

Particle Physics: Quantum Mechanics

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Week 4: February 17, 2017
Columbia University Science Honors Program



Course policies

- Classes from 10:00 AM to 12:30 PM (10 min break at ~ 11:10 AM).
- **Attendance record counts.**
 - Up to four absences
 - Lateness or leaving early counts as half-absence
 - Send email notifications of all absences to shpattendance@columbia.edu
- Please, no cell phones during class
- **Please, ask questions!**
- Lecture materials + Research Opportunities + Resources to become a particle physicist

<https://twiki.nevis.columbia.edu/twiki/bin/view/Main/ScienceHonorsProgram>

Schedule

Month	Day	Lecture	Teacher
January	27	Introduction	Jose
February	3	History of Particle Physics	Jose
	10	Special Relativity	Jose
	17	Quantum Mechanics	Jose
	24	Experimental Methods	Cris
March	3	The Standard Model - Overview	Cris
	10	The Standard Model - Limitations	Cris
	17	No classes, Columbia University spring break	
	24	Neutrino Theory	Cris
	31	No classes, Easter and Passover weekend	
April	7	Neutrino Experiment	Jose
	14	LHC and Experiments	Ines
	21	No classes, SHP break	
	28	The Higgs Boson and Beyond	Ines
May	5	Particle Cosmology	Cris

Topics for Today

- **Quantum phenomena:**

- Quantization: how Nature comes in discrete packets
- Particle-wave duality.
- Understanding quantum phenomena in terms of waves.
- The Schrödinger Equation

- **Interpreting quantum mechanics (QM):**

- The probabilistic interpretation of quantum mechanics
- The Uncertainty Principle and the limits of observation.

- **Understanding the Uncertainty Principle:**

- Building up wave packets from sinusoids.
- Why the Uncertainty Principle is a natural property of waves.

What is Quantum Mechanics (QM)?

- QM is the study of physics at very small scales - specifically, when the energies and momenta of a system are of the order of **Planck's constant**:
$$\hbar = h/2\pi \cong 6.6 \times 10^{-16} \text{eV.s}$$
- On the quantum level, "particles" exhibit a number of non-classical behaviors:
 1. Discretization (quantization) of energy, momentum charge, spin, etc. Most quantities are multiples of e and/or h .
 2. Particles can exhibit wavelike effects: interference, diffraction, ...
 3. Systems can exist in a superposition of states.

Quantization of electric charge

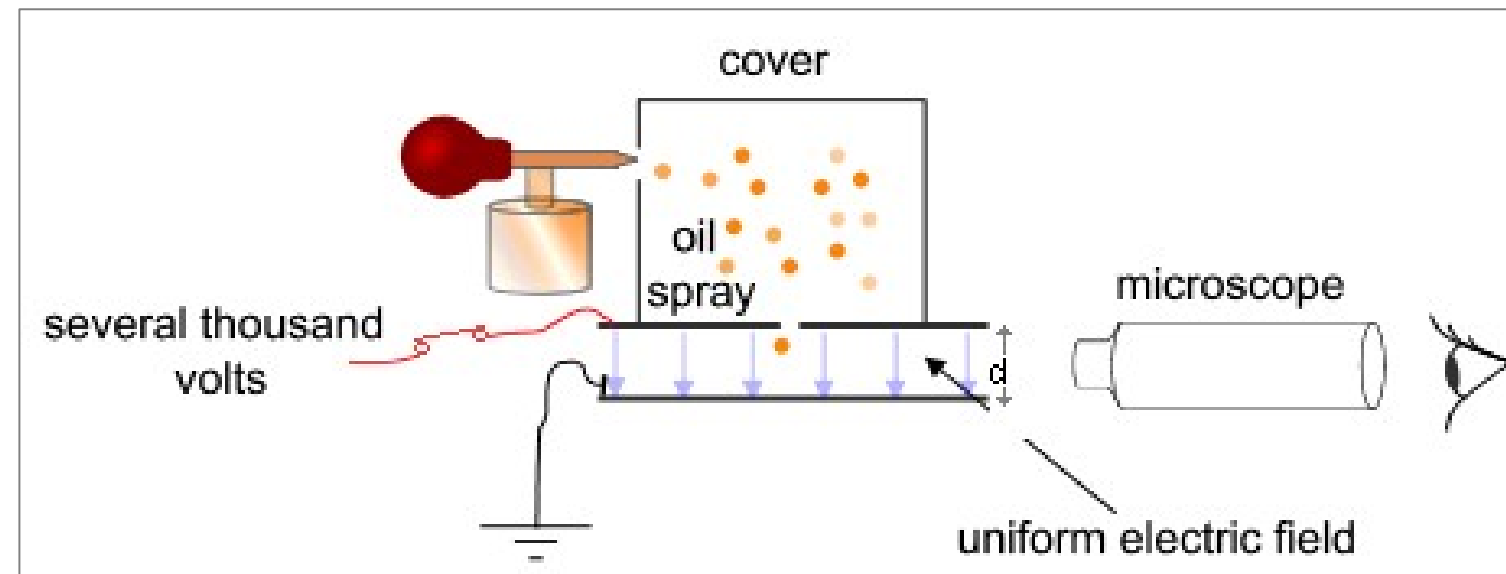
- Recall:
 - J.J. Thomson (1897): electric charge is corpuscular, “stored” in electrons.
 - R. Milikan (1910): electric charge is quantized, always showing up in integral multiples of e .
 - **Milikan's experiment:** measuring the charge on ionized oil droplets.



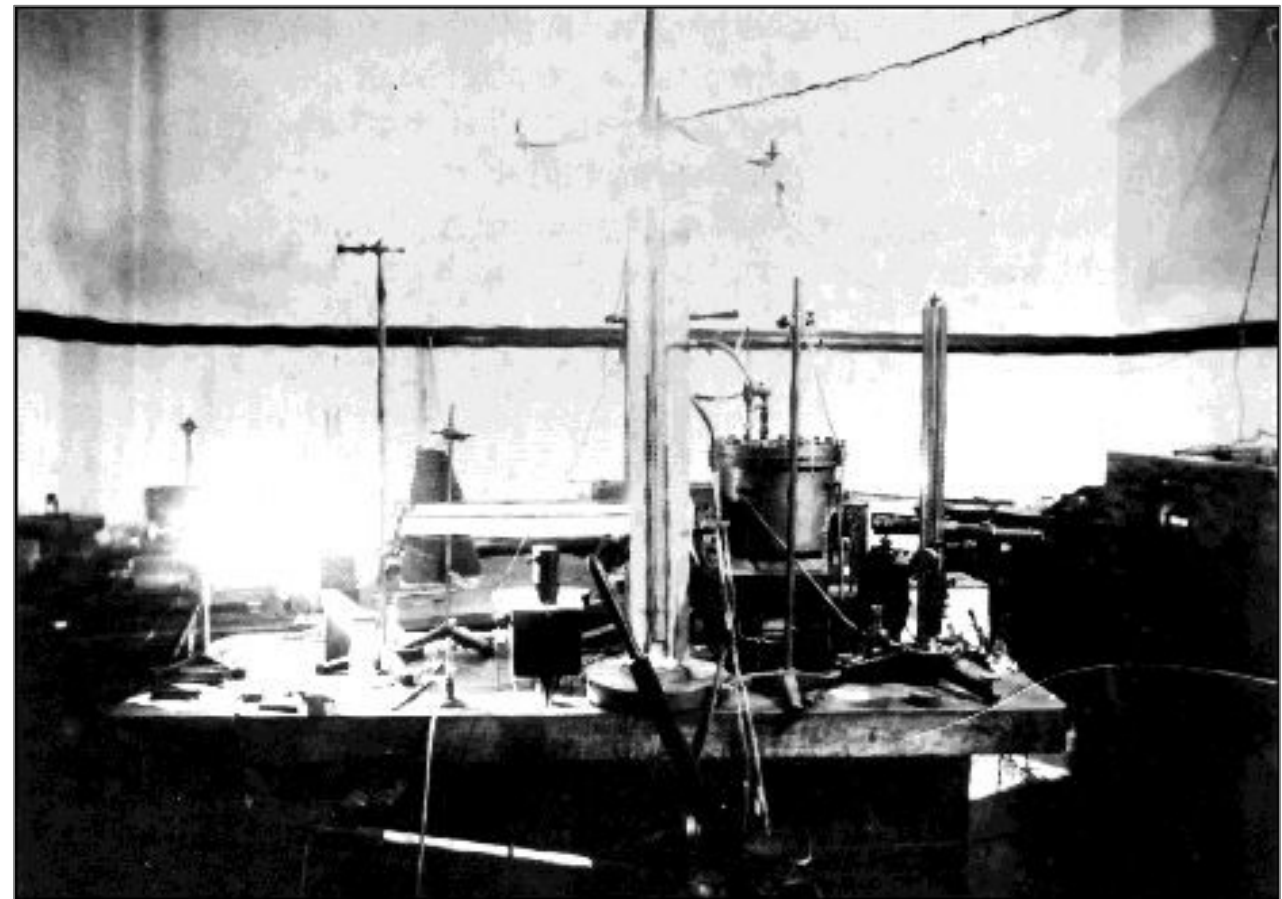
R.A. Millikan
Nobelprize.org

The oil drop experiment

- The experiment entailed balancing the downward gravitational force with the upward buoyant and electric forces on tiny charged droplets of oil suspended between two metal electrodes.
- Since the density of the oil was known, the droplets' masses could be determined from their observed radii.
- Using a known electric field, Millikan and Fletcher could determine the charge on oil droplets in mechanical equilibrium.

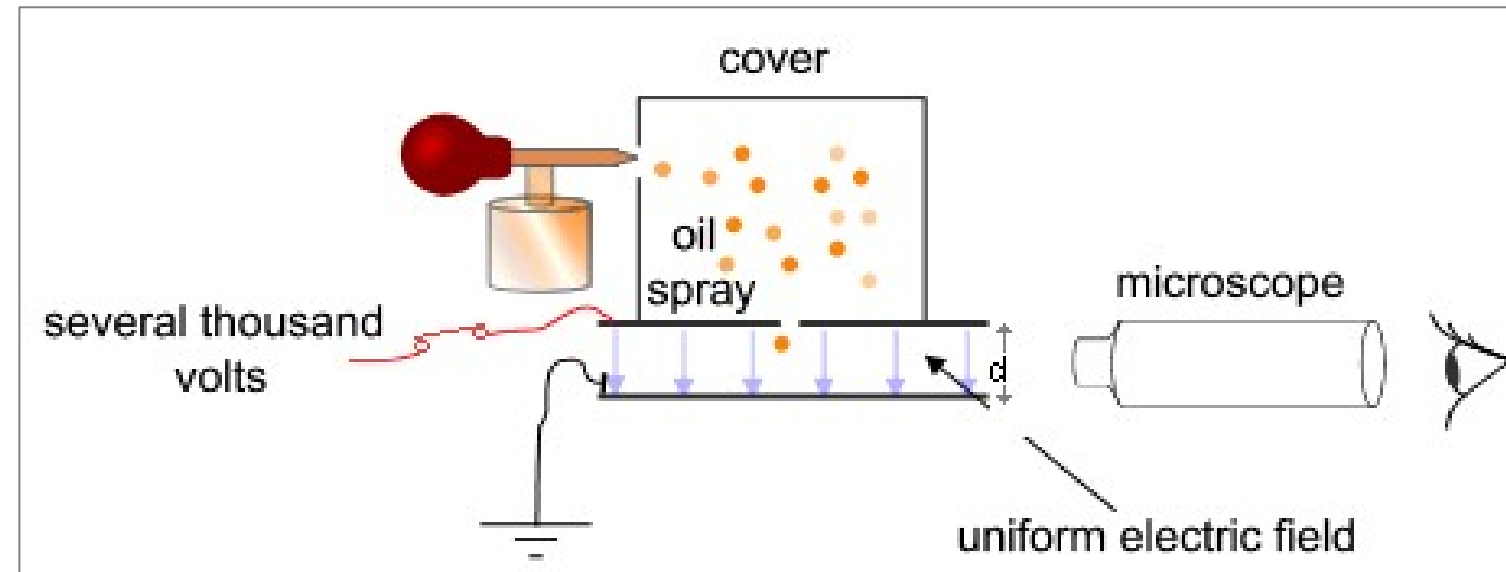


Millikan's setup

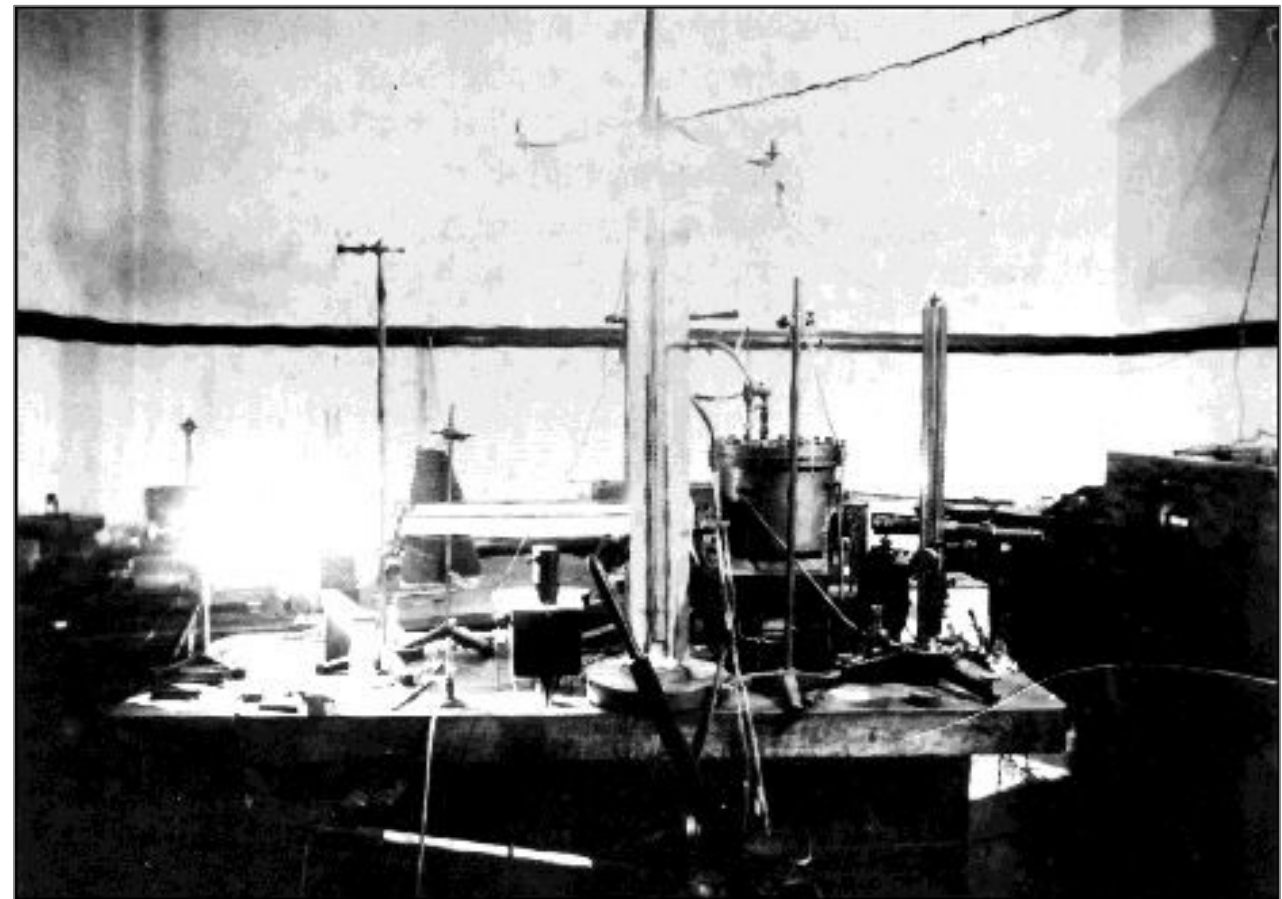


The oil drop experiment

- By repeating the experiment for many droplets, they confirmed that the charges were all multiples of some fundamental value.
- They calculated it to be $1.5924(17) \times 10^{-19} \text{ C}$, within 1% of the currently accepted value of $1.602176487(40) \times 10^{-19} \text{ C}$.
- They proposed that this was the charge of a single electron.



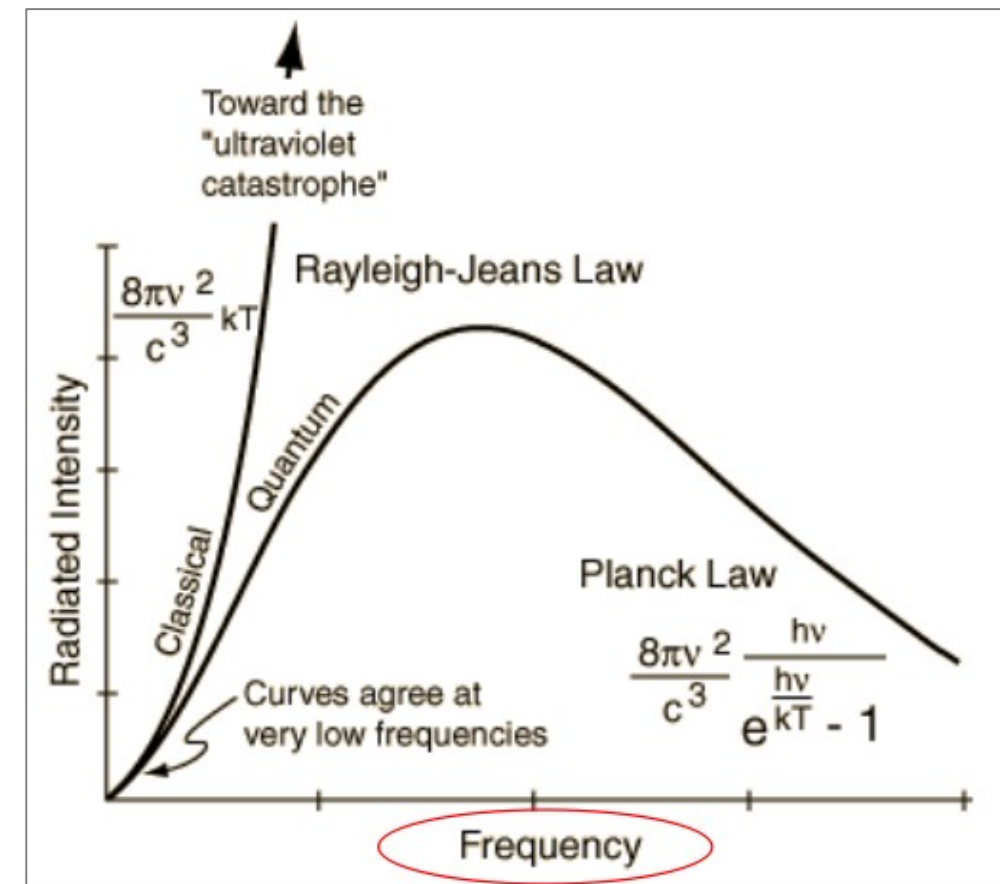
Millikan's setup



Quantization of energy

- Recall:

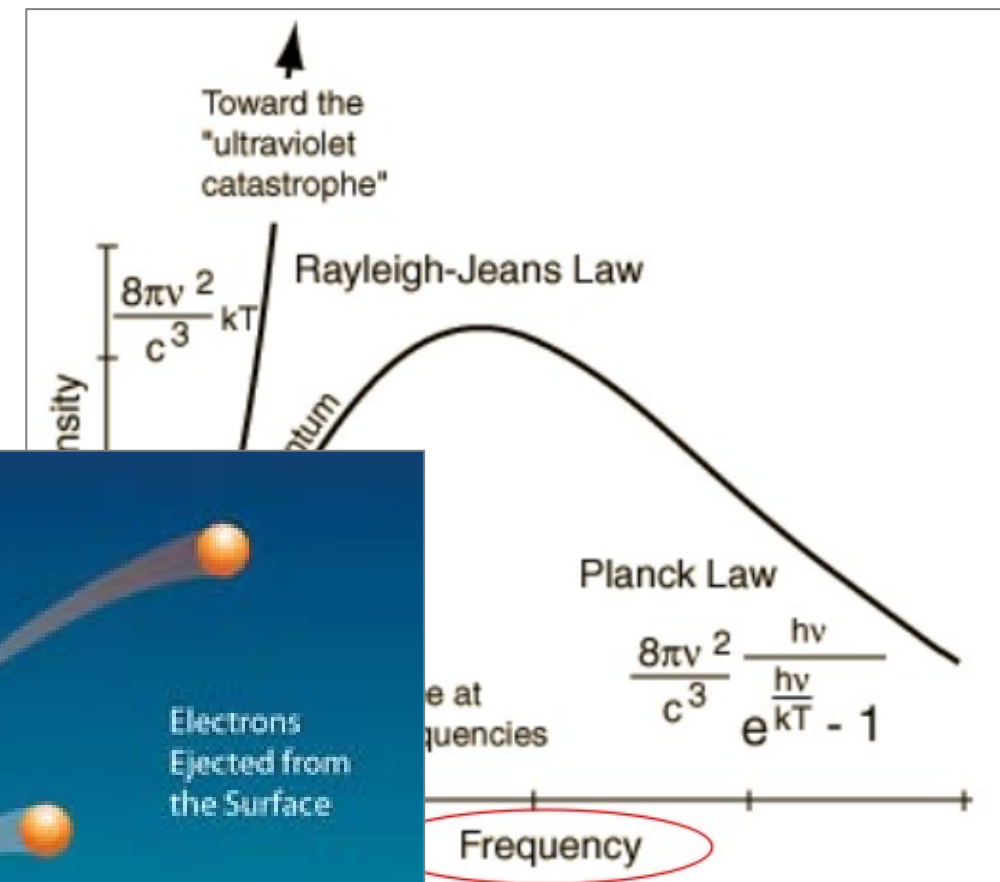
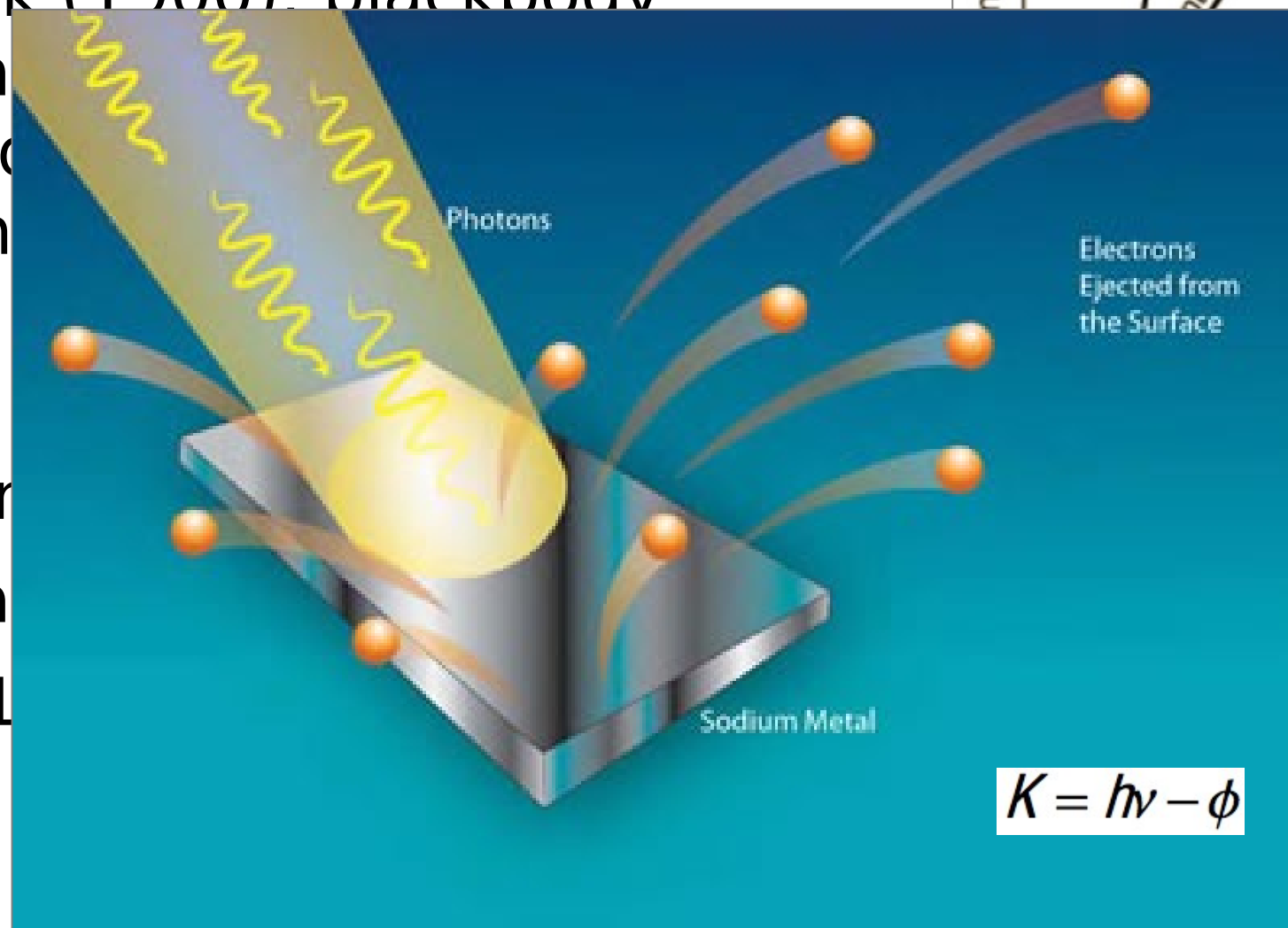
- M. Planck (1900): blackbody radiation spectrum can be explained if light of frequency ν comes in quantized energy packets, with energies of $h\nu$.
- A. Einstein (1905): photoelectric effect can only be theoretically understood if light is corpuscular.
- N. Bohr (1913): discrete energy spectrum of the hydrogen atom can be explained if the electron's angular momentum about the nucleus is quantized.
- In an atom, angular momentum mvr always comes in integral multiples of $\hbar = h/2\pi$.



Quantization of energy

- Recall:

- M. Planck (1900): blackbody radiation explained as energy comes in packets,
- A. Einstein (1905): photoelectric effect explained as energy comes in packets,
- N. Bohr (1913): hydrogen atom can only have certain energy levels about the nucleus.

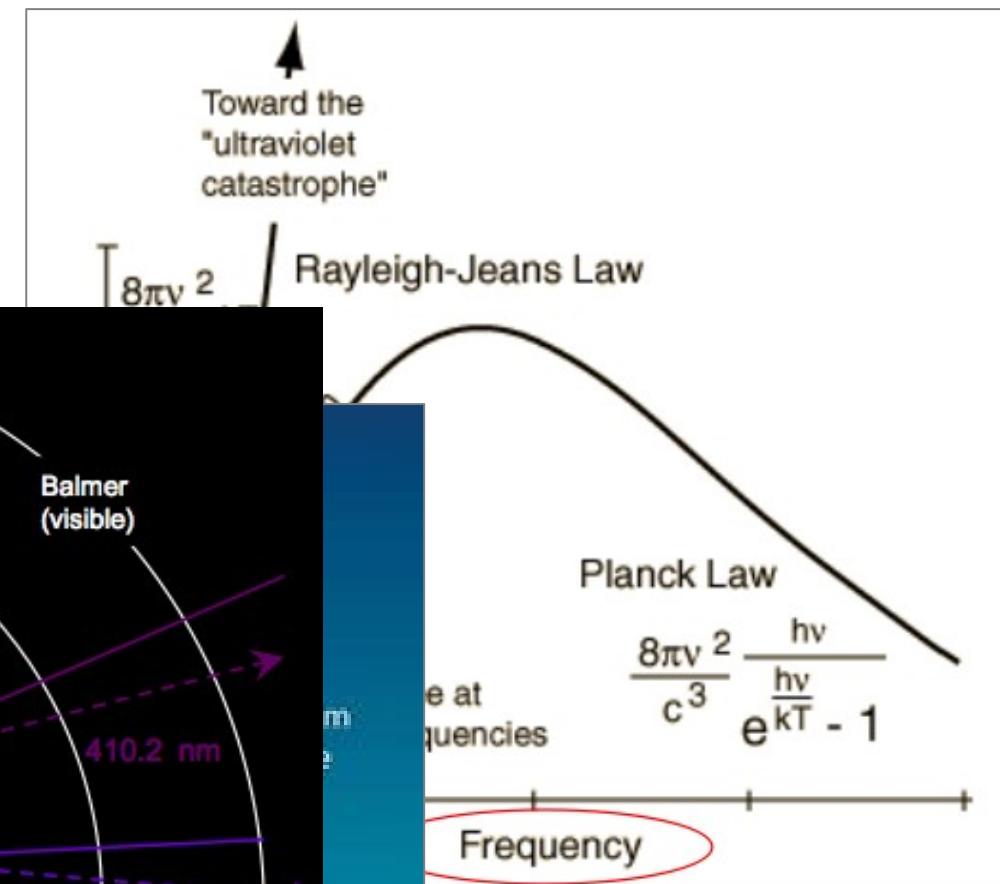
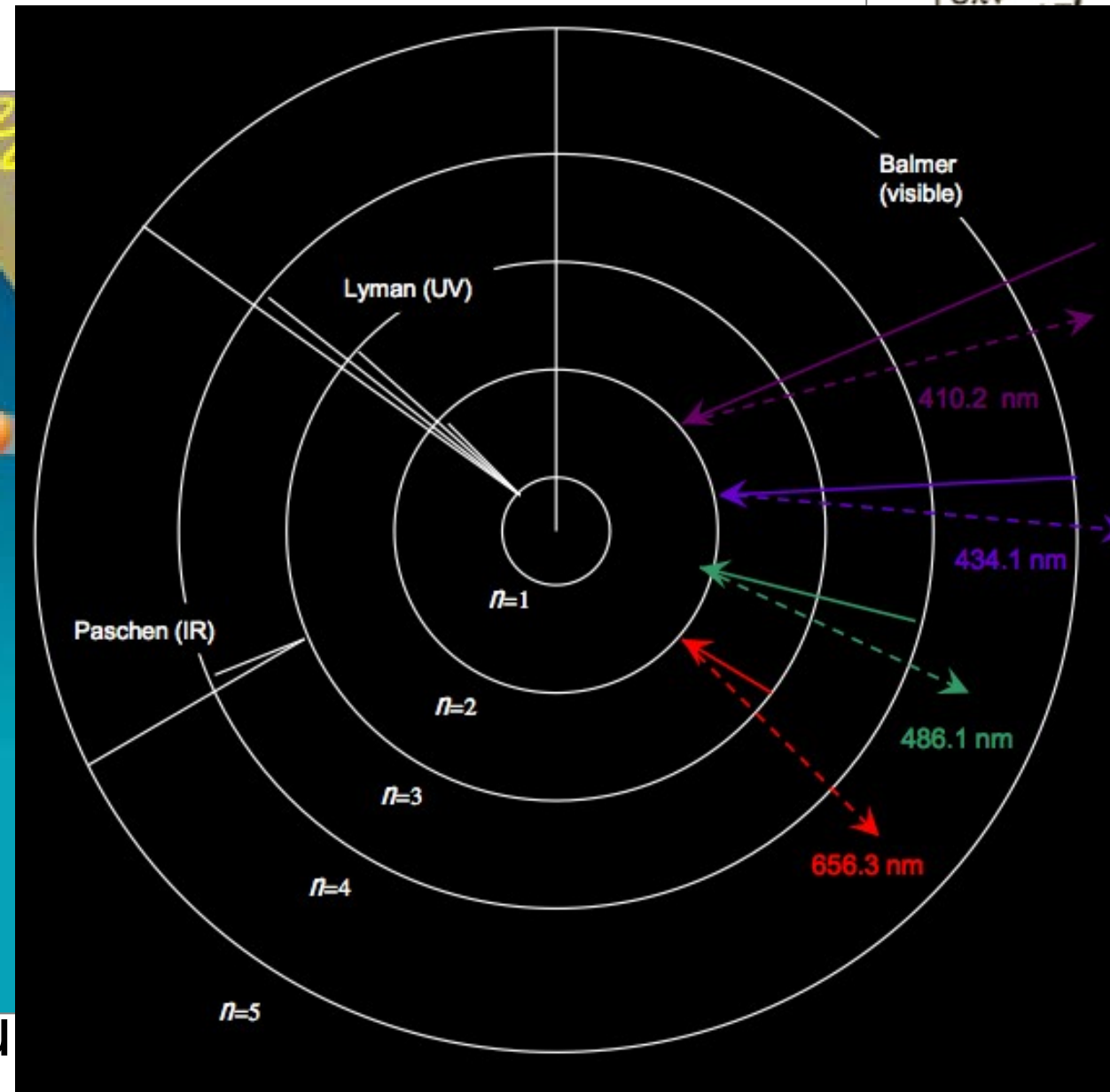


- In an atom, angular momentum mvr always comes in integral multiples of $\hbar = h/2\pi$.

Quantization of energy

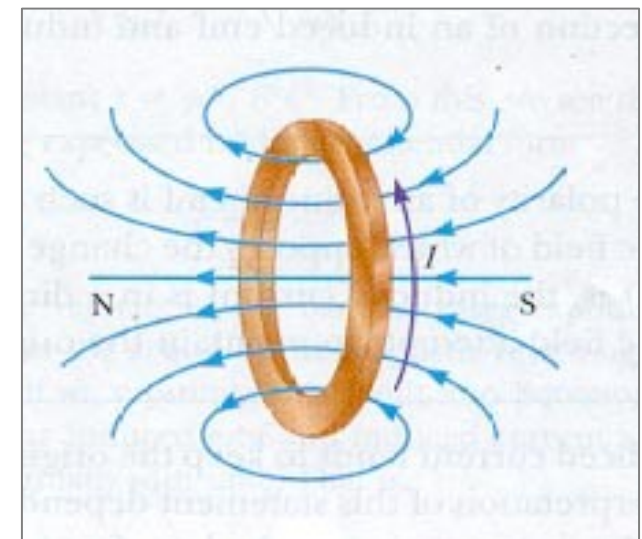
- Recall:

- M. Planck explained that radiation comes in packets,
- A. Einstein explained that light has a theoretical momentum
- N. Bohr (1913) explained that the hydrogen atom can only have certain energy levels about the nucleus
- In an atom, angular momentum mvr always comes in integral multiples of $\hbar = h/2\pi$.



Quantization of spin

- The particle property called *spin*, is also quantized in units of \hbar .
- All particles have spin: it is an inherent property, like electric charge, or mass.
- A magnetic phenomenon, spin is very important.
 - If you understand bar magnets, you (somewhat) understand spin.
- Spin is closely related to Pauli's Exclusion Principle (Spin Statistic Theorem), which relates the change in sign of the QM *wavefunction* when two identical particles are exchanged in a system (more on this later...).

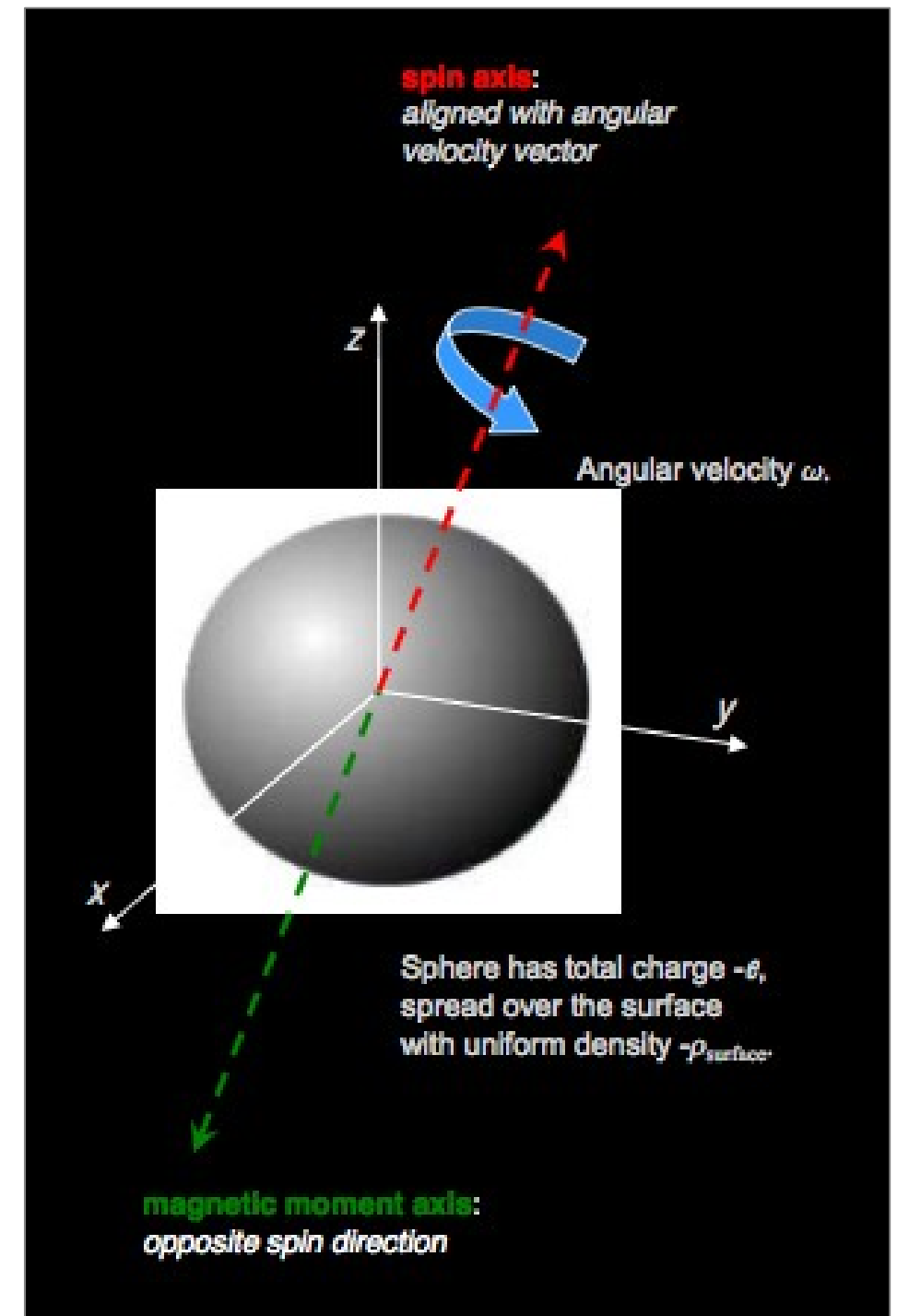


How does spin enter the picture?

- How do particles get a magnetic moment?
- Enter spin. Imagine an electron as a rotating ball of radius R , with its charge distributed over the volume of the sphere.
- The spinning sets up a current loop around the rotation axis, creating a small magnetic dipole: like a bar magnet!
- The spinning ball gets a dipole moment:

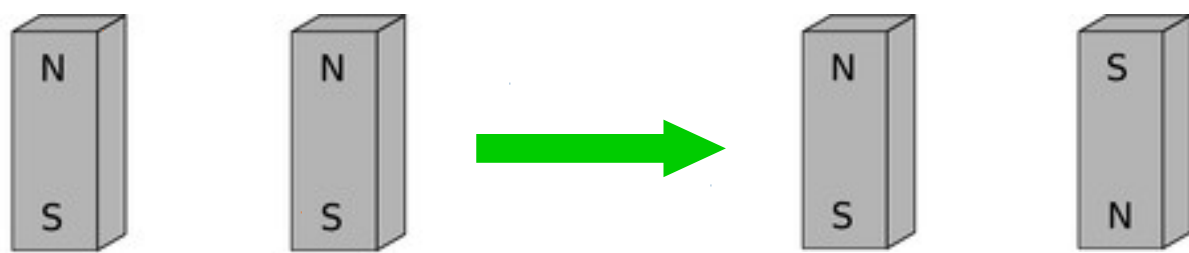
$$\vec{\mu} = -\frac{4}{3}\pi R^4 \rho \vec{\omega} \propto -\vec{S}$$

- Spin S is a vector that points along the axis of rotation (by the “right-hand rule”). The moment μ points in the opposite direction.

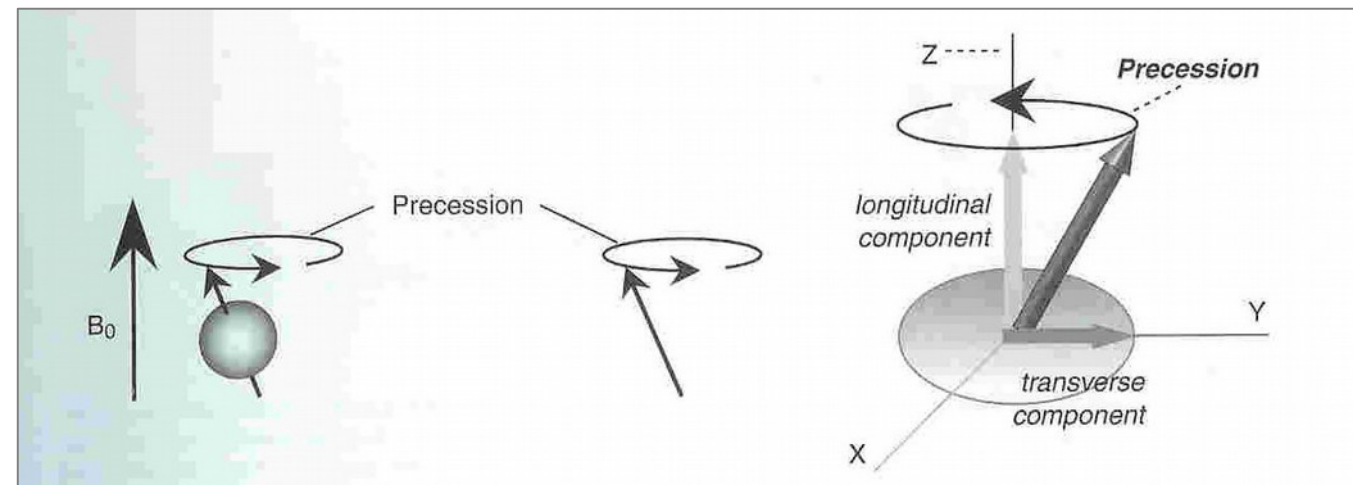


Spin-magnetism analogy

- To some extent, all elementary particles behave like tiny bar magnets, as if they had little N and S magnetic poles.



Two bar magnets set side by side will try to anti-align such that the north and south poles “match up.”

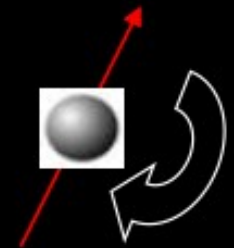


Similarly, an object with a magnetic moment will try to anti-align itself with a magnetic field.

- Jargon: if a particle behaves in this way in a magnetic field, it is said to have a non-zero magnetic dipole moment μ . In a B field, such a particle will feel a force:
$$\vec{F} = \nabla(\vec{\mu} \cdot \vec{B})$$

Quantization of spin

- If an electron is a spinning ball of charge, we understand how its magnetic moment arises.
- This is still classical physics: the spin axis may point in any direction. But, Nature is very different!
- O. Stern and W. Gerlach (1921):
 - While measuring Ag atom spins in a B field, they found that spin always aligns in two opposite directions, “up” and “down”, relative to the field.
 - Moreover, the magnitude of the spin vector is quantized in units of \hbar .
- All elementary particles behave this way: their spins are always quantized, and when measured only point in certain directions (“space quantization”).



Classically, an electron's spin orientation can take on a *continuum* of values; the axis of rotation can point anywhere it likes.

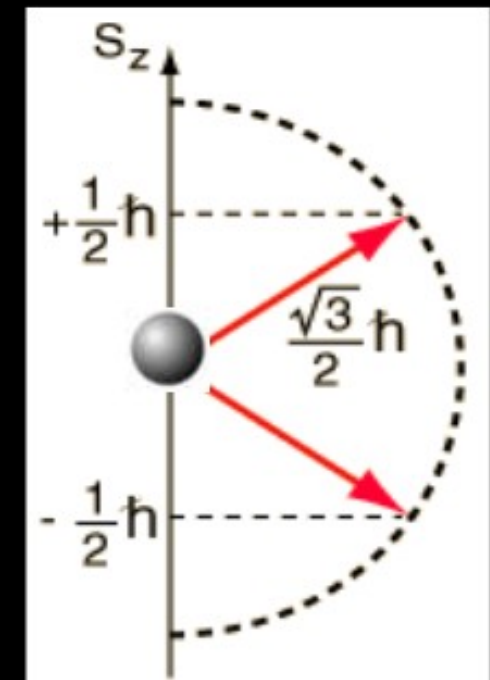


Image © Hyperphysics

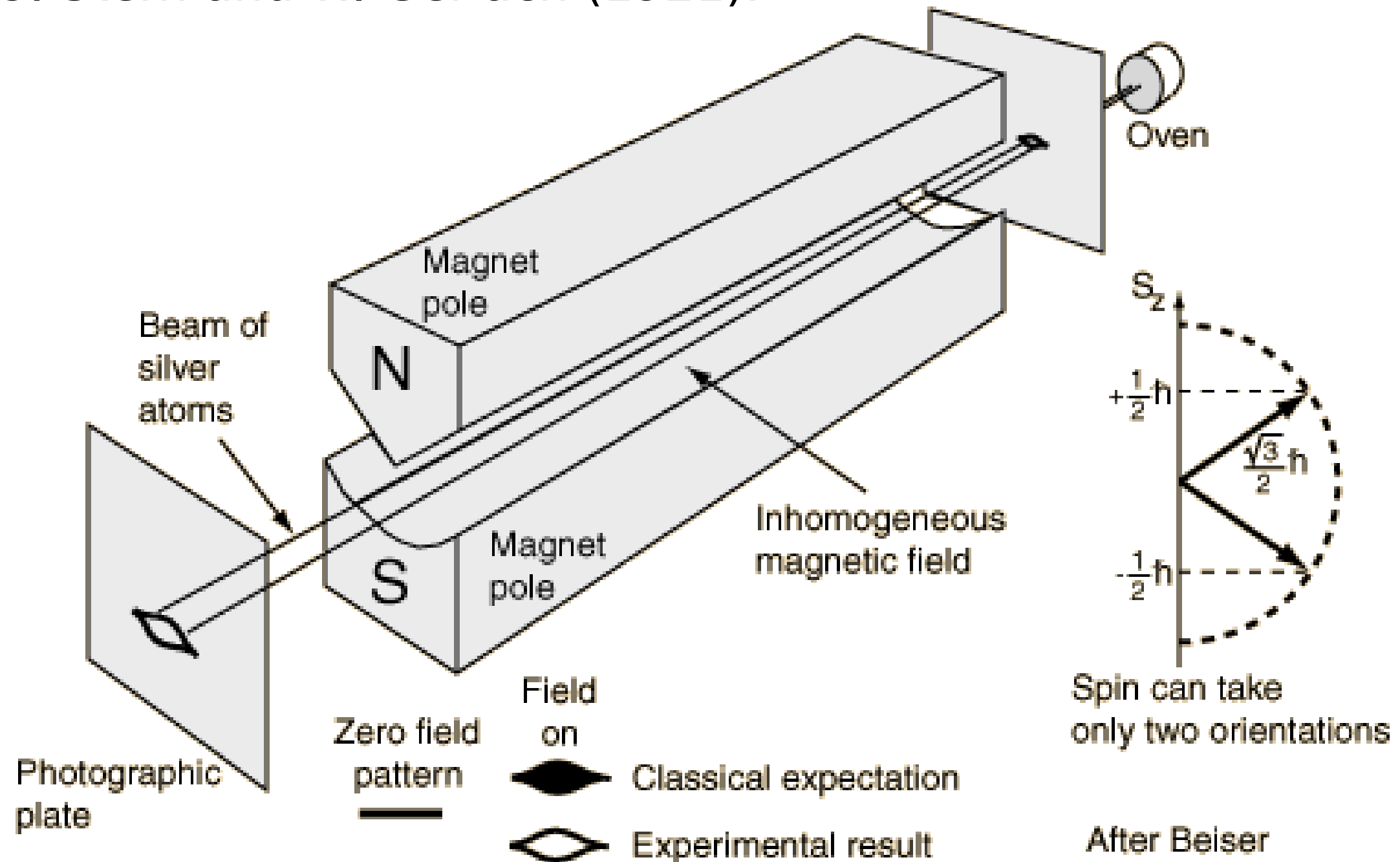
But the classical view is wrong, because measured spins *always* seem to line up in certain preferred directions. This is called a spin-1/2 particle.

Stern & Gerlach experiment

<https://www.youtube.com/watch?v=rg4Fnag4V-E>

Quantization of spin

- O. Stern and W. Gerlach (1921):



Understanding spin

- If elementary particles like the electron are actually little spinning spheres of charge, why should their spins be quantized in magnitude and direction?
- Classically, there is no way to explain this behavior.
- In 1925, S. Goudsmidt and G. Uhlenbeck realized that the classical model just cannot apply: electrons do not spin like tops; magnetic behavior must be explained some other way.
- Modern view: spin is an intrinsic property of all elementary particles, like charge or mass.
 - It is a completely quantum phenomenon, with no classical analog.

Like most other quantum mechanical properties, allowed spin values are restricted to certain numbers proportional to \hbar .

The classically expected continuum of values is not observed.

Quantization summarized

- General rule in QM:
 - Measurable quantities tend to come in integral (or half-integral) multiples of fundamental constants.
- Almost all of the time, **Planck's constant** is involved in the quantized result. **It's truly a universal, fundamental constant of Nature.**
- **Question:** why don't we observe quantization at macroscopic scales?

Quantization summarized

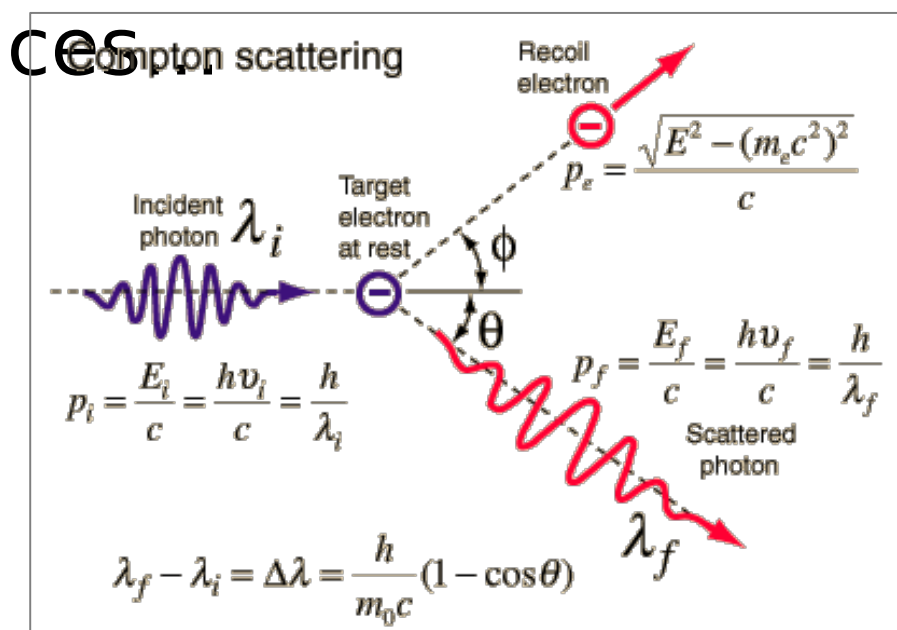
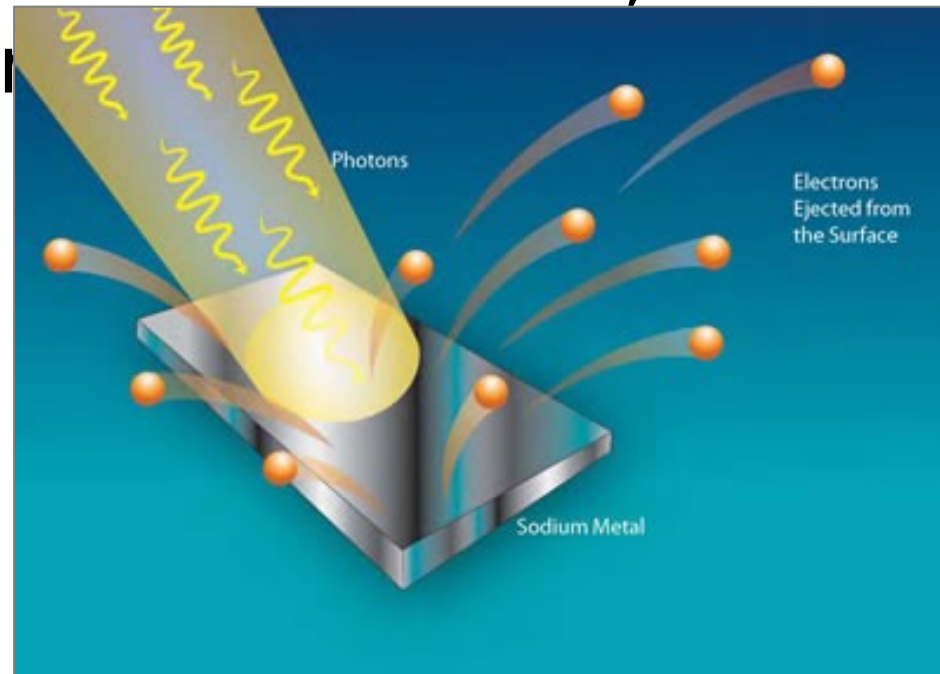
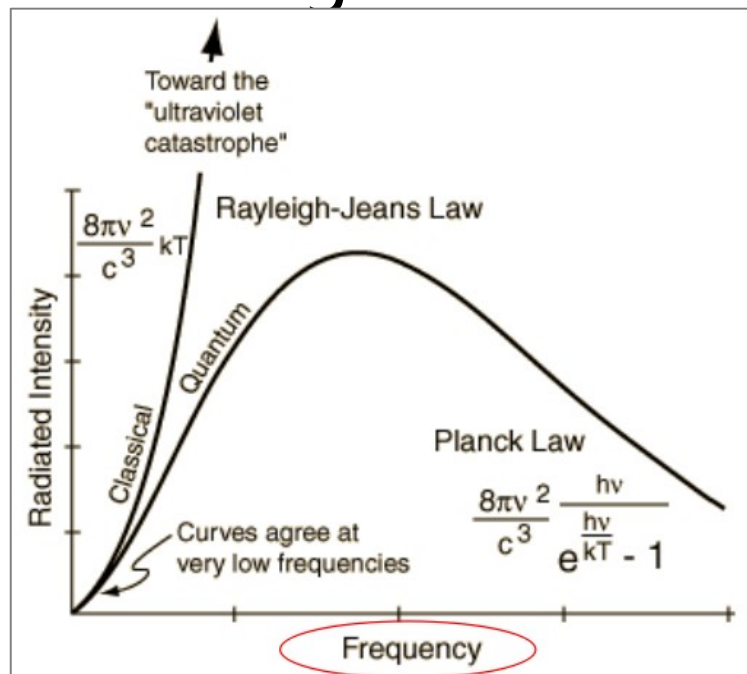
- General rule in QM:
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- Almost all of the time, **Planck's constant** is involved in the quantized result. **It's truly a universal, fundamental constant of Nature.**
- **Question:** why don't we observe quantization at macroscopic scales?
- **Answer:** due to the smallness of Planck's constant.
- This is analogous to Special Relativity, where the small size of the ratio v/c at everyday energy scales prevents us from observing the consequences of SR in the everyday world.

More quantum weirdness

- Observation tells us that physical quantities are not continuous down to the smallest scales, but tend to be discrete.
- But QM has another surprise: if you look small enough, matter - that is, “particles” - start to exhibit *wavelike* behavior.
- We have already seen hints of this idea.
- Light can behave like a wave, and it can behave like a particle, depending on the circumstances...

More quantum weirdness

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More quantum weirdness

- L. de Broglie (1924) suggested that the wave-particle behavior of light might apply equally well to matter.
- Just as a photon is associated with a light wave, so an electron could be associated with a matter wave that governs its motion.



The de Broglie hypothesis

- de Broglie's suggestion was a very bold statement about the symmetry of Nature.
- **Proposal:** the wave aspects of matter are related to its particle aspects in quantitatively *the same way* that the wave and particle aspects of light are related.
- **Hypothesis:** for matter and radiation, the total energy E of a particle is related to the frequency ν of the wave associated with its motion by:

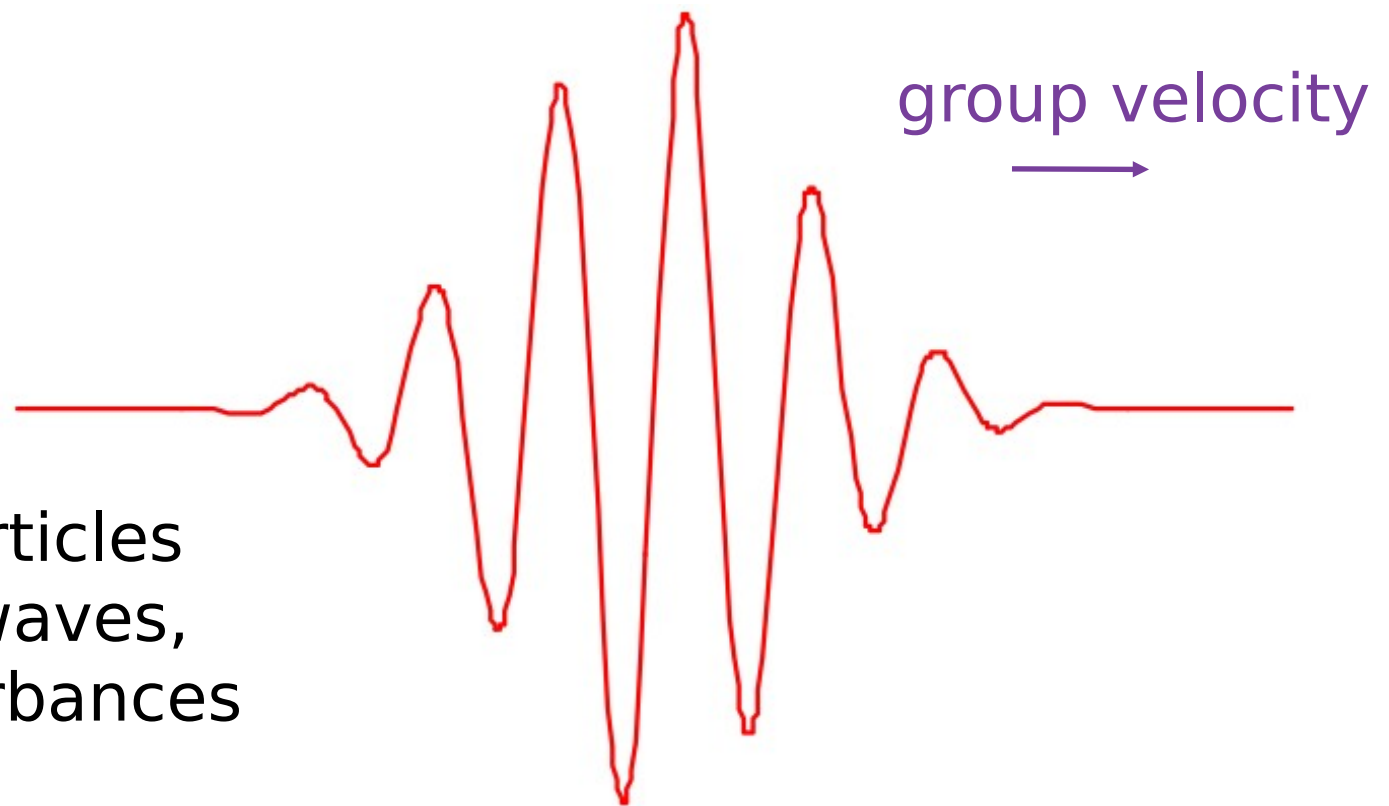
$$E = h\nu$$

- If $E=pc$ (recall SR), then the momentum p of the particle is related to the wavelength λ of the associated wave by the equation:

$$p = h/\lambda$$

The de Broglie hypothesis

- $p = h/\lambda$ or $\lambda = h/p \rightarrow$ **de Broglie relation**
- *It holds even for massive particles.*
- It predicts the de Broglie wavelength of a matter wave associated with a material particle of momentum p .



de Broglie hypothesis: particles are also associated with waves, which are extended disturbances in space and *time*.

Matter waves and the classical limit

- Question: if the de Broglie hypothesis is correct, why don't macroscopic bodies exhibit wavelike behaviors?

Matter waves and the classical limit

- Question: if the de Broglie hypothesis is correct, why don't macroscopic bodies exhibit wavelike behaviors?
 - Smallness of h !

Try yourself:

- What is your de Broglie wavelength?
- What is the de Broglie wavelength of a 100 eV/c electron?

$$h = 6.62606957(29) \times 10^{-34} \text{ J s}$$

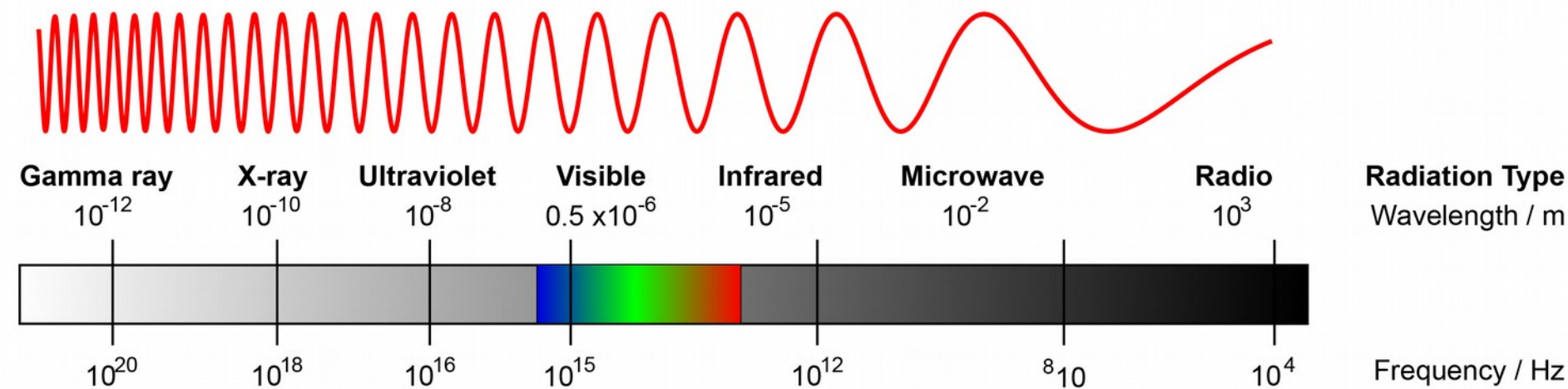
$$h = 4.13566733(10) \times 10^{-15} \text{ eV s}$$

Testing the wave nature of matter

- Macroscopic particles do not have measurable de Broglie wavelengths, but **electron wavelengths are about the characteristic size of X-rays**.
- So, we have an easy test of the de Broglie hypothesis:
 - Check if electrons exhibit wavelike behavior (diffraction, interference, ...) under the same circumstances that X-rays do.
- First, let's talk a little bit about X-rays and X-ray diffraction.

X-rays

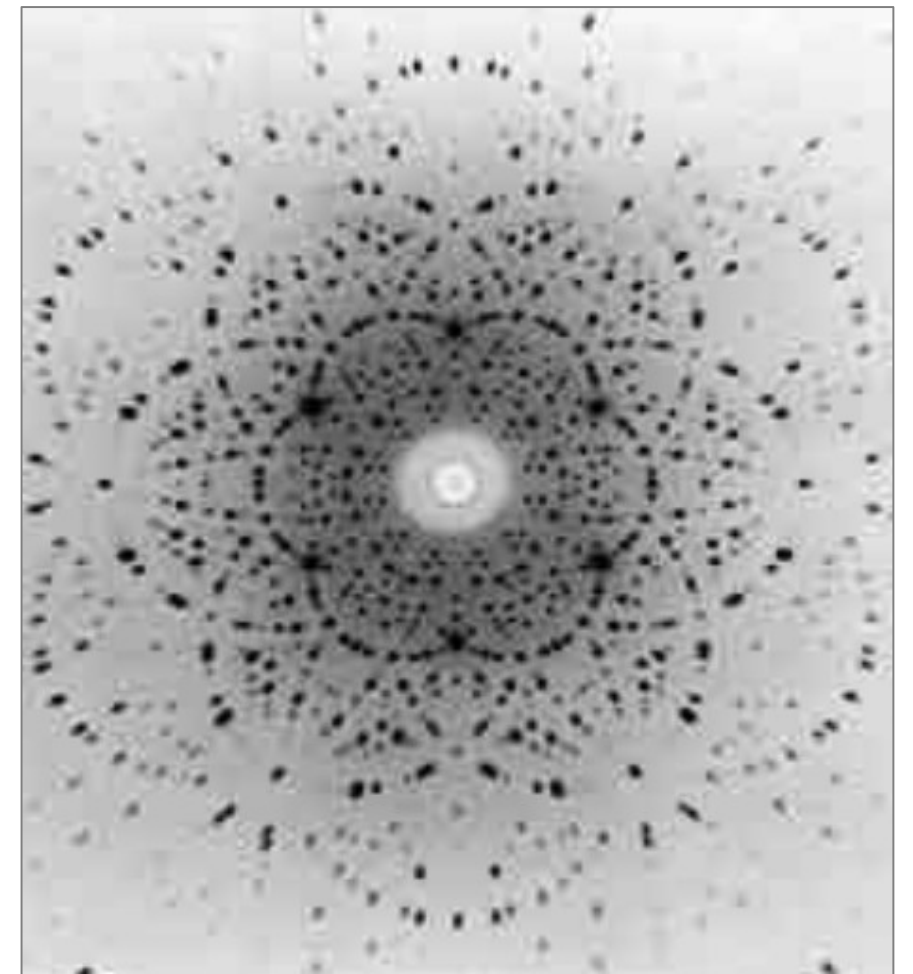
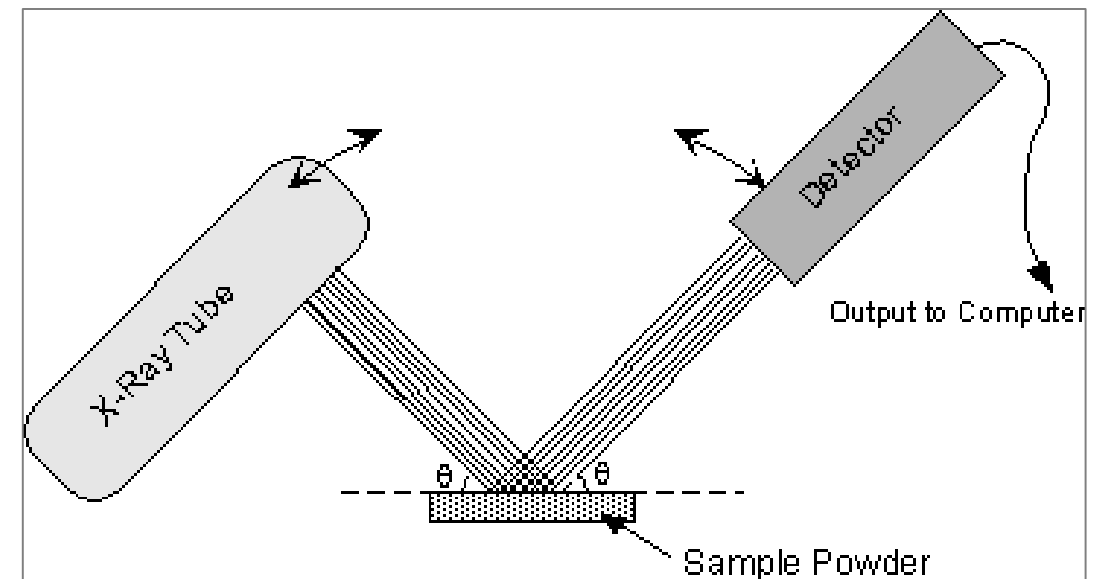
The first medical
röntgengram:
a hand with buckshot,
1896.



- W. Röntgen (1895): discovers X-rays, high energy photons with typical wavelengths near 0.1 nm.
- Compared to visible light (λ between 400 and 750 nm), X-rays have extremely short wavelengths and high penetrating power.
- Early in the 20th century, physicists proved that X-rays are light waves by observing X-ray diffraction from crystals.

X-ray crystallography

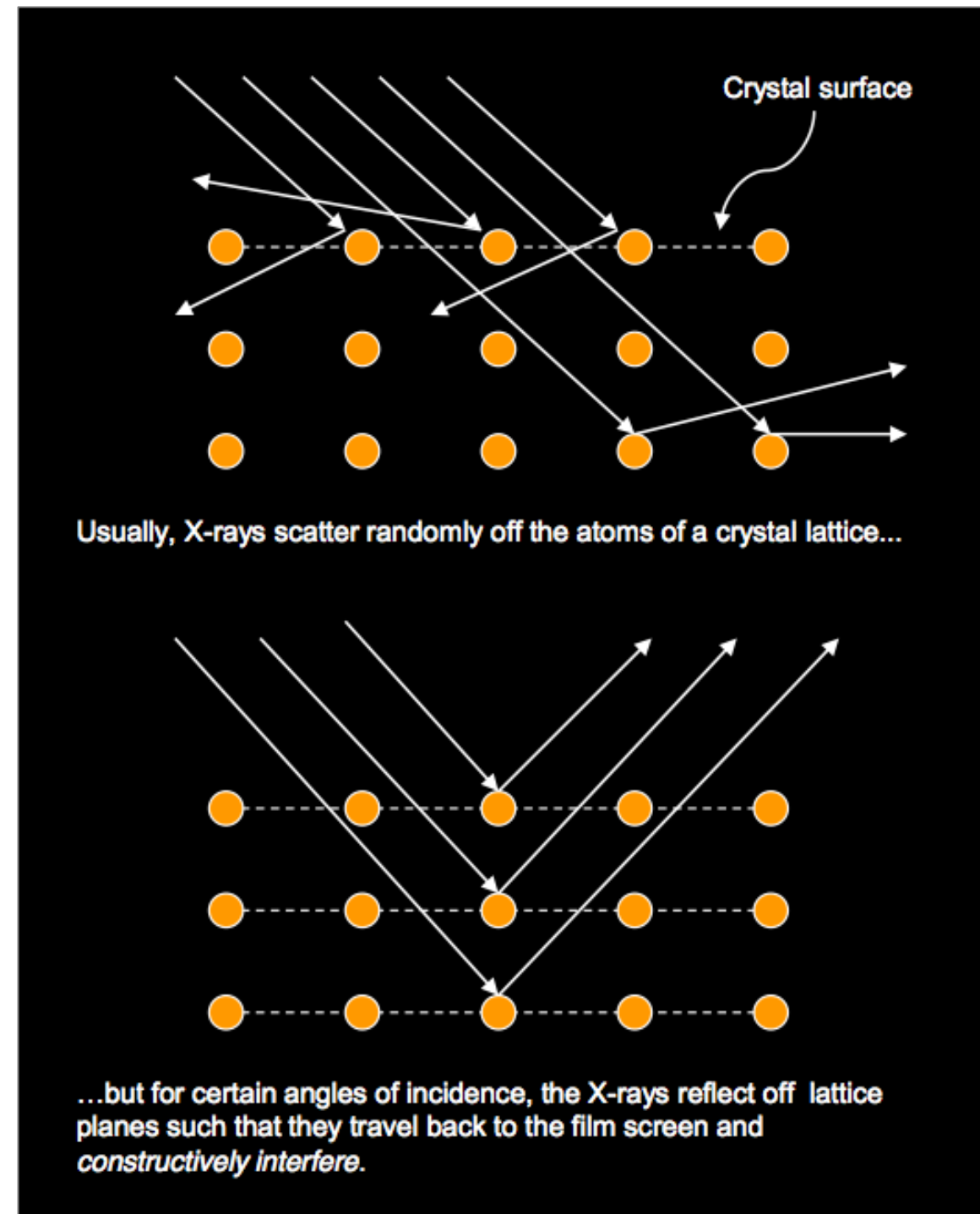
- How it's done: take X-rays and shoot them onto a crystal specimen.
 - Some of the X-rays will scatter backwards off the crystal.
- When film is exposed to the backscattered X-rays, geometric patterns emerge.
- In this image, the dark spots correspond to regions of high intensity (more scattered X-rays).
- The geometry of the pattern is characteristic of the structure of the crystal specimen.
 - Different crystals will create different scattering patterns.



Negative image of an X-ray diffraction pattern from a beryllium aluminum silicate crystal. The X-rays seem to scatter only in preferred directions

X-ray diffraction

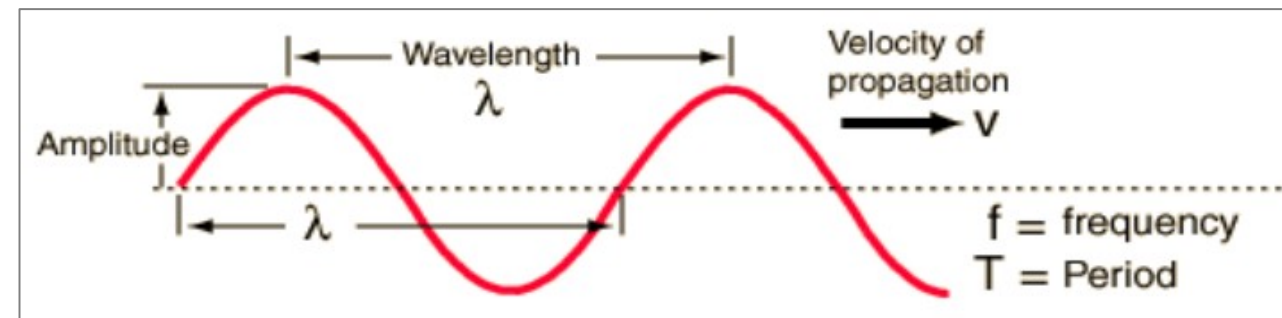
- What is the process behind X-ray crystallography?
- Why do X-rays appear to scatter off crystals in only certain directions?
 - This behavior can only be understood if X-rays are waves.
- The idea: think of the crystal as a set of semi-reflective planes.
- The X-rays reflect from different planes in the crystal, and then constructively and destructively interfere at the film screen.



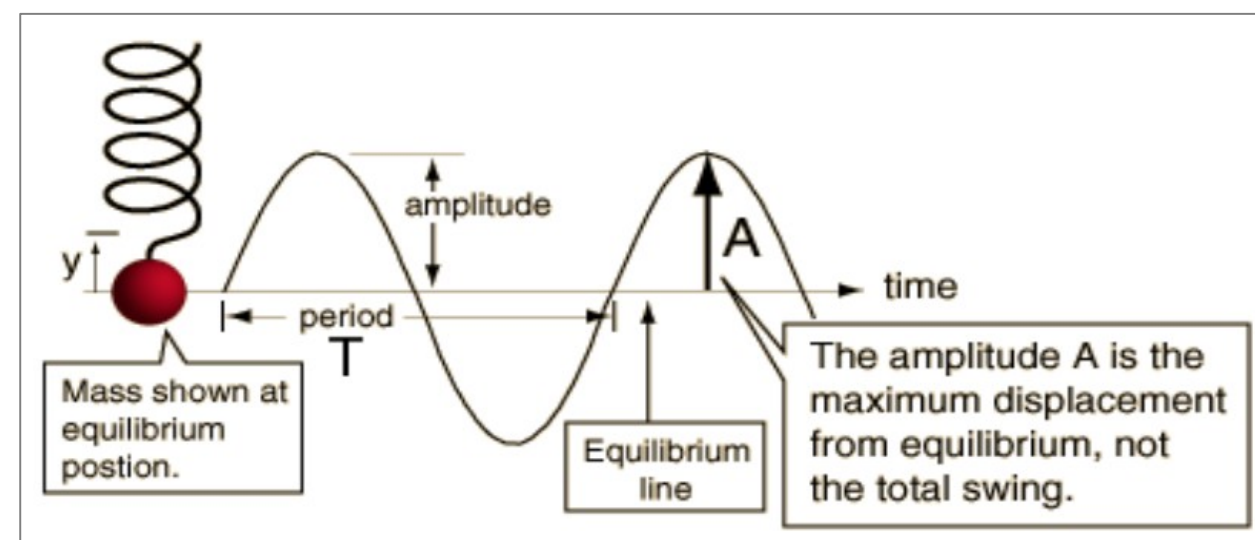
Basic wave concepts

- To understand waves in QM, let's review some basic wave concepts.
- Wavelength λ : the repeat distance of a wave in space.
- Period T : the “repeat distance” of a wave in time.
- Frequency ν : the inverse of the period; $\nu = 1/T$.
- Amplitude A : the wave's maximum displacement from equilibrium.
- Classically, this determines a wave's intensity.

Wavelength is easy to visualize; it is the distance over which the wave starts to repeat.

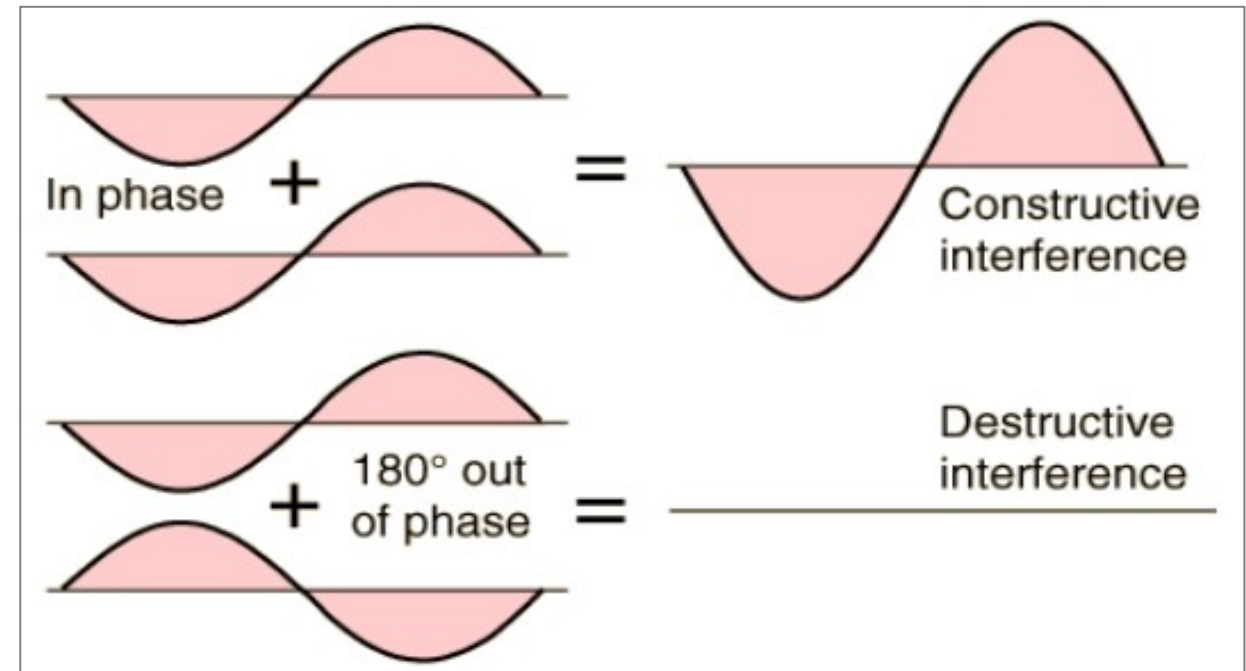
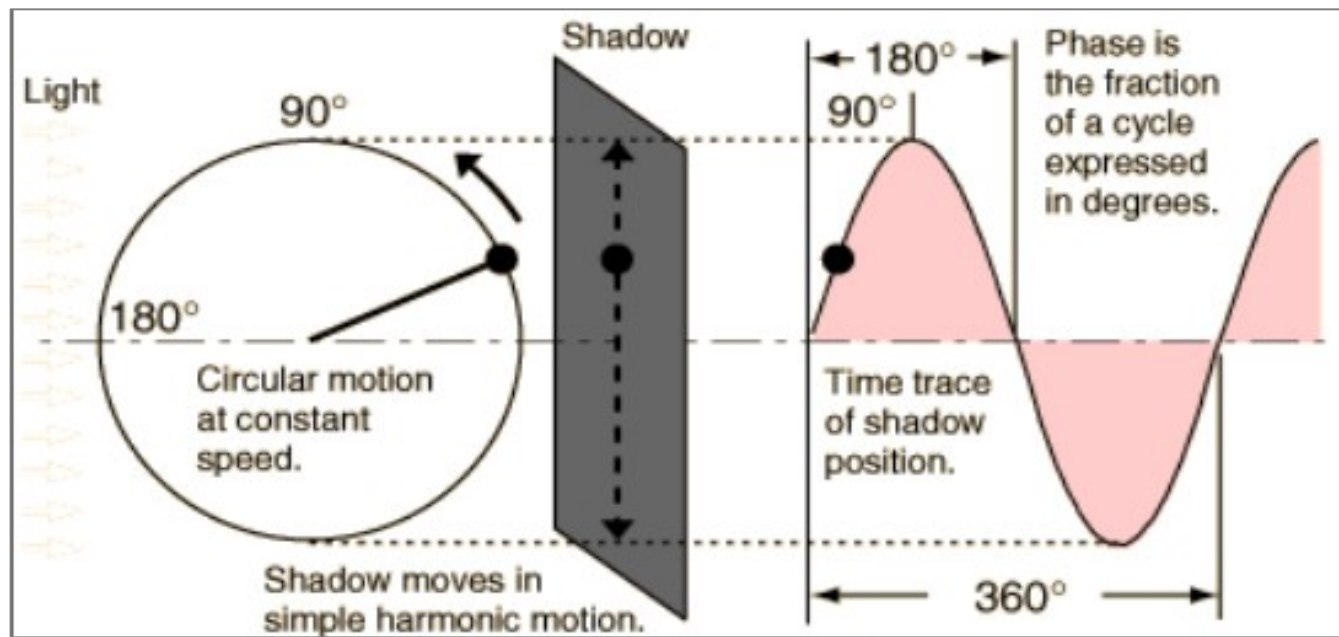


For an object executing periodic motion, like a mass on a spring, the period is just the time interval over which the wave starts to repeat.



Basic concept: interference

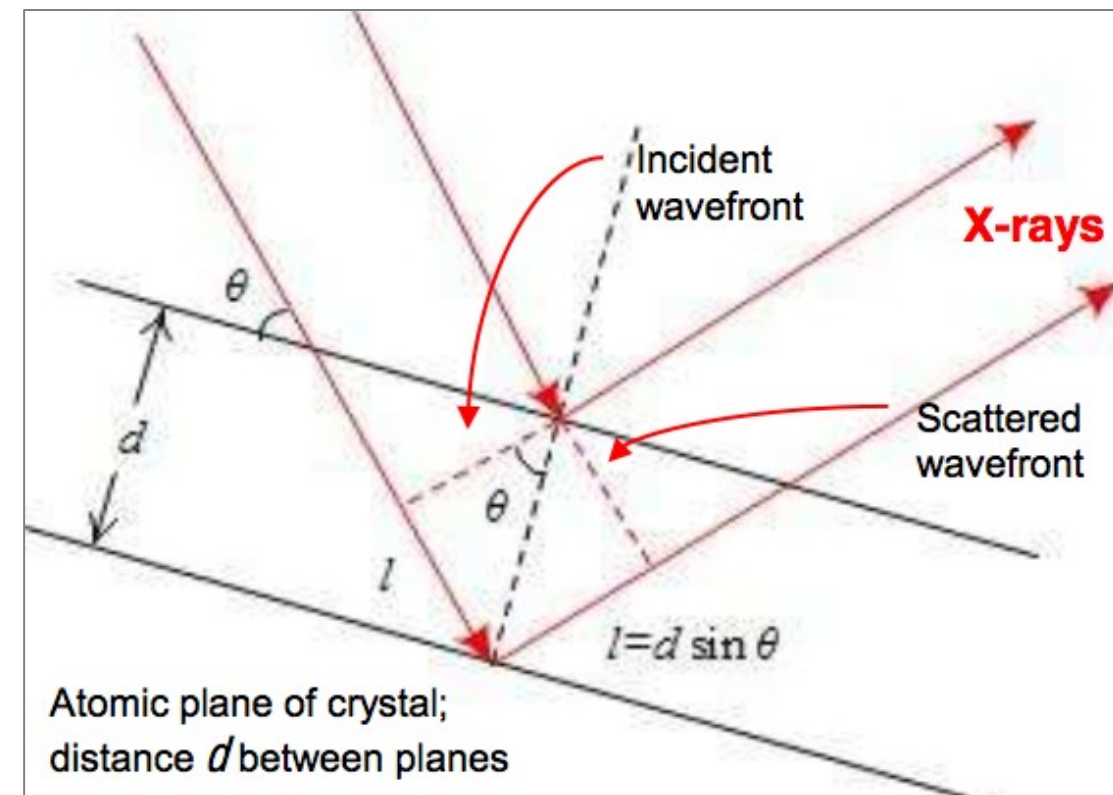
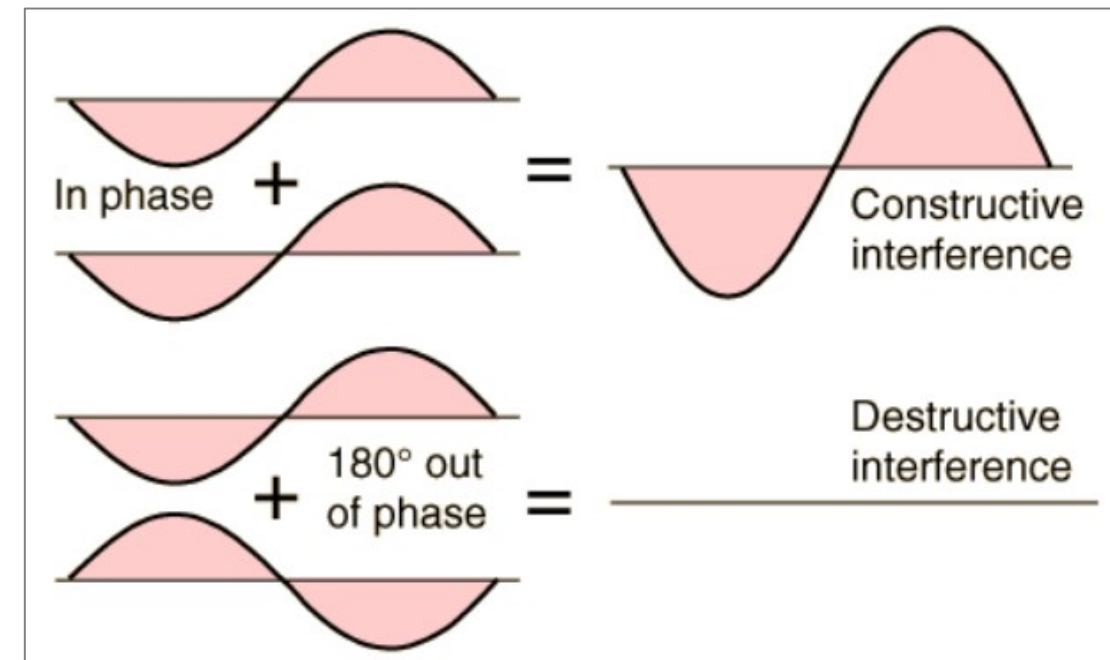
Waves can add or cancel, depending on their relative phase. You can visualize phase by imagining uniform circular motion on a unit circle.



- **Principle of Superposition:** you can add up any number of waves (sinusoids) to get another wave.
- The resultant wave may be larger or smaller than its components, depending on their relative phase angles.
- **Phases can be understood in terms of motion on the unit circle.**
 - Hence, waves interfere, canceling each other at certain locations.
- **Interference is what gives rise to the light and dark spots in the X-ray diffraction pattern.**

How X-ray interference works

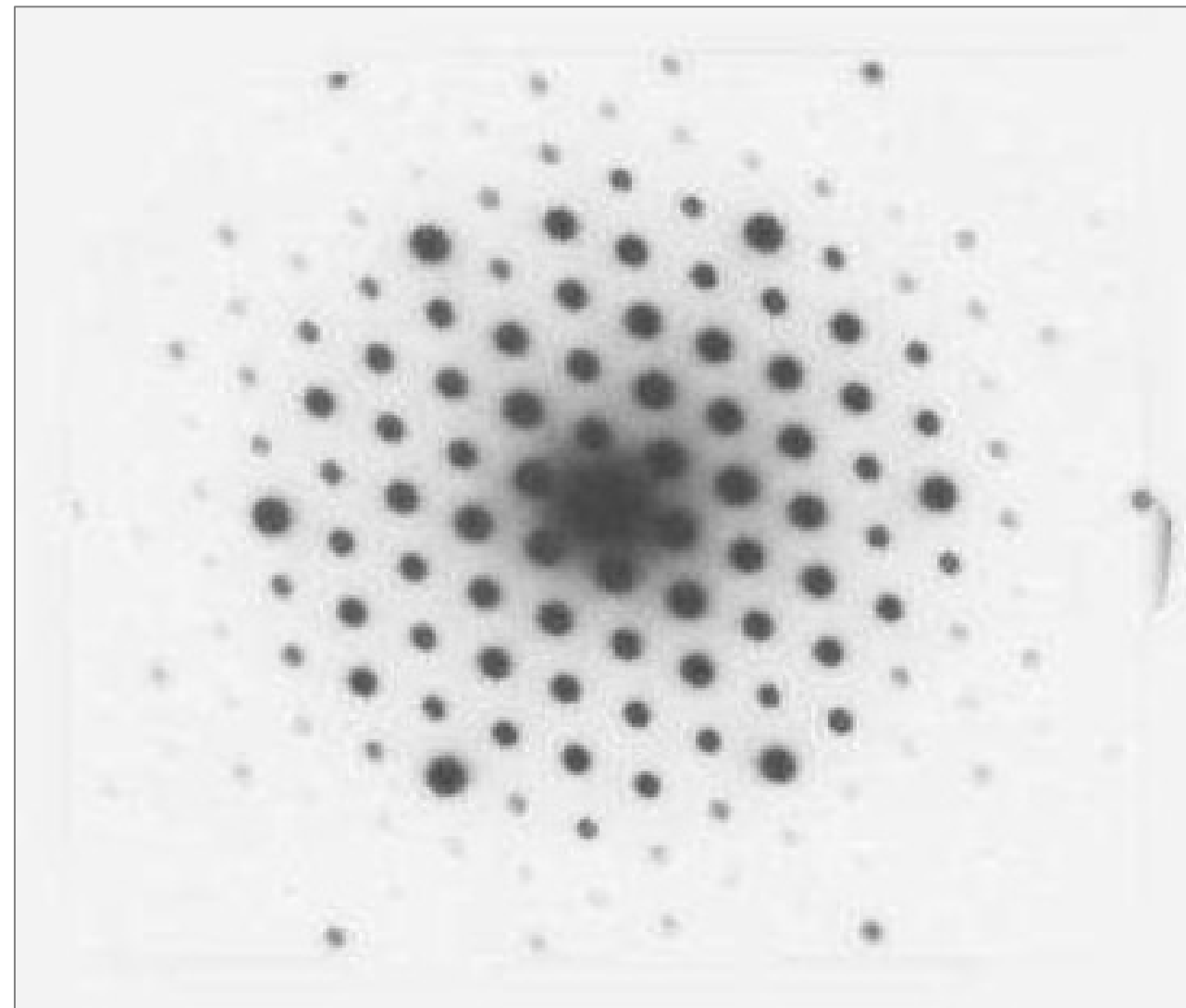
- Consider two X-ray beams reflecting from two crystal planes a distance $d \gg 0.1\text{nm}$ apart.
- The wave reflecting from the lower plane travels a distance $2l$ farther than the upper wave.
- If an integral number of wavelengths $n\lambda$ just fits into the distance $2l$, the two beams will be in phase and will constructively interfere.
- Only certain incident angles lead to constructive interference: $n\lambda = 2l = 2d \sin \theta$



Electron diffraction

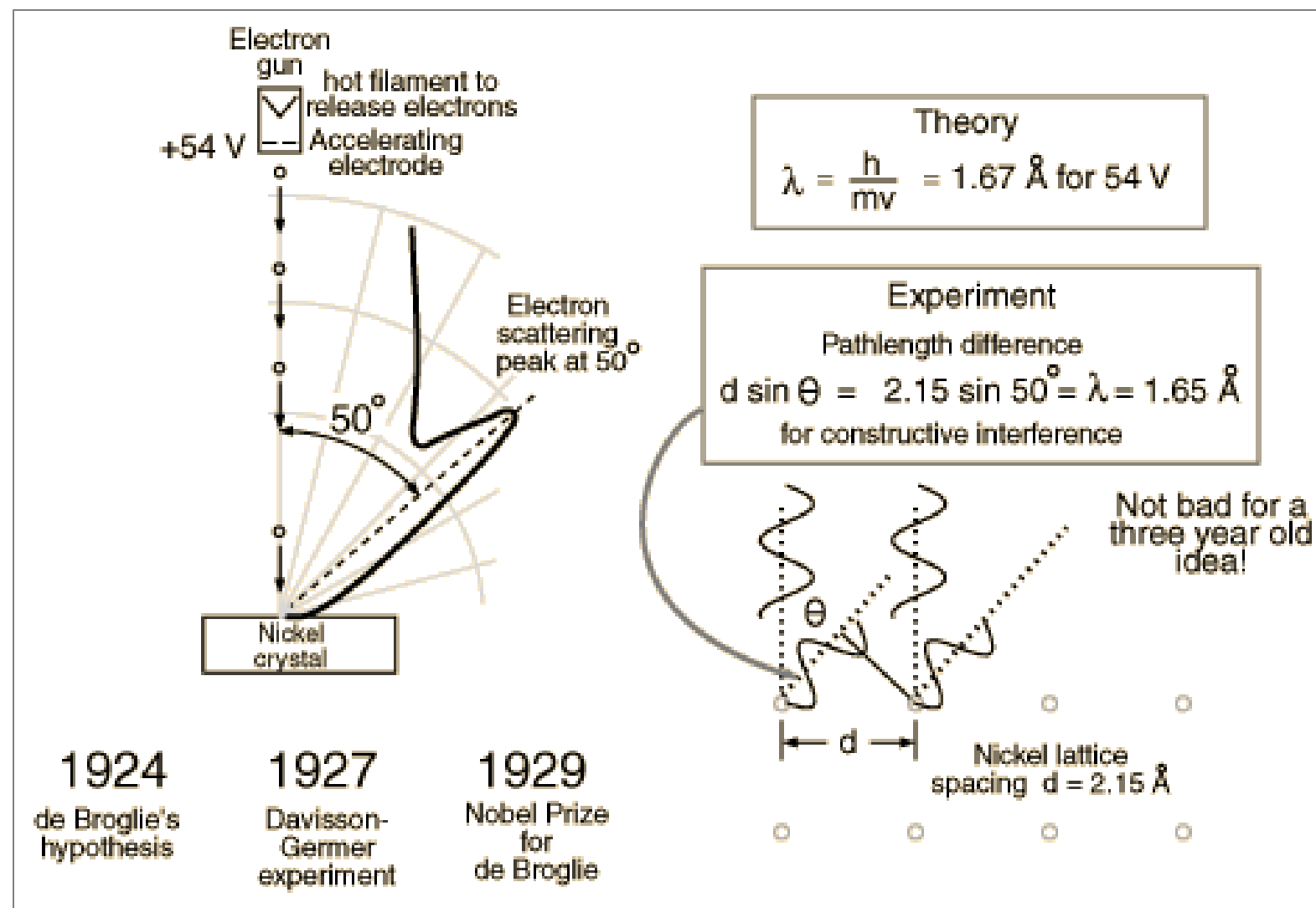
- Scattering electrons off crystals also creates a diffraction pattern!
- Electron diffraction is only possible if electrons are waves.
- Hence, electrons (matter particles) can also behave as waves.

Diffraction pattern created by scattering electrons off a crystal. (This is a negative image, so the dark spots are actually regions of constructive interference.) Electron diffraction is only possible if electrons are waves.



Observation of electron diffraction

- Electron diffraction was first observed in a famous 1927 experiment by Davisson and Germer.
- They fired 54 eV electrons at a nickel target and observed diffraction peaks consistent with de Broglie's hypothesis.



Electron diffraction demo

<https://www.youtube.com/watch?v=IYnU4T3jbgA>

Understanding matter waves

- Let's think a little more about de Broglie waves.
- In classical physics, energy is transported either by waves or by particles.
 - Particle: a definite, localized bundle of energy and momentum, like a bullet that transfers energy from gun to target.
 - Wave: a periodic disturbance spread over space and time, like water waves carrying energy on the surface of the ocean.
- In quantum mechanics, the same entity can be described by both a wave and a particle model:
 - Electrons scatter like localized particles, but they can also diffract like extended waves.

The wave function

- Particles \leftrightarrow waves.
 - How do we represent wave-particles mathematically?
- Classically: a particle can be represented solely by its position x and momentum p . If it is acted on by an external force, we can find x and p at any time by solving a differential equation like:

$$m \frac{d^2 x}{dt^2} = \frac{dp}{dt} = F \quad (\text{Newton's 2nd Law})$$

- A wave is an extended object, so we can't represent it by the pair of numbers (x, p) .
- Instead, for a wave moving at velocity $c = \lambda \nu$, we define a “wave function” $\psi(x, t)$ that describes the wave's extended motion in time and space.
- To find $\psi(x, t)$ at any position and time, we solve a differential equation like:

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} \quad (\text{wave equation})$$

Something similar applies to particles...

Wave equation for quantum waves

- Time evolution of waves can usually be found by solving a wave equation like:

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2}$$

- E. Schrödinger (1926) found that quantum particles moving in a 1D potential $V(x)$ obey the partial differential equation:

$$i\hbar \frac{\partial \psi(x, t)}{\partial t} = \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right) \psi(x, t)$$

- ✓ This expression involves \hbar , the usual QM parameter.
- ✓ It also involves the number i , which means that the waves $\psi(x, t)$ are complex.

The de Broglie wave

- Particles can be described by de Broglie waves of wavelength $\lambda = h/p$.
- For a particle moving in the x direction with momentum $p = h/\lambda$ and energy $E = h\nu$, the wave function can be written as a simple sinusoid of amplitude A :

$$\psi(x, t) = A \sin 2\pi \left(\frac{x}{\lambda} - \nu t \right)$$

- The propagation velocity c of this wave is:

$$c = \lambda \nu$$

Quantum waves summarized

- An elementary particle like a photon can act like a particle (Compton effect, photoelectric effect) or a wave (diffraction), depending on the type of experiment / observation.
- If it's acting like a particle, the photon can be described by its position and momentum x and p . If it's acting like a wave, we must describe the photon with a wave function $\psi(x,t)$.
- Two waves can always superpose to form a third: $\psi = \psi_1 + \psi_2$.
 - This is what gives rise to interference effects like diffraction.
- The wave function for a moving particle (a traveling wave) has a simple sinusoidal form.
 - But how do we interpret $\psi(x,t)$ physically?

Interpretation of the wave function

- We have seen that any wave can be described by a wave function $\psi(x,t)$.
- For any wave, we define the wave's intensity I to be:

$$I = |\psi(x,t)|^2 \equiv \int dx \psi(x,t)^* \psi(x,t)$$

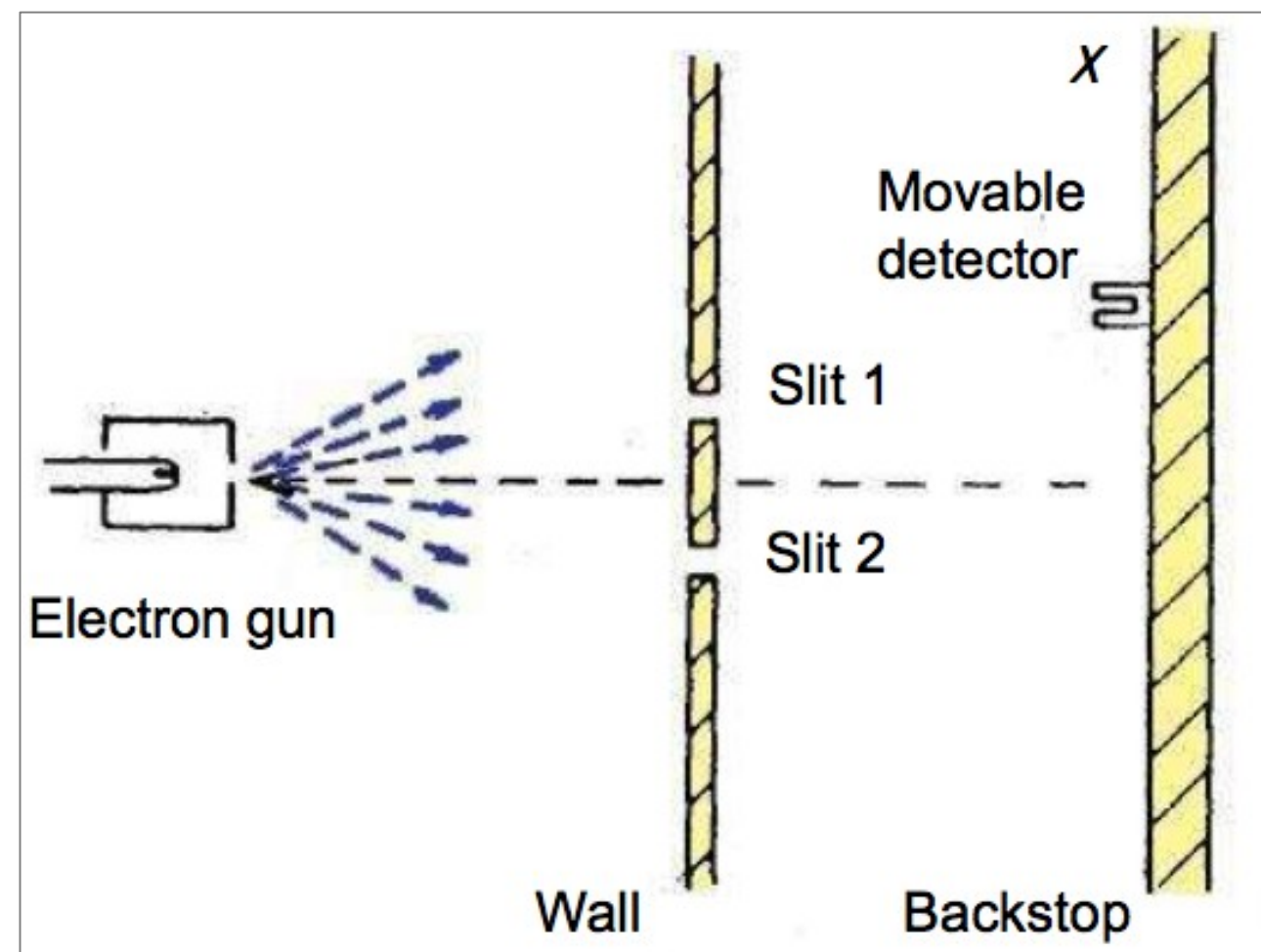
where the asterisk signifies complex conjugation. Note that for a plane wave this is the square of A (a constant):

$$I = |\psi(x,t)|^2 = |A|^2$$

- Let's use the concept of intensity and a simple thought experiment to get some intuition about the physical meaning of the de Broglie wave function.

The double slit experiment

- Experiment: a device sprays an electron beam at a wall with two small holes in it. The size of the holes is close to the electrons' wavelength λ_{dB} .
- Behind the wall is a screen with an electron detector.
- As electrons reach the screen, the detector counts up how many electrons strike each point on the wall.
- Using the data, we plot the intensity $I(x)$: the number of electrons arriving per second at position x .

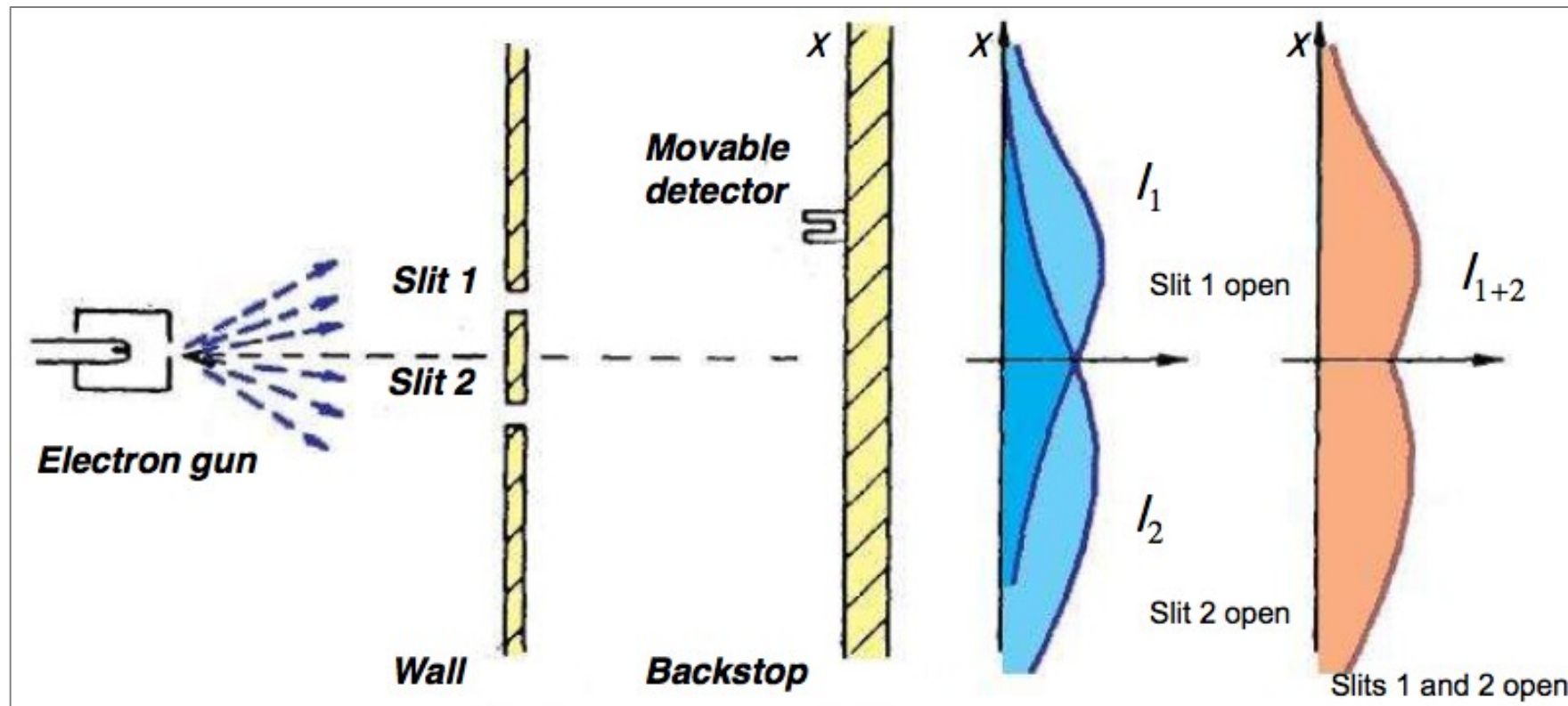


The double slit experiment

<https://www.youtube.com/watch?v=qCmtegdqOOA>

The double slit experiment

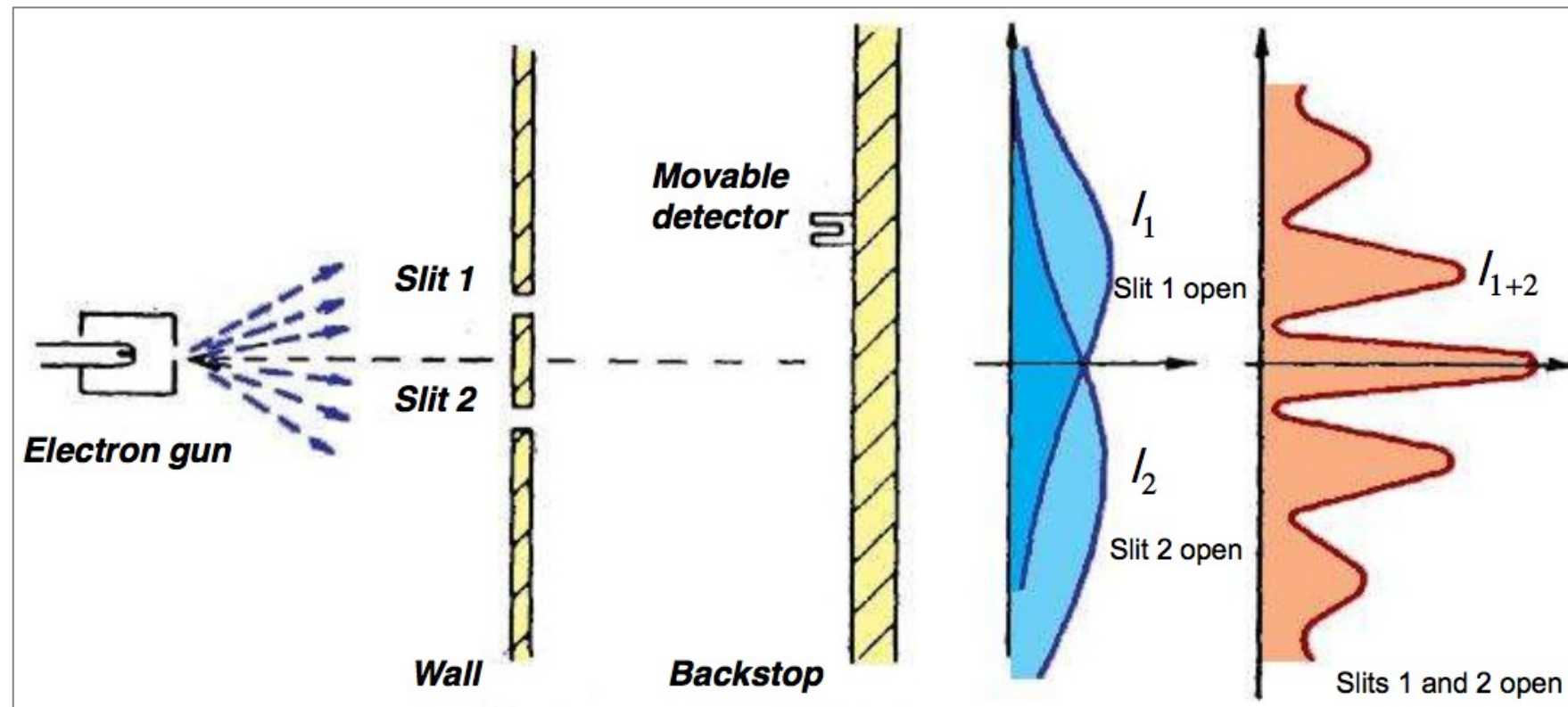
- The classical result:



- If electrons are classical particles, we expect the intensity in front of each slit to look like a bell curve, peaked directly in front of each slit opening.
- When both slits are open, the total intensity I_{1+2} on the screen should just be the sum of the intensities I_1 and I_2 when only one or the other slit is open.

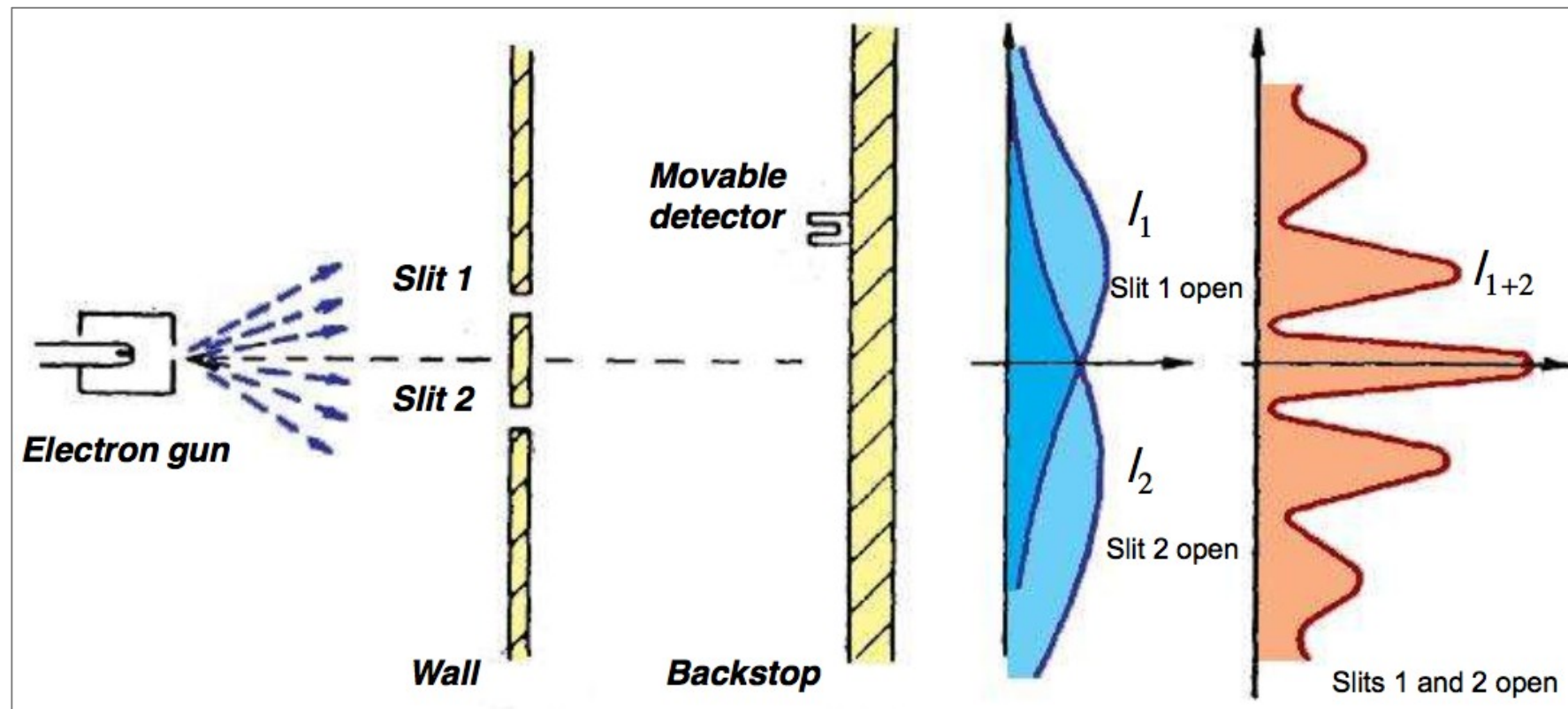
The double slit experiment

- In practice:



- When we perform the real experiment, a strange thing happens.
- When only one slit is open, we get the expected intensity distributions.
- But when both slits are open, a wavelike diffraction pattern appears.
- Apparently, the electrons are acting like waves in this experiment.

Wave function interpretation

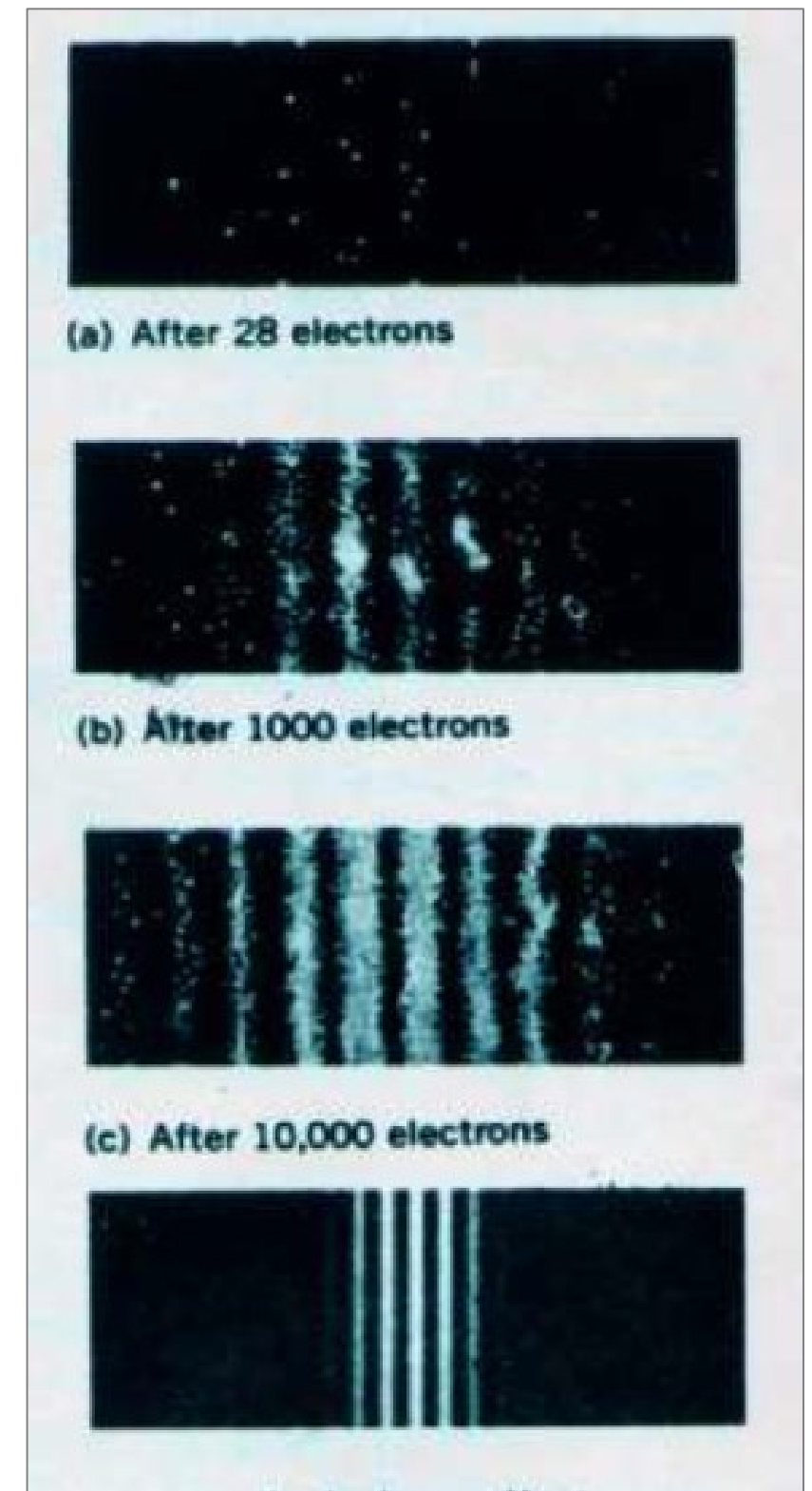


- In terms of waves, the wave function of an electron at the screen when only slit 1 is open is ψ_1 , and when only slit 2 is open it's ψ_2 .
 - Hence, the intensity when only one slit is open is either $I_1 = |\psi_1|^2$ or $I_2 = |\psi_2|^2$.
- With both slits open, the intensity at the screen is $I_{1+2} = |\psi_1 + \psi_2|^2$, not just $I_1 + I_2$!
- When we add the wave functions, we get constructive and destructive interference; this is what creates the diffraction pattern.

Wave function and probability

- If electrons create a diffraction pattern, they must be waves. But when they hit the screen, they are detected as particles.
 - How do we interpret this?
- M. Born: Intensity $I(x)=|\psi(x,t)|^2$ actually refers to **probability** a given electron hits the screen at position x .
- In QM, we don't specify the exact location of an electron at a given time, but instead state by $I(x)=|\psi(x,t)|^2$ the probability of finding a particle at a certain location at a given time.

The probabilistic interpretation gives physical meaning to $\psi(x,t)$: it is the “probability amplitude” of finding a particle at x at time t .



Diffraction pattern from an actual electron double slit experiment. Notice how the interference pattern builds particle by particle.

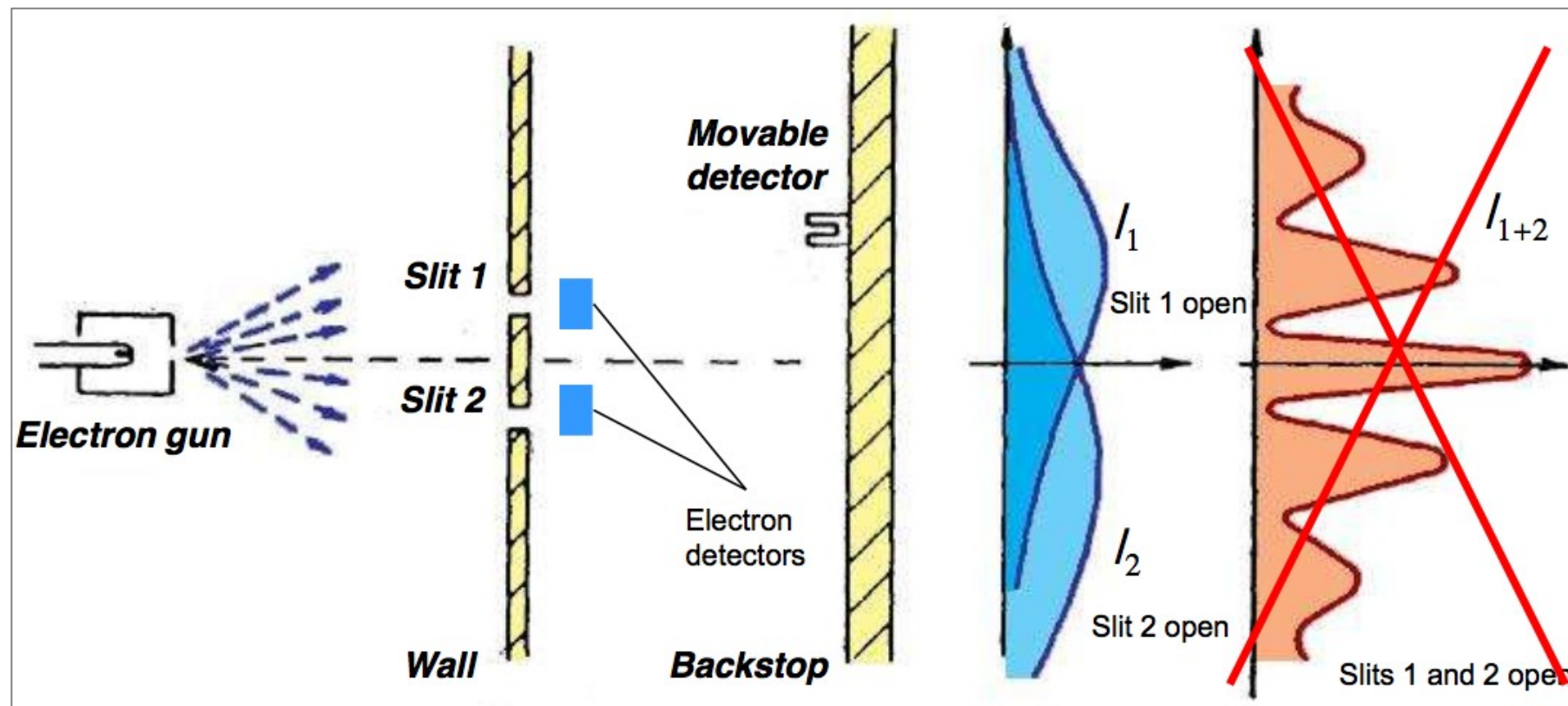
Is there a contradiction?

- We now understand the wave function in terms of the probability of a particle being someplace at some time.
 - However, there is another problem to think about.
- Since the electrons diffract, they show wavelike behavior; but when they hit the screen, they interact like particles. And if they are particles, then shouldn't they only go through one slit at a time?
- If this is the case, how can an electron's wave undergo double slit interference when the electron only goes through one slit?
 - Seems impossible...

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 - Seems impossible...
- To test what's going on, suppose we slow down the electron gun so that only one electron at a time hits the wall.
- We then insert a device over each slit that tells us if the electron definitely went through one slit or the other.

Destroying the interference pattern

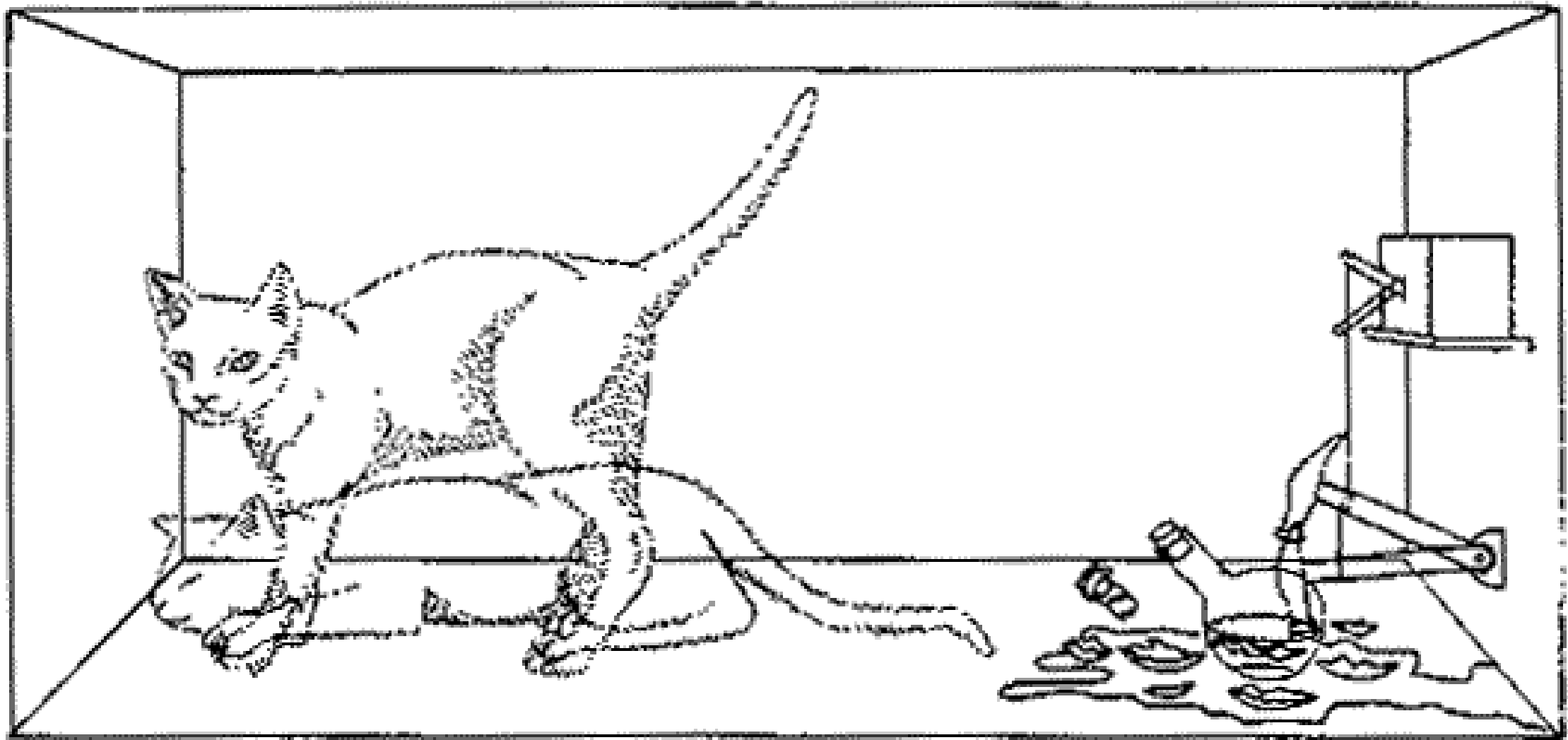


- We add electron detectors that shine light across each slit.
- When an electron passes through one of the slits, it breaks the beam, allowing us to see whether it traveled through slit 1 or slit 2 on its way to the backstop.
- **Result:** if we try to detect the electrons at one of the two slits in this way, the interference pattern is *destroyed*! In fact, the pattern now looks like the one expected for classical particles.

Complementarity

- Why did the interference pattern disappear?
- Apparently, when we used the light beam to localize the electron at one slit, we destroyed something in the wave function $\psi(x,t)$, that contributes to interference.
- **Principle of Complementarity (N. Bohr):** if a measurement proves the wave character of radiation or matter, it is impossible to prove the particle character in the same measurement, and vice-versa.
- The link between the wave and particle models is provided by the probability interpretation of $\psi(x,t)$, in which an entity's wave gives the probability of finding its particle at some position.

Schrodinger's cat



The Uncertainty Principle

- There seems to be some fundamental constraint on QM that prevents matter from acting wave-like and particle-like simultaneously.
- Moreover, it appears that our measurements can directly affect whether we observe particle or wavelike behavior.
- These effects are encapsulated in the Uncertainty Principle.
- Heisenberg: quantum observations are fundamentally limited in accuracy.

The Uncertainty Principle (1)

- According to classical physics, we can (at the same instant) measure the position x and momentum p_x of a particle to infinite accuracy if we like. We're only limited by our equipment.
- However, Heisenberg's uncertainty principle states that experiment cannot simultaneously determine the exact values of x and p_x .
- Quantitatively, the principle states that if we know a particle's momentum p_x to an accuracy Δp_x , and its position x to within some Δx , the precision of our measurement is **inherently** limited such that:

$$\Delta p_x \Delta x \geq \hbar / 2$$

Using the Uncertainty Principle

- Does the Uncertainty Principle (UP) mean that we can't measure position or momentum to arbitrary accuracy?
- No. The restriction is not on the accuracy to which x and p_x can be measured, but rather on the product $\Delta p_x \Delta x$ in a *simultaneous* measurement of both.
- The UP implies that the more accurately we know one variable, the less we know the other. If we could measure a particle's p_x to infinite precision, so that $\Delta p_x = 0$, then the uncertainty principle states:

$$\Delta x \geq \frac{\hbar / 2}{\Delta p_x} = \frac{\hbar / 2}{0} = \infty$$

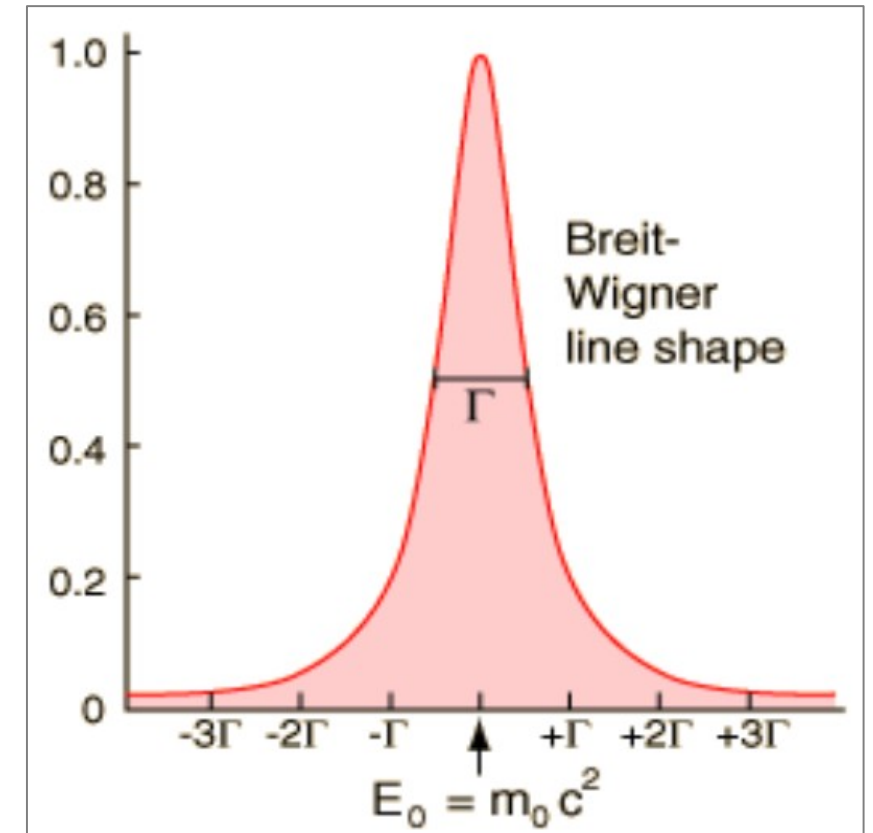
- In other words, after our measurement of the particle's direction (momentum), we lose all information about its position!

The Uncertainty Principle (2)

- The Uncertainty Principle has a second part related to measurements of energy E and the time t needed for the measurements. It states that:

$$\Delta E \Delta t \geq \hbar / 2$$

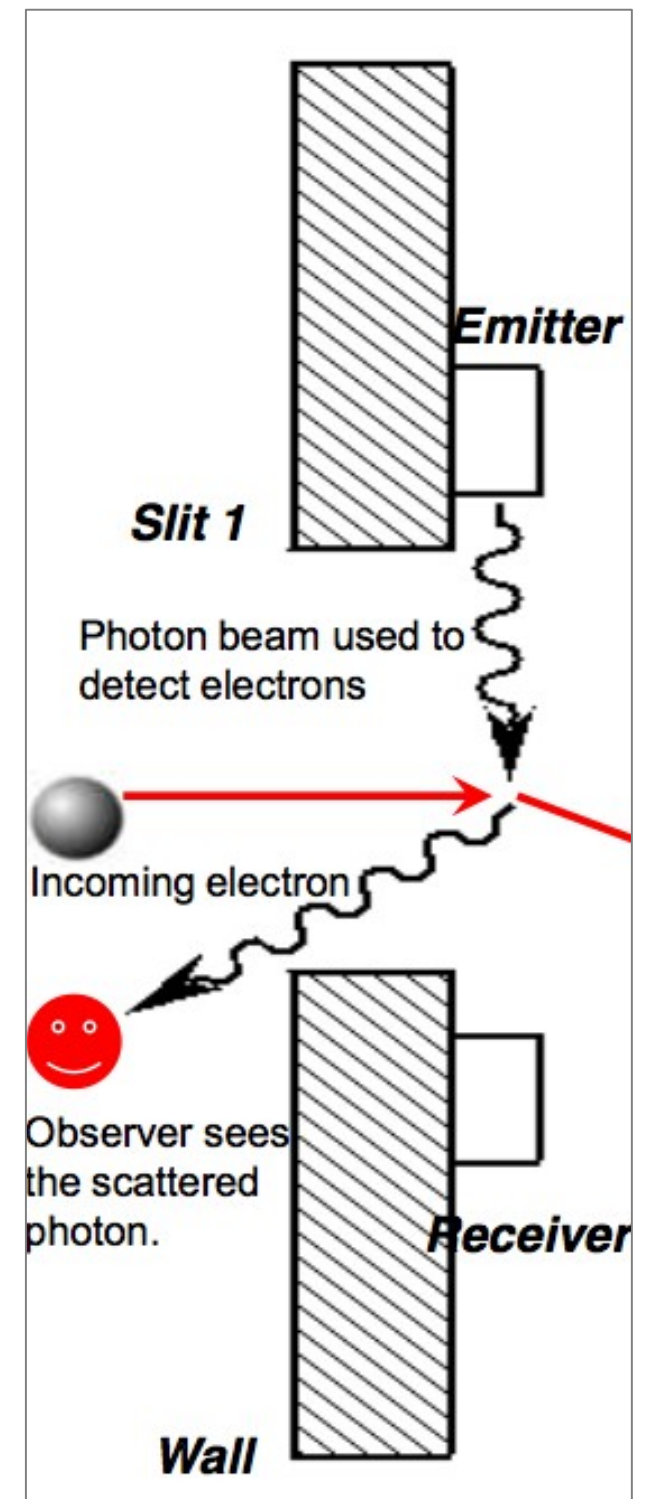
- **Example:** estimate the mass of virtual particles confined to the nucleus (see Lecture 2 on estimating Yukawa's meson mass).
- **Example:** Δt could be the time during which a photon of energy spread ΔE is emitted from an atom.
- **This effect causes spectral lines in excited atoms to have a finite uncertainty $\Delta\lambda$ ("natural width") in their wavelengths.**



Atomic spectral lines, the result of transitions that take a finite time, are not thin "delta function" spikes, but actually have a natural width due to the Uncertainty Principle.

Uncertainty and the double slit

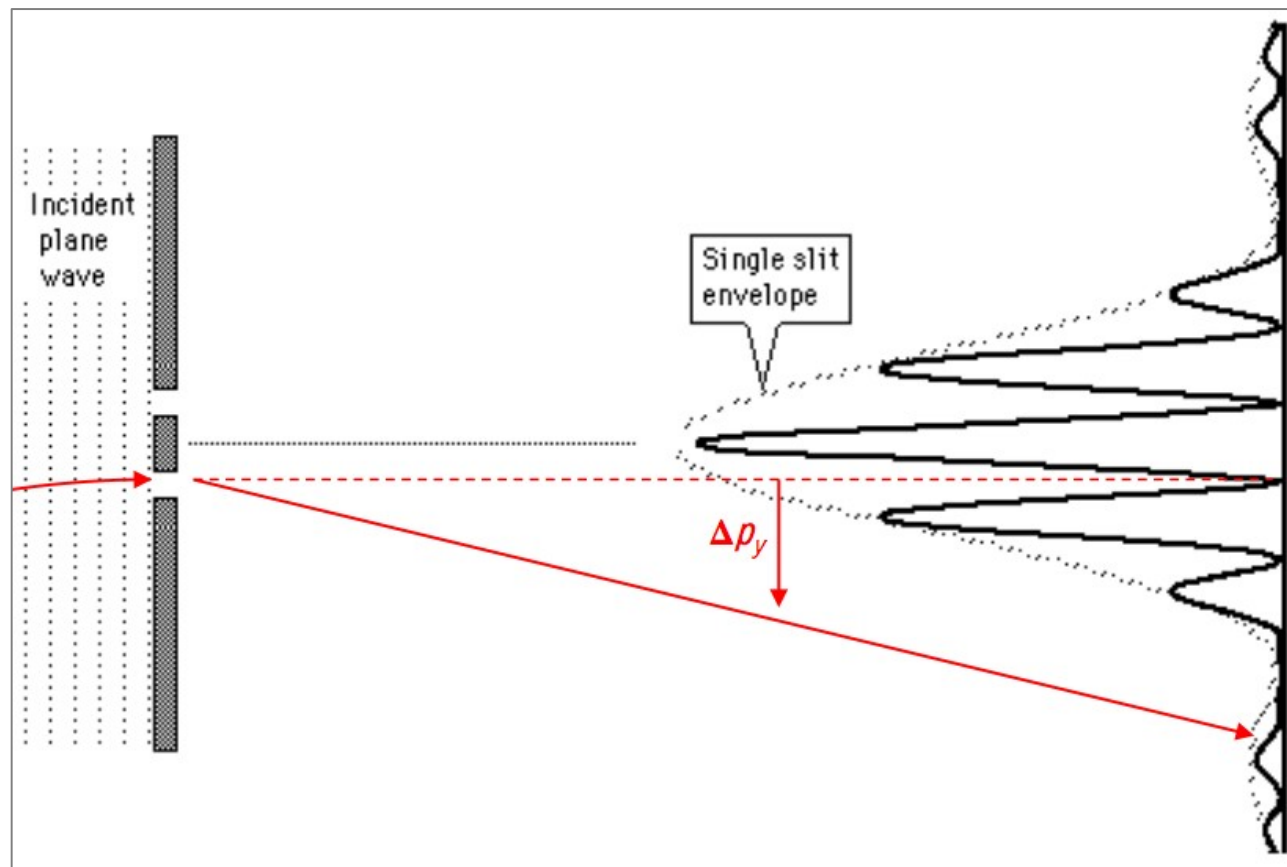
- Now that we know the UP, we can understand why we can't "beat" the double slit experiment and simultaneously observe wave and particle behavior.
- Basically, our electron detector Compton scatters light off of incoming electrons.
- When an electron uses one of the slits, we observe the scattered photon and know that an electron went through that slit.
- Unfortunately, when this happens, the photon transfers some of its momentum to the electron, **changing its momentum**.



Detection photon
Compton scatters off an
incoming electron,
transferring its
momentum.

Uncertainty and the double slit

- If we want to know the slit used, the photon's wavelength must be smaller than the spacing between the two slits.
- Hence, the photon has to have a large momentum (remember, $\lambda = h/p$).
- As a result, a lot of momentum gets transferred to the electron - enough to effectively destroy the diffraction pattern.



If the wavelength of the scattering photon is small enough to pinpoint the electron at one of the slits, the resulting momentum transfer is large enough to “push” the electron out of the interference minima and maxima. The diffraction pattern is destroyed.

Uncertainty and the double slit

- If we try to pinpoint the electron at one of the slits, we change its motion, and the interference pattern vanishes.
- Can you think of a way to get around this problem?

Uncertainty and the double slit

- If we try to pinpoint the electron at one of the slits, we change its motion, and the interference pattern vanishes.
- Can you think of a way to get around this problem?
- We could use **lower energy photons...**
- It turns out, that just when the photon momentum gets low enough that electron diffraction reappears, **the photon wavelength becomes larger than the separation between the two slits** (see *Feynman Lectures on Physics*, Vol. 3).
- **This means we can no longer tell which slit the electron went through!**

Why the Uncertainty Principle

- Let's review what we have said so far about matter waves and quantum mechanics.
- Elementary particles have associated matter waves $\psi(x,t)$. The intensity of the wave, $I(x)=|\psi(x,t)|^2$, gives the probability of finding the particle at position x at time t .
- However, QM places a firm constraint on the simultaneous measurements we can make of a particle's position and momentum: $\Delta p_x \Delta x \geq \hbar/2$.
- This last concept, the Uncertainty Principle, probably seems very mysterious. However, it turns out that it is just a natural consequence of the wave nature of matter, **for all waves obey an Uncertainty Principle!**
- Let's take a look...

Probabilities and waves, again

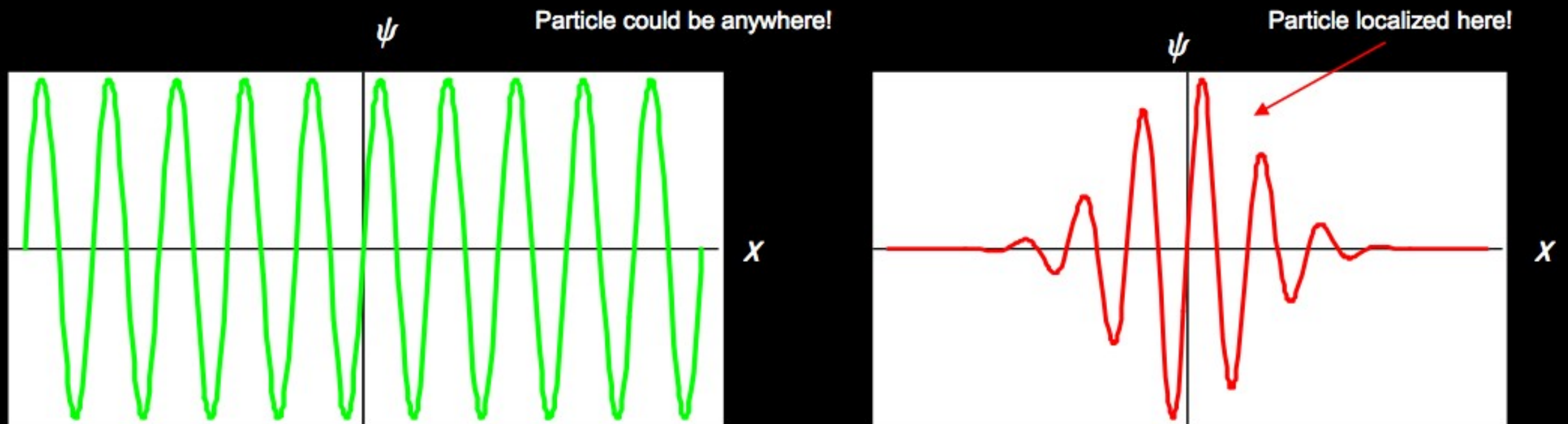
- Consider a particle with de Broglie wavelength λ .
- Defining the wave number $k = 2\pi/\lambda$ and the angular frequency
- $\omega = 2\pi\nu$, the wave function for such a particle could be (conveniently) written:

$$\psi(x, t) = A \sin(kx - \omega t)$$

- If the wavelength has a definite value λ there is no uncertainty $\Delta\lambda$, and hence $p = h/\lambda$ is also definite.
 - Such a wave is a sinusoid that extends over all values of x with constant amplitude.
- If this is the case, then the probability of finding the particle should be equally likely for any x ; in other words, the location of the particle is totally unknown. Of course, this should happen, according to the Uncertainty Principle.

Wave functions should be finite

- If a wavefunction is infinite in extent, like $\psi = A \sin 2\pi(x/\lambda - vt)$, the probability interpretation suggests that the particle could be anywhere: $\Delta x = \infty$.
- If we want particles to be **localized to some smaller Δx** , we need one whose amplitude varies with x and t , so that it vanishes for most values of x . But how do we create a function like this?



Left: a wave function with constant amplitude means that its corresponding particle is equally likely to be at any value of x .
Right: a finite wave function, or "wave packet," corresponds to a localized particle with $\Delta x \neq \infty$.

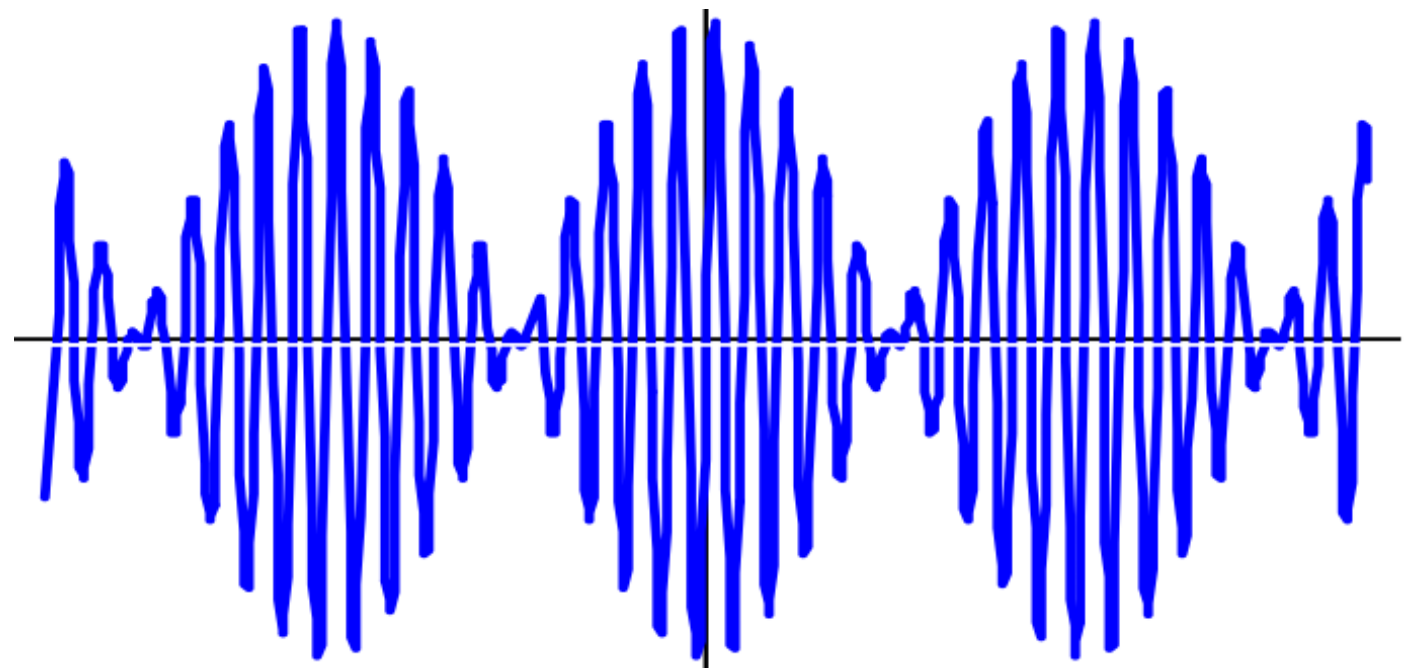
Building wave packets

- In order to get localized particles, their corresponding wave functions need to go to zero as $x \rightarrow \pm\infty$. Such a wave function is called a wave packet.
- It turns out to be rather easy to generate wave packets: all we have to do is superimpose, or add up, several sinusoids of different wavelengths or frequencies.
- **Recall the Principle of Superposition:** any wave ψ can be built up by adding two or more other waves.
- If we pick the right combination of sinusoids, they will cancel at every x other than some finite interval (Fourier Theorem).

Superposition example: beats

- You may already know that adding two sine waves of slightly different wavelengths causes the phenomenon of beats (see below).
- A typical example: using a tuning fork to tune a piano string. If you hear beats when you strike the string and the tuning fork simultaneously, you know that the string is slightly out of tune.

The sum of two sine waves of slightly different wavelengths results in beats. The beat frequency is related to the difference between the wavelengths. This is a demonstration of how adding two sinusoids creates a wave of varying amplitude.



Superimposing more sinusoids

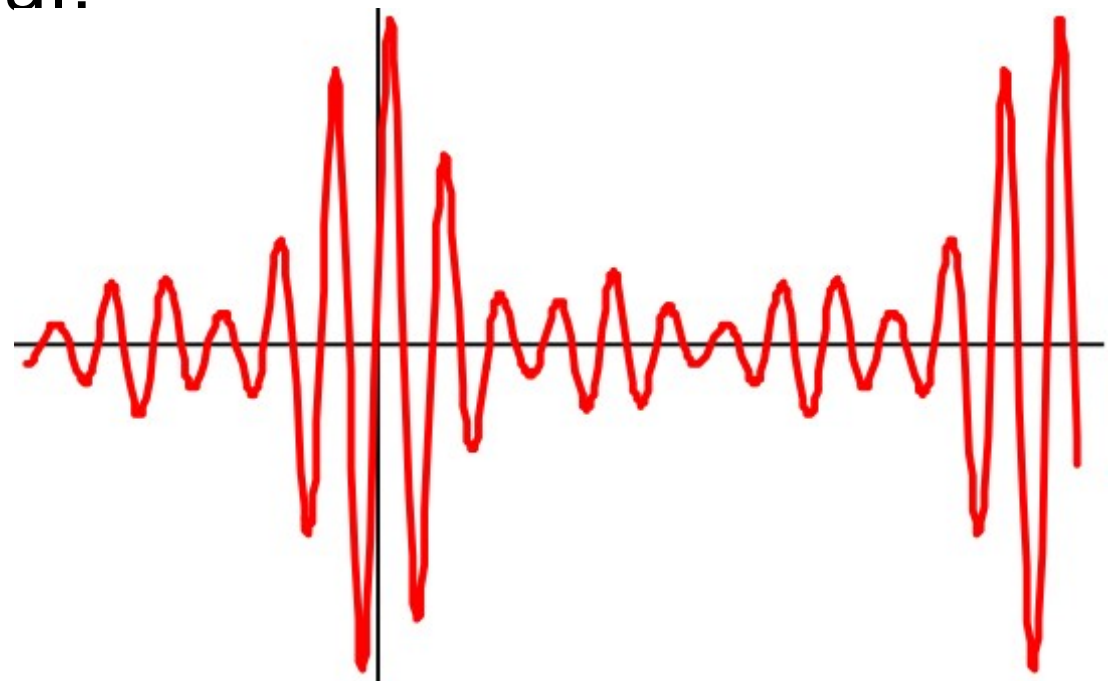
- Now, let's sum seven sinusoidal waves of the form:

$$\psi = \sum_k \psi_k = \sum_{k=9*2\pi}^{15*2\pi} A_k \cos(kx)$$

where $k=2\pi/\lambda$ and $A_9 = A_{15} = 1/4$, $A_{10} = A_{14} = 1/3$, $A_{11} = A_{13} = 1/2$, and $A_{12} = 1$.

We get a function that is starting to look like a wave packet; however, as you can see, it's still periodic, although in a more complicated way than usual:

Summing up sinusoids to get another periodic function. This is an example of Fourier's Theorem: any periodic function may be generated by summing up sines and cosines.



The continuum limit

- So, evaluating a sum of sinusoids like:

$$\psi = \sum_k A_k \cos(kx)$$

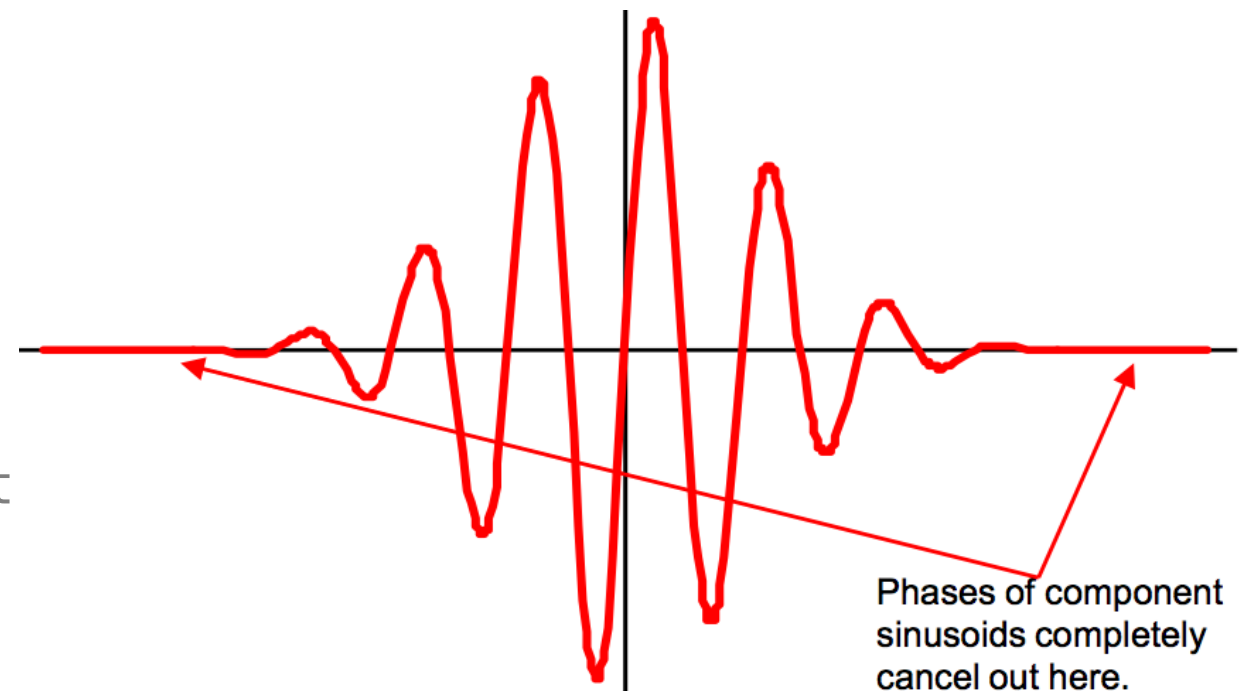
where k is an integer that runs between k_1 and k_2 , we can reproduce any periodic function.

- If we let k run over all values between k_1 and k_2 – that is, we make it a continuous variable – then we can finally reproduce a finite wave packet.

By summing over all values of k in an interval $dk = k_2 - k_1$, we are essentially evaluating the integral

$$\psi = \int_{k_1}^{k_2} dk A(k) \cos(kx)$$

In such a “continuous sum”, the component sinusoids are all in phase near $x=0$. Away from this point, in either direction, the components begin to get out of phase with each other. If we go far enough out, the phases of the infinite number of components become totally random, and cancel out phases of component sinusoids completely.



Connection to uncertainty

- So, we can sum up sinusoids to get wave packets.
 - What does this actually mean?
- **Intuition:** we want a wave function that is non-zero only over some finite interval Δx .
- To build such a function, we start adding sinusoids whose inverse wavelengths, $k=2\pi/\lambda$, take on values in some finite interval Δk .
- **Here's the point:** as we make the interval Δk bigger, the width of the wave packet Δx gets smaller. This sounds like the Uncertainty Principle!

Wave-related uncertainties

- As we sum over sinusoids, making the range of k values Δk larger, we decrease the width of the resulting function.
- In fact, there is a fundamental limit here that looks just like the position-momentum uncertainty relation.
- For any wave, the minimum width Δx of a wave packet composed from sinusoids with range Δk is $\Delta x = 1/(2\Delta k)$, or:

$$\Delta x \Delta k \geq 1/2$$

- There is a similar relation between time and frequency:

$$\Delta t \Delta \omega \geq 1/2$$

Connection to QM

- If $k=2\pi/\lambda$, and $\lambda=h/p$, then the uncertainty relation for waves in general tells us:

$$\Delta x \Delta k \geq 1/2$$

$$= \Delta x \frac{2\pi}{\Delta \lambda} = 2\pi \Delta x \frac{\Delta p}{h}$$

$$\Delta x \Delta p \geq h/4\pi = \hbar/2$$

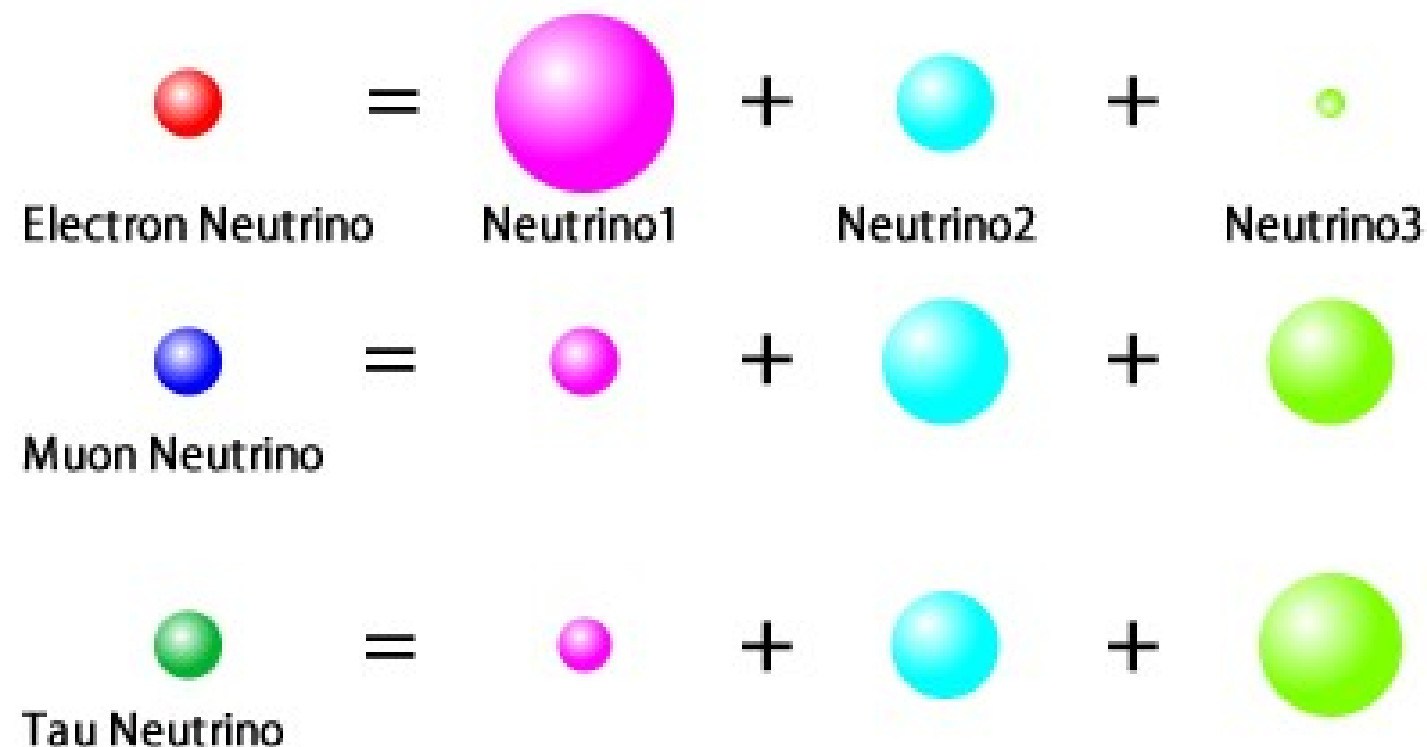
- We recover the Heisenberg uncertainty relation!
- By a similar argument, we can show that the frequency-time uncertainty relation for waves implies the energy-time uncertainty of quantum mechanics.
- The Uncertainty Principle arises as a consequence of the wave nature of particles!

Applications to Particle Physics

- A particle can be described as a superposition of other particles (e.g. neutrino oscillations).
- Since trajectories are not well defined, we need to consider all processes (Feynman diagrams) consistent with the observed result. The amplitudes of these processes can interfere, enhancing or suppressing the total probability.
- The production of new particles from collisions or decays of other particles is subject to the probabilistic nature of Quantum Mechanics. We call this probabilities branching ratios.
- Schrödinger's equation is valid for non-relativistic particles. In Particle Physics we use the Dirac equation for spin = $\frac{1}{2}$ particles and the Klein-Gordon equation for spin = 0 particles.

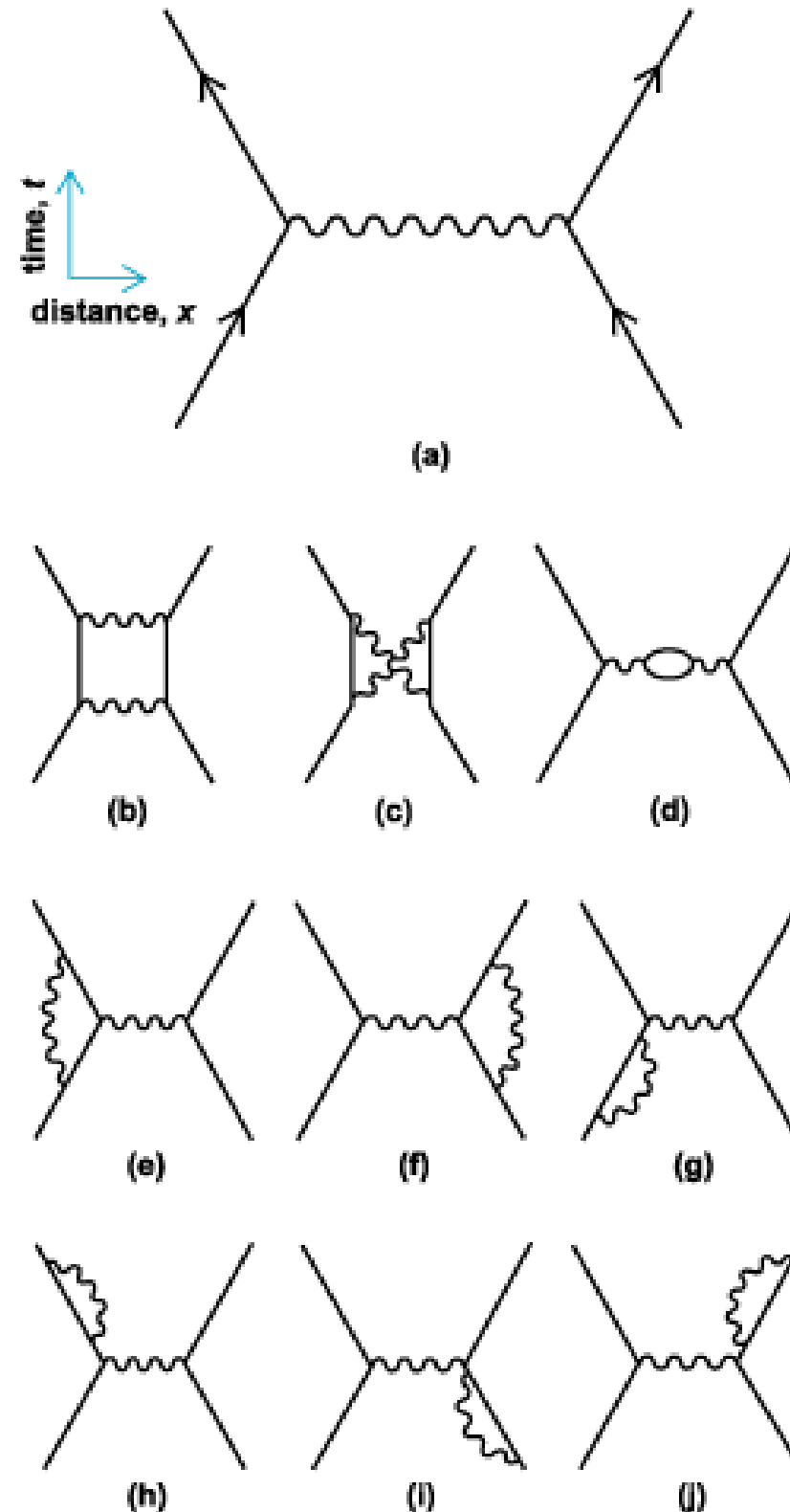
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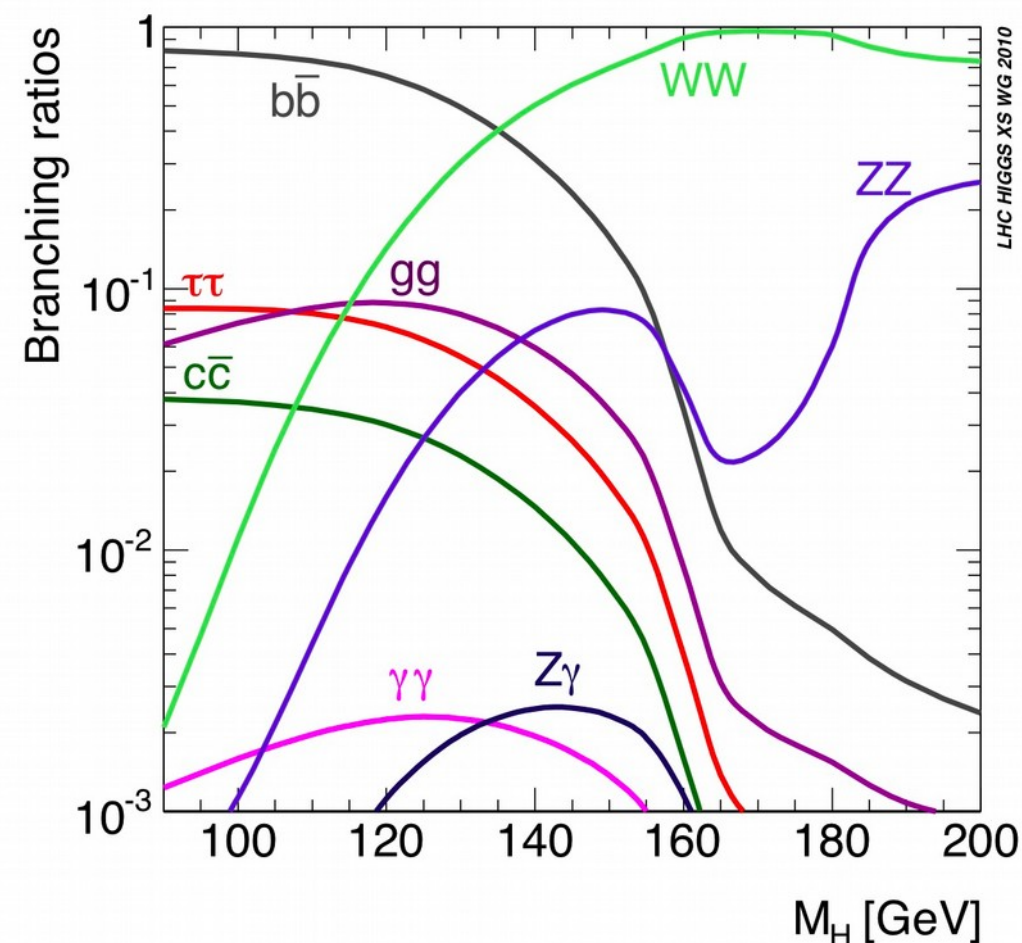
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Dirac equation

$$i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0$$

where there is an [implied summation](#) over the values of the twice-repeated index $\mu = 0, 1, 2, 3$, and ∂_μ is the 4-gradient. In practice one often writes the [gamma matrices](#) in terms of 2×2 sub-matrices taken from the [Pauli matrices](#) and the 2×2 [identity matrix](#). Explicitly the [standard representation](#) is

$$\gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}, \gamma^1 = \begin{pmatrix} 0 & \sigma_x \\ -\sigma_x & 0 \end{pmatrix}, \gamma^2 = \begin{pmatrix} 0 & \sigma_y \\ -\sigma_y & 0 \end{pmatrix}, \gamma^3 = \begin{pmatrix} 0 & \sigma_z \\ -\sigma_z & 0 \end{pmatrix}.$$

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \psi - \nabla^2 \psi + \frac{m^2 c^2}{\hbar^2} \psi = 0.$$

Summary

- Quantum mechanics is the physics of small objects.
 - Its typical energy scale is given by **Planck's constant**.
- In QM, variables like position, momentum, energy, etc. tend to take on **discrete values** (often proportional to h).
- Matter and radiation can have both particle and wavelike properties, depending on the type of observation.
- But by the Uncertainty Principle, objects can never be wavelike or particle-like simultaneously.
- Moreover, it is the act of observation that determines whether matter behaves like a wave or a particle.