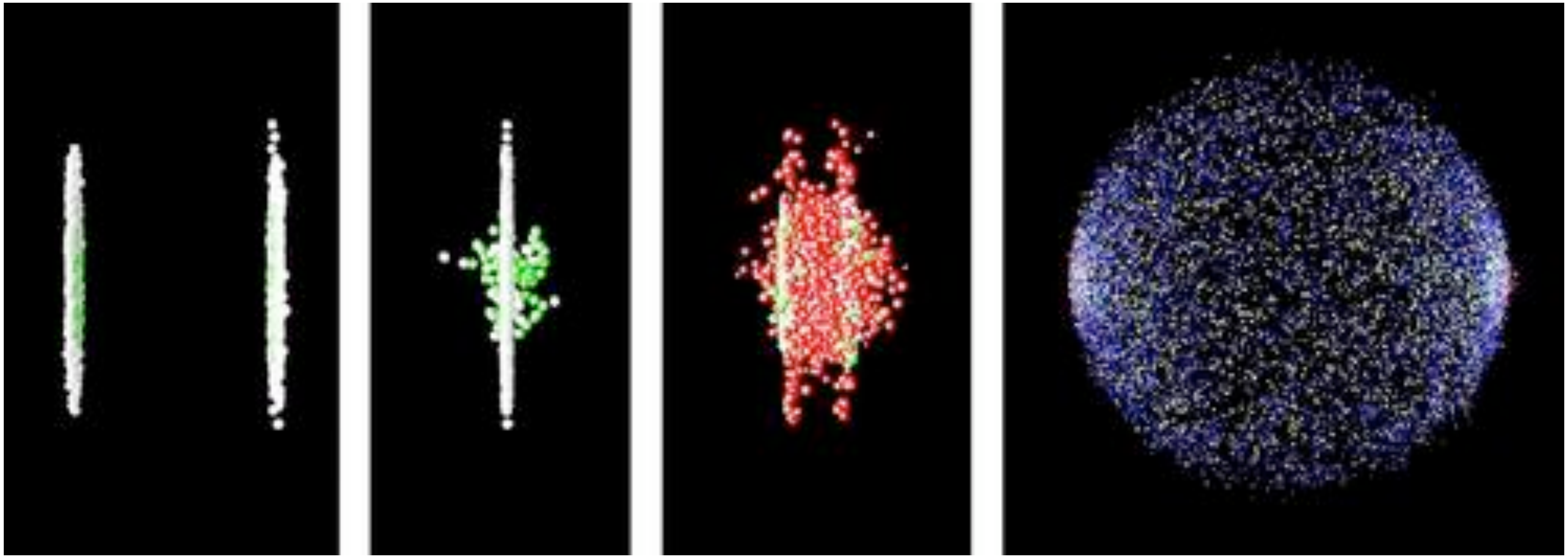
$$\text{moving} \quad L' = \gamma \ L \quad \text{at rest}$$

$$L = L' / \gamma$$

This is the **Lorentz contraction**: if an object has length L when it is at rest, then when it moves with speed v in a direction parallel to its length, an observer at rest will measure its length as the shorter value L/γ.

Lorentz Contraction

- An example of Lorentz contraction in the case of collisions of two gold nuclei at the RHIC collider at Brookhaven Lab on Long Island:



- In typical collisions (200 GeV), nuclei have a Lorentz factor of O(200).

Velocity addition

- Finally, let's briefly derive the rule for addition of relativistic velocities (we will need to use the boost equations…)
- Suppose a particle is moving in the x direction at speed u' with respect to observer O'. What is its speed u with respect to O?
  - Since the particle travels a distance $\Delta x=\gamma(\Delta x'+v\Delta t')$–an "inverse" boost –in time $\Delta t=\gamma\,(\Delta t'+(v/c^2)\,\Delta x')$, the velocity in frame O is:

$$\boxed{\frac{\Delta x}{\Delta t}=\frac{\Delta x'+v\Delta t'}{\Delta t'+(v/c^2)\Delta x'}=\frac{(\Delta x'/\Delta t')+v}{1+(v/c^2)(\Delta x'/\Delta t')}}$$

where v is the relative velocity of the two inertial frames.

  - Since $u=\Delta x/\Delta t$ and $u'=\Delta x'/\Delta t'$, we get the addition rule:

$$\boxed{u=\frac{u'+v}{1+(u'v/c^2)};\quad \text{compare to}\quad u=u'+v}$$

Four-vector notation

- This is a way to simplify notation for all we've talked about so far.
- Soon after Einstein published his papers on Special Relativity, Minkowski noticed that **regarding t and (x,y,z) as simply four coordinates in a 4-D space ("space-time")** really simplified many calculations.
- In this spirit, we can introduce a position-time four-vector x^μ, where $\mu=0,1,2,3$, as follows:

$$\boxed{x^0 = ct, \quad x^1 = x, \quad x^2 = y, \quad x^3 = z}$$

Lorentz boosts in four-vector notation

- In terms of the 4-vector x^μ, a Lorentz boost along the x^1 (that is, the x) direction looks like:

$$
\begin{aligned}
x^0{}' &= \gamma(x^0 - \beta x^1) \\
x^1{}' &= \gamma(x^1 - \beta x^0) \\
x^2{}' &= x^2 \\
x^3{}' &= x^3
\end{aligned}
\quad , \quad \text{where } \beta = v/c
$$

- As an exercise, you can show that the above equations recover the Lorentz boosts we discussed earlier.
- FYI, this set of equations also has a very nice and useful matrix form.

Lorentz boosts in matrix form

- Using 4-vectors, we can write the Lorentz boost transformation as a matrix equation:

$$\begin{pmatrix} x^{0\prime} \\ x^{1\prime} \\ x^{2\prime} \\ x^{3\prime} \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

- Looks very similar to the 3-D rotation!
- Mathematically, boosts and rotations are actually very close "cousins". We can understand this connection using the ideas of group theory.

Invariant quantities

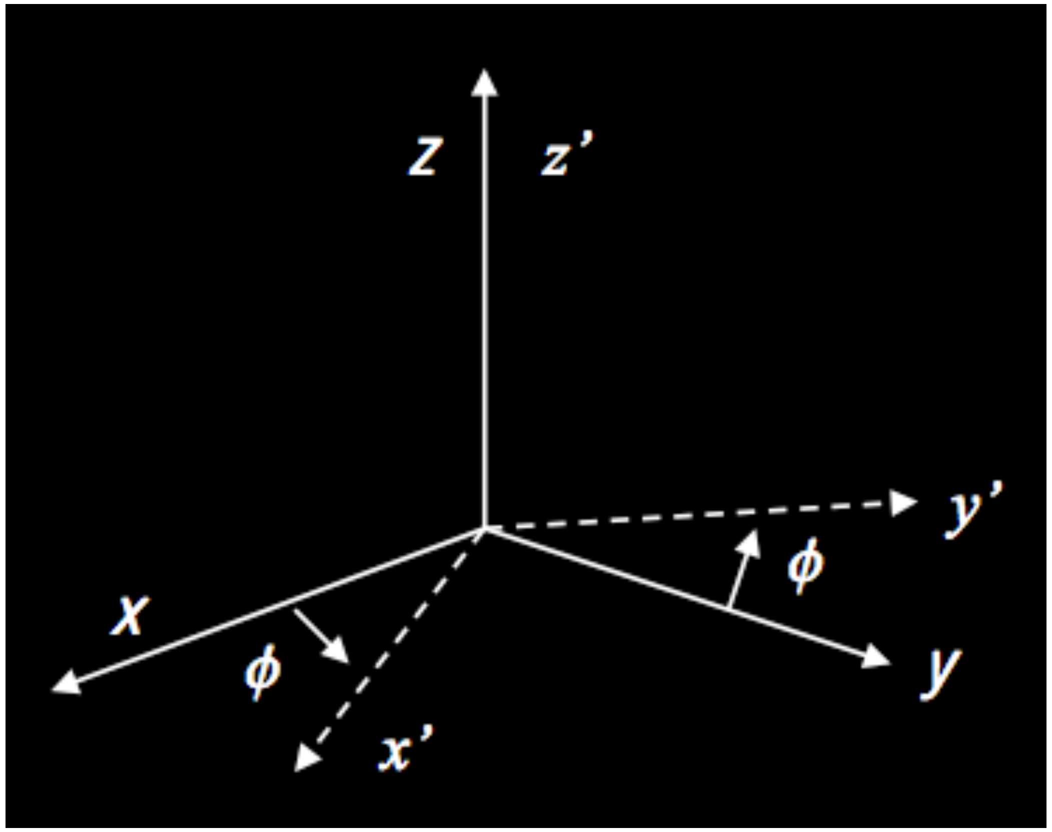
- The utility of 4-vectors comes in when we start to talk about invariant quantities.
- **Definition: a quantity is called invariant if it has the same value in any inertial system.**
- Recall: the laws of physics are always the same in any inertial coordinate system (this is the definition of an inertial observer).
  - Therefore, these laws are invariants, in a sense.
- The identification of invariants in a system is often the best way to understand its physical behavior.

Example of invariant quantity

- Think of a 3-vector (x,y,z). An example of an invariant is its square magnitude: $r^2=x^2+y^2+z^2$, whose value does not change under coordinate rotations.
- Consider a rotation about the z-axis:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{aligned} r'^2 &= x'^2 + y'^2 + z'^2 \\ &= (x\cos\varphi + y\sin\varphi)^2 + (-x\sin\varphi + y\cos\varphi)^2 + z^2 \\ &= (\cos^2\varphi + \sin^2\varphi)(x^2 + y^2) + z^2 \\ &= r^2 \end{aligned}$$



4-vector scalar product

- The quantity Δs^2, given by:

$$\Delta s^2 = x^0 x^0 - x^1 x^1 - x^2 x^2 - x^3 x^3 = x^0 x^0 - \vec{x} \cdot \vec{x}$$
$$= (ct)^2 - x^2$$

is called the scalar product of x^μ with itself.

- It has the same value in any coordinate system (just like any scalar).
  - This spacetime interval is often called the proper length.
- To denote the scalar product of two arbitrary 4-vectors a^μ and b^μ, it is convenient to drop the Greek index and just write:

$$a \cdot b = a^0 b^0 - \vec{a} \cdot \vec{b}$$

- In this case, the 4-vectors *a* and *b* are distinguished from their spatial 3-vector components, by the little arrow overbar.

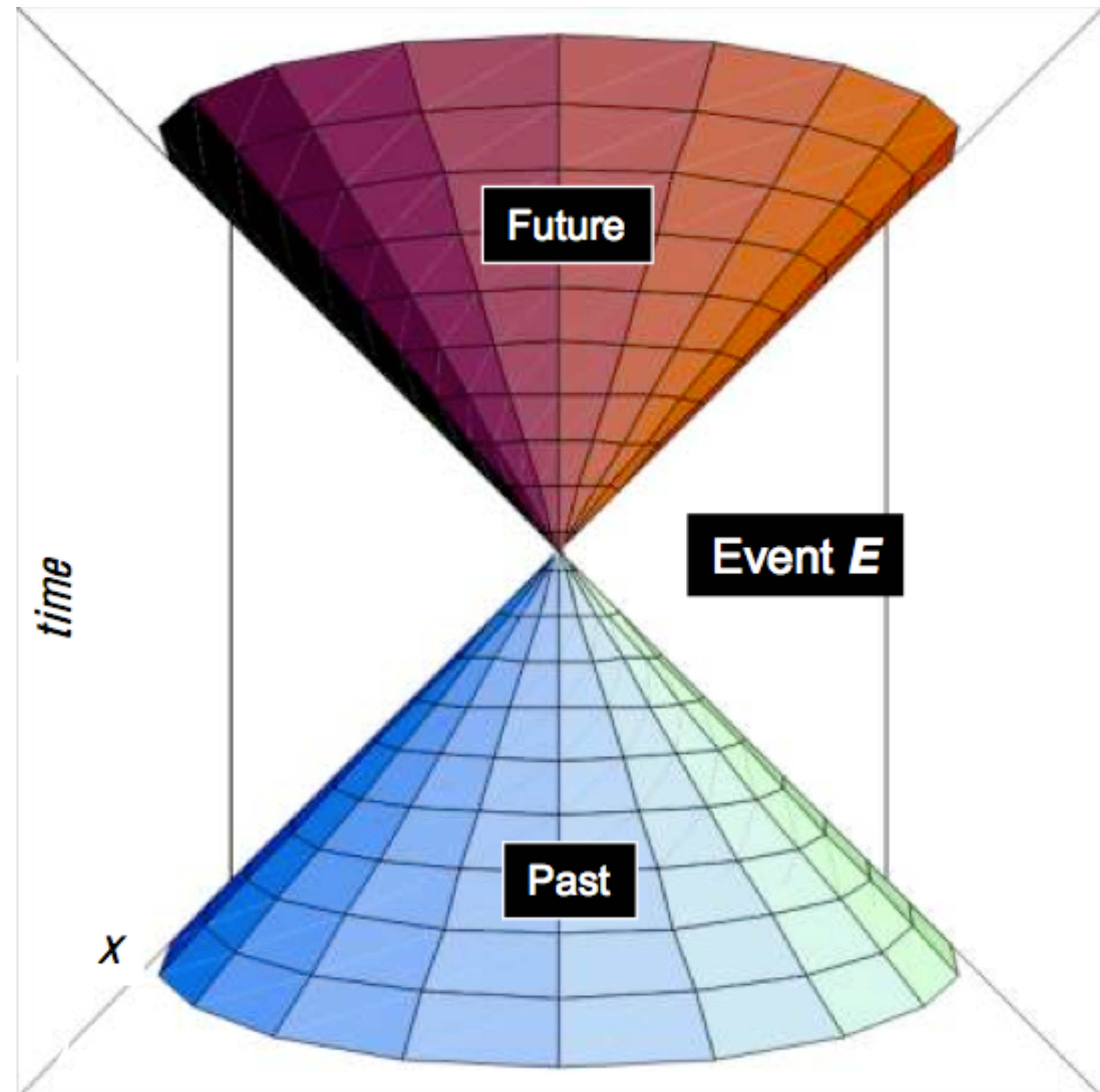
4-vector scalar product

- Terminology: any arbitrary 4-vector a^μ can be classified by the sign of its scalar product a^2:

1. If $a^2>0$, a^μ is called *time-like* because the time component dominates the scalar product.
2. If $a^2<0$, a^μ is called *space-like* because the spatial components dominate a^2.
3. If $a^2 = 0$, a^μ is called *light-like* or null because, as with photons, the time and space components of a^μ cancel.

The light cone, revisited

- A set of points all connected to a single event E by lines moving at the speed of light is called the **light cone**.
- The set of points inside the light cone are time-like separated from E.
- The set of points outside the cone are space-like separated from E.
- Points outside the cone cannot casually affect (or be affected by) the event E.
  - Signal from these points cannot make it to the event.



Past and future light cones for an event E, with z dimension suppressed, and units defined such that c=1.

Back to particle physics…

- Why is relativity so prevalent and fundamental in this field?

SR in particle physics

- We will talk about relativistic kinematics - the physics of particle collisions and decays.
- In the context of what we have discussed so far, we start to think of particles as moving "observers", and scientists as stationary observers.
  - The reference frame of particles is often called the "particle rest frame", while the frame in which the scientist sits at rest, studying the particle, is called the "lab frame".
- To begin, let's *define* (not derive) the notions of relativistic energy, momentum, and the mass-energy relation.
  - These should reduce to classical expressions when velocities are very low (classical limit).

We will be applying the algebra of 4-vectors to particle physics.

Relativistic momentum

- The relativistic momentum (a three-vector) of a particle is similar to the momentum you're familiar with, except for one of those factors of γ:

$$\vec{p} = \gamma m\vec{v} = \frac{m\vec{v}}{\sqrt{1 - v^2 / c^2}}$$

- The relativistic momentum agrees with the more familiar expression in the so-called "classical regime" where *v* is a small fraction of *c*.
- In this case:

$$\vec{p} = m\vec{v}\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \ldots\right) \approx m\vec{v}$$

(Taylor expansion)

Relativistic energy

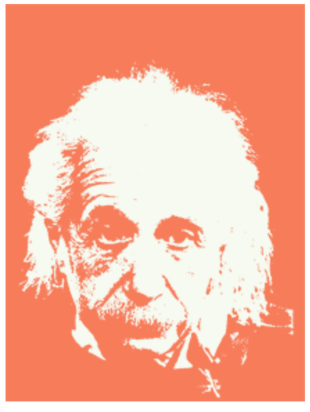
- The relativistic energy (excluding particle interactions) is quite a bit different from the classical expression:

$$\boxed{E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}}$$

- When the particle velocity v is much smaller than c, we can expand the denominator to get:

$$E = mc^2\left(1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \ldots\right) = mc^2 + \frac{1}{2}mv^2 + \frac{3}{8}m\frac{v^4}{c^2} + \ldots$$

- The second term here corresponds to the classical kinetic energy, while the leading term is a constant.
- This is not a contradiction in the classical limit, because in classical mechanics we can offset particle energies by arbitrary amounts.
- The constant term is called the rest energy of the particle and it is Einstein's famous equation: $\boxed{E_{\text{rest}} = mc^2}$



Energy-momentum four-vector

- It is convenient to combine the relativistic energy and momentum into a single 4-vector called the four-momentum.

$$p^{\mu} = \left(\frac{E}{c}, \vec{p}\right) = (\gamma mc, \gamma m\vec{v})$$

- The four-momentum, denoted p^{μ} or just p, is defined by:

The scalar product of the four-momentum with itself gives us an invariant that depends on the mass of the particle under study.

- Squaring p^{μ} yields the famous relativistic energy-momentum relation (also called the mass-shell formula):

$$p \cdot p = \frac{E^2}{c^2} - \vec{p}^2 = m^2 c^2$$

$$E = \sqrt{|\vec{p}|^2 c^2 + m^2 c^4}$$

The Lorentz-invariant quantity that results from squaring 4-momentum is called the invariant mass.

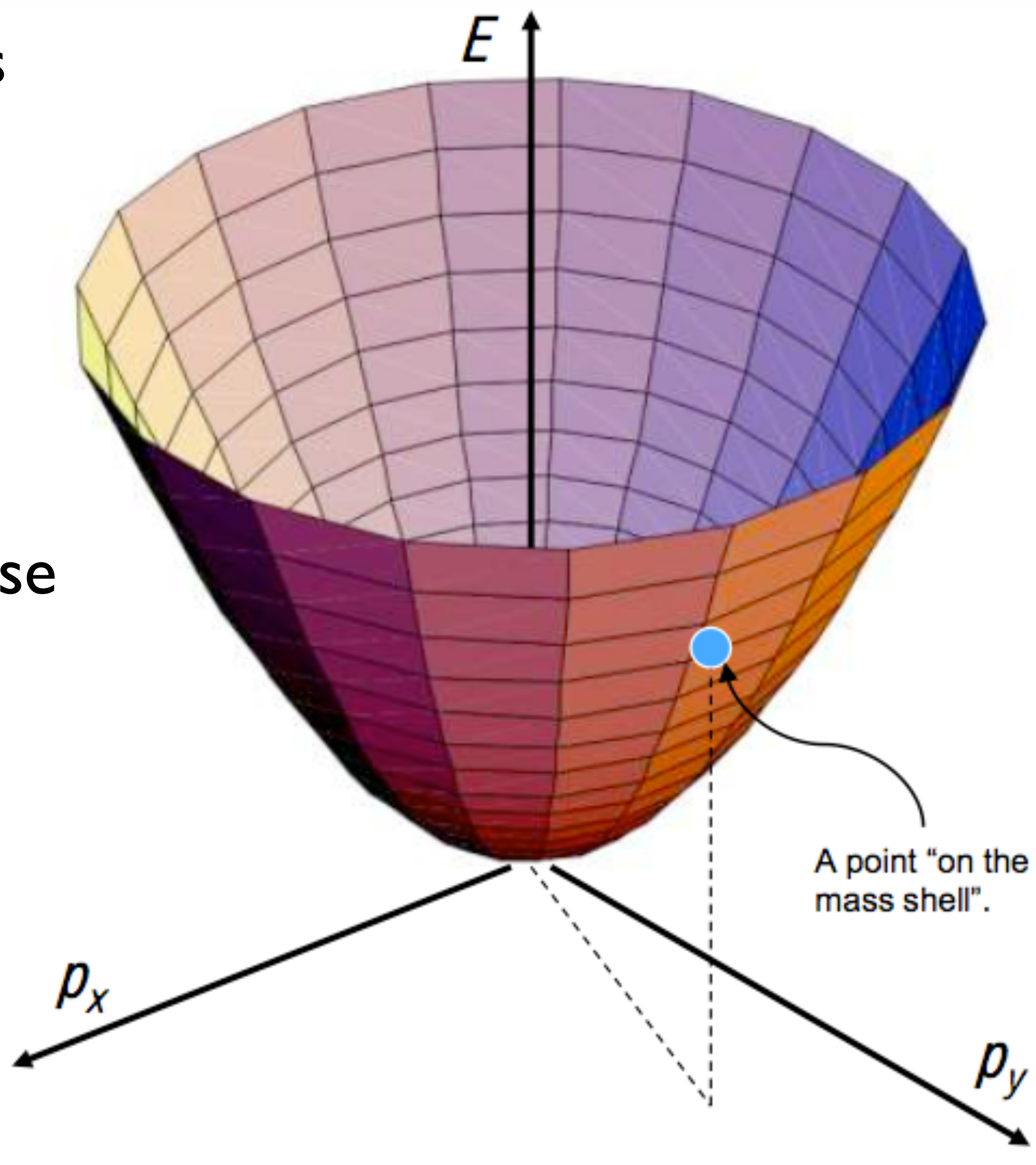
Classical vs. relativistic mass shell

- In *classical physics*, the mass-shell relation is quadratic in the momentum:

$$E = \frac{\vec{p}^2}{2m} = \frac{\left(p_x^{\ 2} + p_y^{\ 2} + p_z^{\ 2}\right)}{2m}$$

- This is called the mass-shell formula because if one plots E vs p in two dimensions, the function looks like a parabolic shell.
- Jargon: particles that obey the relativistic mass-shell relation are said to be "on mass shell":

$$E = \sqrt{\left|\vec{p}\right|^2 c^2 + m^2 c^4}$$

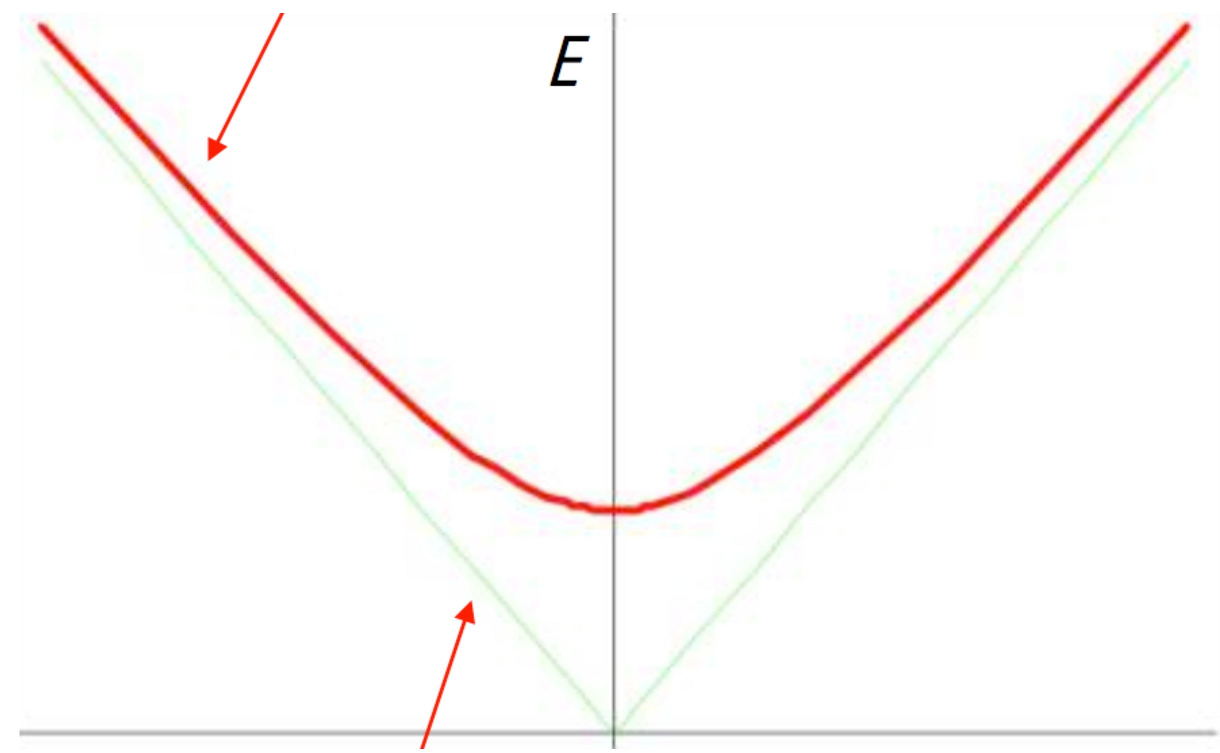


Classical mass shell relation for a 2D system.

Classical vs. relativistic mass shell

- The *relativistic* mass shell, due to the presence of the rest energy, looks like a hyperbola.

I Relativistic mass shell for 1D motion (m≠0).



Relativistic mass shell for 1D motion (m=0) (boundary of the light cone).

- Unlike classical mechanics, zero-mass particles are allowed if they travel at the speed of light.

- In the case of zero mass, the mass-shell relation reduces to:

$$E = |\vec{p}|c$$

Collisions and kinematics

- Why have we introduced energy and momentum?
  - **These quantities are conserved in any physical process (true in any inertial frame!).**
- The cleanest application of these conservation laws in particle physics is to collisions.
- The collisions we will discuss are somewhat idealized; we essentially treat particles like billiard balls, ignoring external forces like gravity or electromagnetic interactions.
- Is this a good approximation? Well, if the collisions occur fast enough, we can ignore the effects of external interactions (these make the calculation much harder!).

Classical vs. relativistic collisions

- In classical mechanics, recall the usual conservation laws:

1. Mass is conserved;
2. Momentum is conserved;
3. Kinetic energy may or may not be conserved.

Classical vs. relativistic collisions

- In ~~classical~~ relativistic mechanics, recall the usual conservation laws:

1. ~~Mass~~ is conserved; Relativistic energy
2. ~~Momentum~~ is conserved; Relativistic momentum
3. Kinetic energy may or may not be conserved.

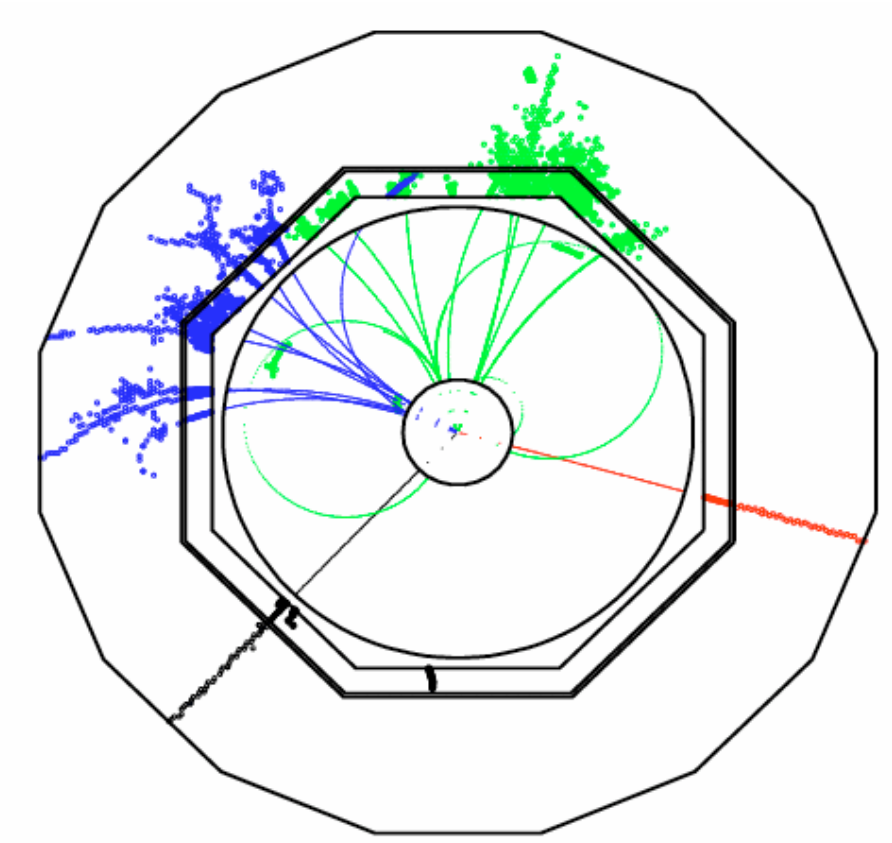
Note: conservation of energy and momentum can be encompassed into conservation of four-momentum.

Inelastic collisions

- There is a difference in interpretation between classical and relativistic inelastic collisions.
- In the classical case, inelastic collisions mean that kinetic energy is converted into "internal energy" in the system (e.g., heat).
- In special relativity, we say that the kinetic energy goes into rest energy.

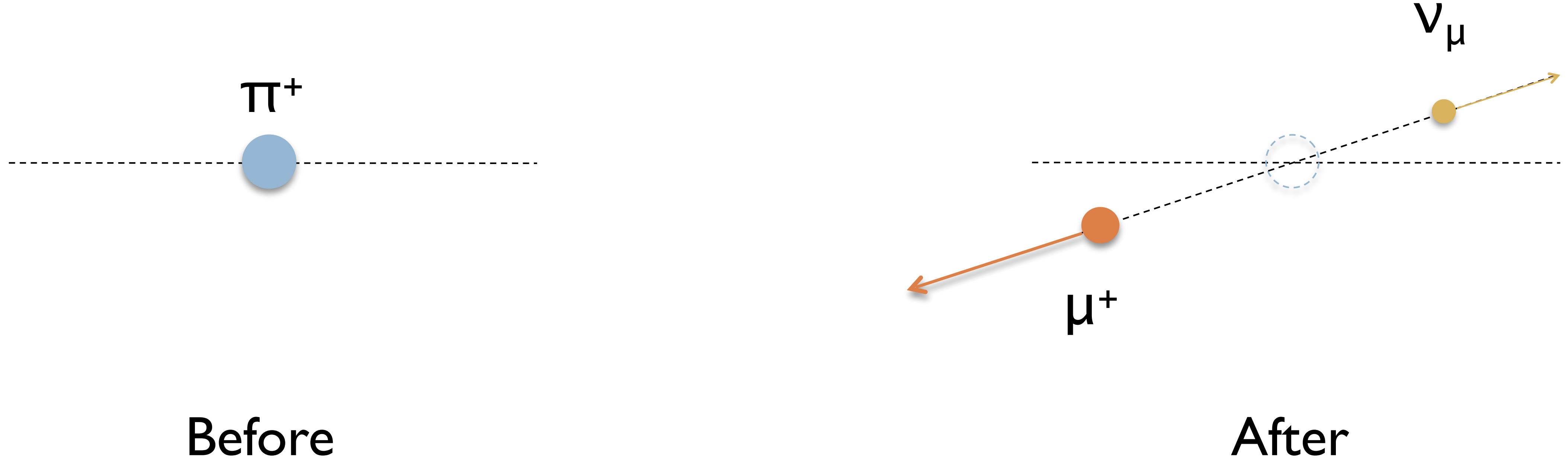
- Is there a contradiction?
  - No, because the energy-masse relation $E=mc^2$, tells us that all "internal" forms of energy are manifested in the rest energy of an object.
  - In other words, hot objects weigh more than cold objects. But this is not a measurable effect even on the atomic scale!

Mass-energy equivalence



SR in particle physics

- Consider the decay $\pi^+ \rightarrow \mu^+ \nu^{\mu}$:



Summary

- **Lorentz boosts** to and from a moving reference frame:

$$\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - vx/c^2) \end{aligned} \quad , \quad \begin{aligned} x &= \gamma(x' + vt') \\ y &= y' \\ z &= z' \\ t &= \gamma(t' + vx'/c^2) \end{aligned} \quad , \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \geq 1$$

- Relativistic momentum and energy:

$\vec{p} = \gamma m \vec{v},$ relativistic momentum

$E = \gamma m c^2,$ relativistic energy

$E_{\text{rest}} = mc^2,$ rest energy

$T = E - E_{\text{rest}} = (\gamma - 1) mc^2,$ relativistic kinetic energy

$E^2 = |\vec{p}|^2 c^2 + m^2 c^4,$ mass-shell relation

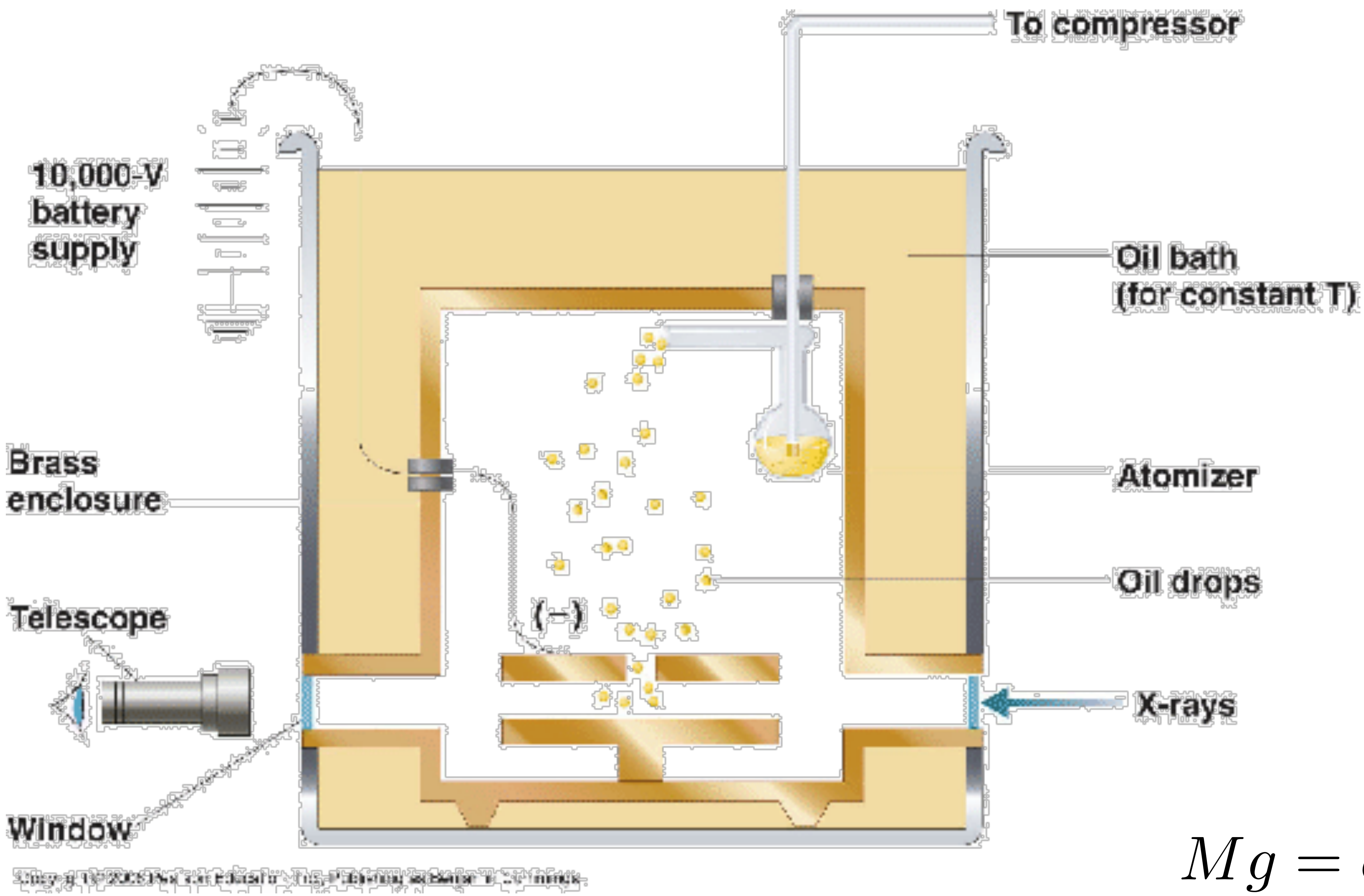
That’s all for this week…

- **Next week:** Quantum Mechanics

Bonus material

Electron charge

- Robert Milikan's oil drop experiment (1906):



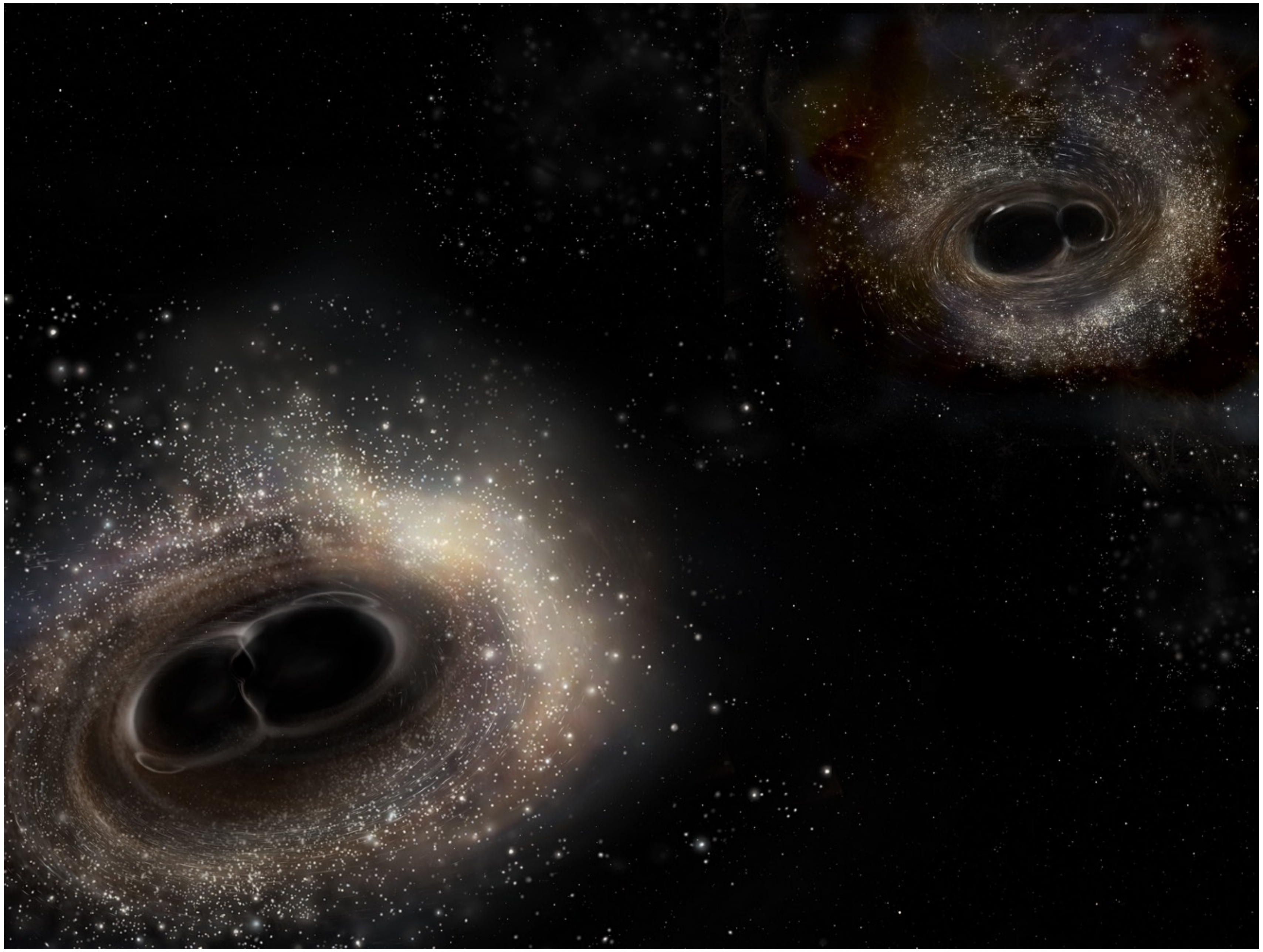
$$Mg = qE$$

General relativity

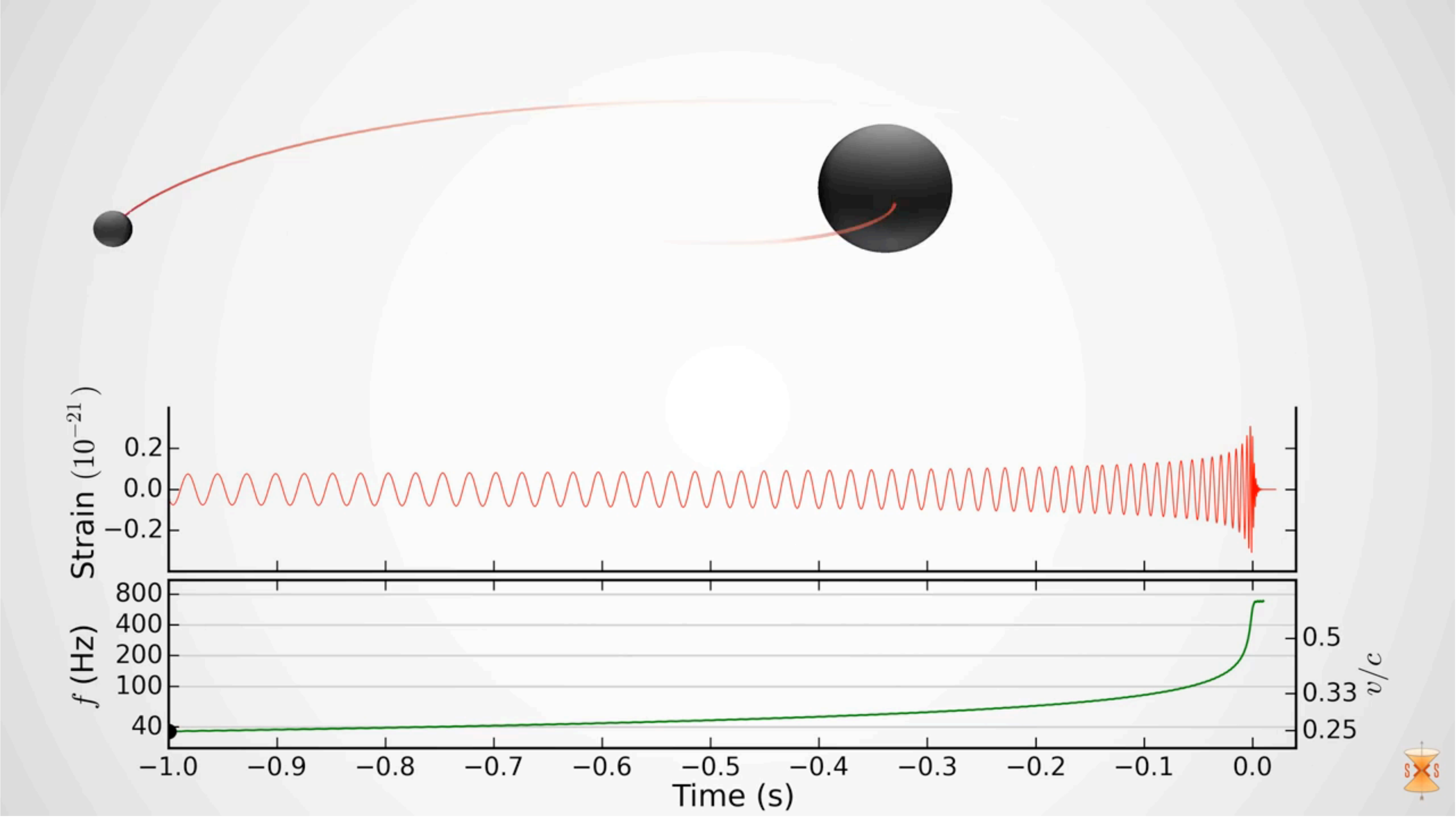
- The idea that gravity is the geometry of curved spacetime.
- It predicts that ripples in the spacetime curvature can propagate with the speed of light through otherwise empty space: **gravitational waves**.
  - Direct evidence of gravitational waves did not exist… until January of 2016, when the LIGO collaboration announced the detection of one wave from a pair of merging black holes.
  - This year's Nobel Prize in Physics!
- Another prediction of general relativity is **gravitation lensing**.
  - Electromagnetic radiation is bent by large distributions of matter.

Gravitational waves!

- Coalescing black holes at the heart of merging galaxies

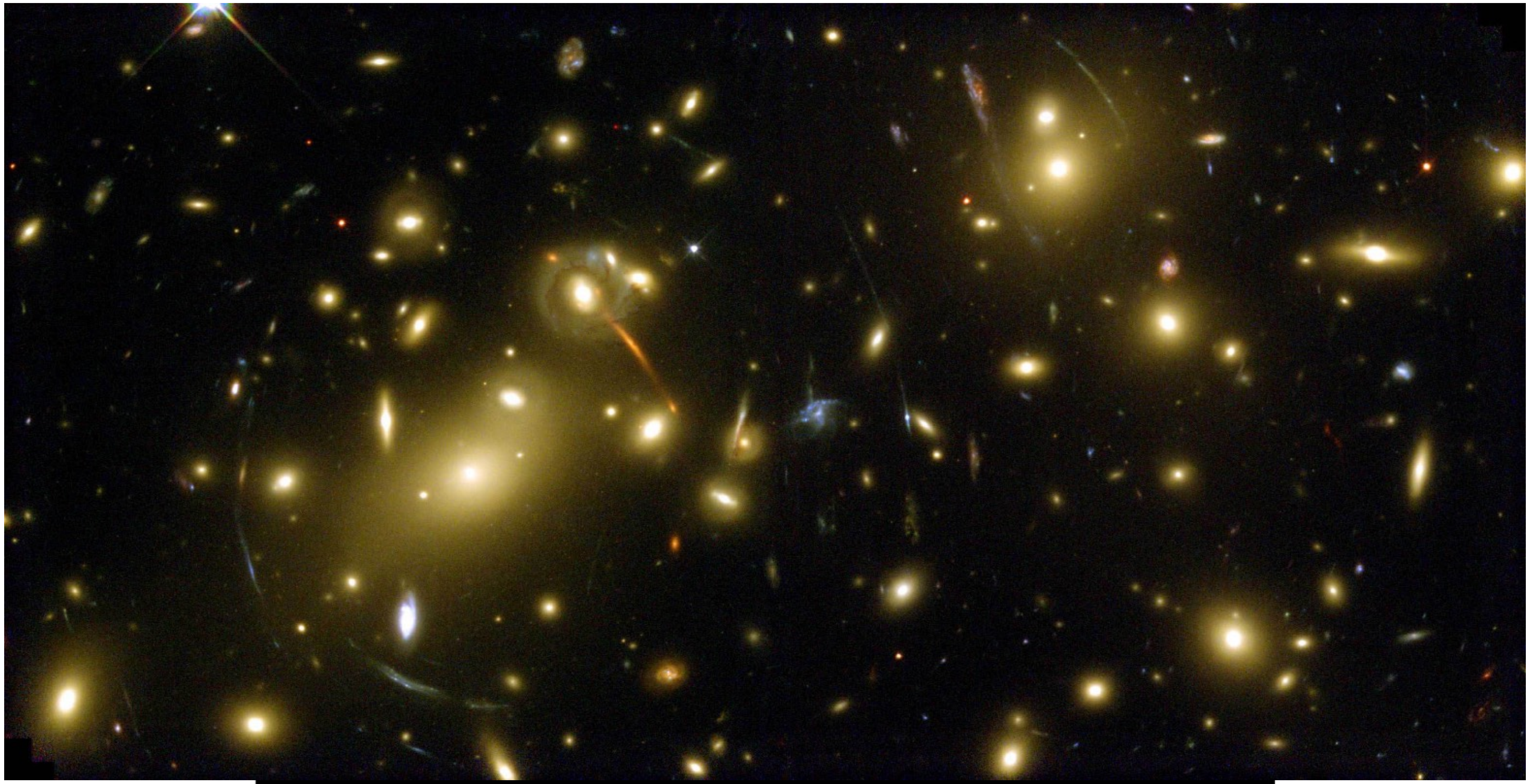


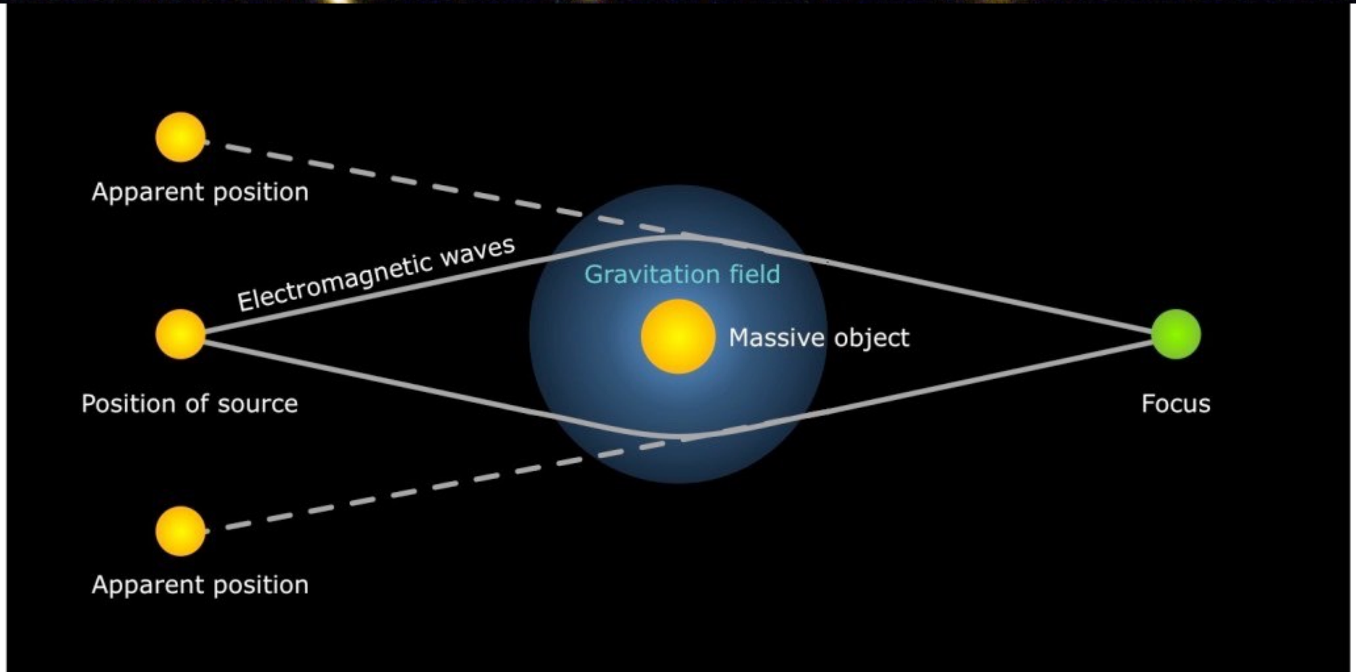
Gravitational waves!



LIGO collaboration

Gravitational lensing





Perturbation theory

- Use a power series in a parameter ε (such that ε << 1) - known as perturbation series - as an approximation to the full solution.
- For example:

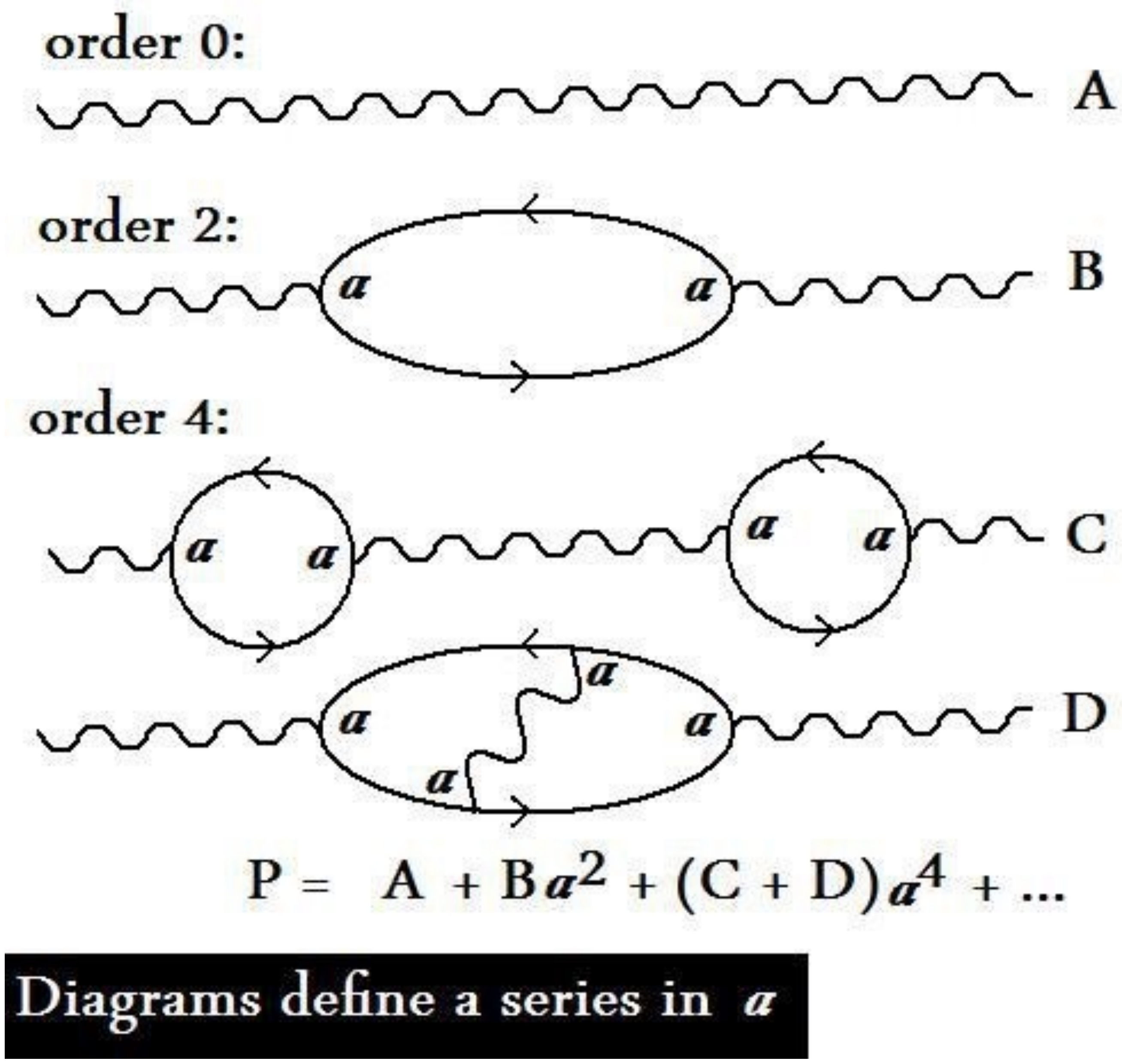
$$A = A_0 + \epsilon A_1 + \epsilon^2 A_2 + \ldots$$

- In this example, A_0 is the "leading order" solution, while A_1, A_2, … represent higher order terms.
- **Note:** if ε is small, the higher-order terms in the series become successively smaller.
- Approximation:

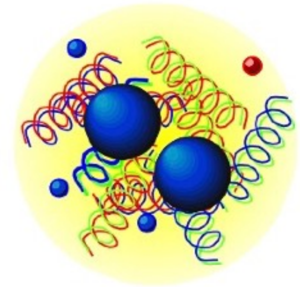
$$A \approx A_0 + \epsilon A_1$$

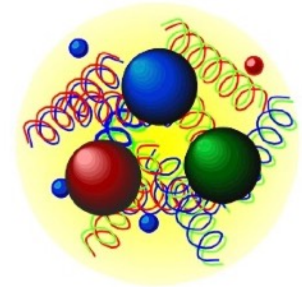
Perturbation theory in QFT

- Perturbation theory allows for well-defined predictions in quantum field theories (as long as they obey certain requirements).
- Quantum Electrodynamics (QED) is one of those theories.
- Feynman diagrams correspond to the terms in the perturbation series!



Quantum Chromodynamics (QCD)





- Theory of strong interactions. Recall: gluons are the force carriers.
- Confinement: why we don't see free quarks.
- Asymptotic freedom: at very high energies, the interaction scale is smaller than at low energies, and we're in the perturbative regime.